Busy period, time of the first loss of a customer and the number of the customers in the $\mathrm{M}^{\varkappa}|\mathrm{G}^{\delta}|1|\mathrm{B}$ queuing system

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Abstract

Queueing systems with batch arrivals and finite buffer have wide applications in the performance evaluation, telecommunications, and manufacturing systems. One of the crucial performance issues of the single-server queue with finite buffer room) is losses, namely, customers (packets, cells, jobs) that were not allowed to enter the system due to the buffer overflow. This issue is especially important in the analysis of telecommunication networks. Motivated by this fact, we derived the most important performance measurements of several queuing systems of this type. More precisely, we considered the $M^{\varkappa}|G^{\delta}|1|B$ and $G^{\delta}|M^{\varkappa}|1|B$ queuing systems with finite buffer and their modifications. Evolution of the number of the customers in such systems is described by a process with two reflecting boundaries. In general case this process is a difference of two renewal processes. Reflections from the upper boundary are generated by the supremum (infimum) of the process. Reflections from the lower boundary govern the server's behavior.

In general such processes are not Markovians, but by adding a complementary linear component (in some literature called age process), we obtain a Markov process, which describes functioning of the queuing system. Studying main characteristics of the system results to the investigating of the two-boundary functionals of the governing process. We applied the solutions of the two-sided exit problem for the governing process to obtain the performance measures. For the queuing systems of $M^{\varkappa}|G^{\delta}|1|B$, $G^{\delta}|M^{\varkappa}|1|B$ type the governing process is the difference of the compound Poisson process and the compound renewal process complemented with the age process.

We determine the Laplace transforms of busy period, time of the first loss of the customer and the number of the customers in the system. The results are given in term of the resolvent sequences of the governing process. Additionally, we consider a special case, when the governing process has unit negative jumps. It means that the customers arrive not in batches but one-by-one. We also study $G^{\delta}|M^{\varkappa}|1|B$ system, for which we determine the busy period, time of the first loss and the number of the lost customers, distribution of the number of the customers in the system and the virtual waiting time. Partial case is treated separately.

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