$x \in \arg\min_z \; f(z) - y^T z$

$\Longleftrightarrow \; 0 \in \partial f(x) - y$

$\Longleftrightarrow \; y \in \partial f(x).$

$x \in \partial f^*(-A^T u) \Longleftrightarrow x \in \arg\min_z \; f^*(z) - z^T(-A^T u)$

$= \underline{\underline{\arg\min_z \; f(z) + u^T A z}}$

$\partial g(u) = A\, \underline{\partial f^*(-A^T u)} - b.$

$g(y) \geq g(x) + \underbrace{\nabla g(x)^T}_{0}(y - x) + \frac{m}{2}\|y - x\|_2^2$

$= \underbrace{\cancel{g(x)}}_{g(x)} + \frac{m}{2}\|y - x\|_2^2$

$g_u(x) = f(x) - u^T x$. minimizer: $x_u = \nabla f^*(u)$.

for any y:

$f(y) - u^T y \geq f(x_u) - u^T x_u + \frac{m}{2}\|y - x_u\|_2^2 \leftarrow$ try $y = x_v$

$g_v(x) = f(x) - v^T x$. minimizer $x_v = \nabla f^*(v)$

$f(y) - v^T y \geq f(x_v) - v^T x_v + \frac{m}{2}\|y - x_v\|_2^2 \leftarrow$ try $y = x_u$

$\cancel{f(x_v)} - u^T x_v + \cancel{f(x_u)} - v^T x_u \geq$

$\cancel{f(x_u)} - u^T x_u + \cancel{f(x_v)} - v^T x_v + m\|x_u - x_v\|_2^2$

$$\Rightarrow (u-v)^T (x_u - x_v) \geq m \|x_u - x_v\|_2^2$$

use Cauchy Schwartz on LHS

$$\|x_u - x_v\|_2 \leq \frac{1}{m} \|u-v\|_2$$

$$\|\nabla f^*(u) - \nabla f^*(v)\|_2 \leq \frac{1}{m} \|u-v\|_2 . \checkmark.$$

$$f(x) = \frac{1}{2} x^T Q x \qquad f^*(y) = \frac{1}{2} y^T Q^{-1} y.$$

$$\min \sum_i f_i(x_i) + u^T A x$$

$$\Longleftrightarrow \min \sum_i f_i(x_i) + \sum u^T A_i x_i$$

$$\Longleftrightarrow \min \sum_i \left(f_i(x_i) + u^T A_i x_i \right)$$

$$\frac{\rho}{2} \|Ax + Bz - c + w\|_2^2 - \frac{\rho}{2} \|w\|_2^2 \qquad w = u/\rho.$$

$$= \underline{\rho w^T} (Ax + Bz - c) + \frac{\rho}{2} \|Ax + Bz - c\|_2^2$$

$$= u^T (Ax + Bz - c) + \frac{\rho}{2} \|Ax + Bz - c\|_2^2 .$$