Metropolis algorithm and distorted Brownian motion

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Abstract

Given a bounded domain D in R^n and a continuous function ρ that is bounded between two positive constants. Consider the following Metropolis Markov chain $\{X_j, j \neq 1\}$ on D. At time $j+1$, pick a point y at random from the ball with radius h centered at X_j. If y is in D and $\rho(y) \geq \rho(x)$, let $X_{j+1} = y$; if y is in D but $\rho(y) < \rho(x)$, flip a coin with head probability $\rho(y)/\rho(x)$ and let $X_{j+1} = y$ if head appears; in all other cases, let $X_{j+1} = X_j$.

We show that after appropriate scaling in time and step h, the Metropolis chain converges weakly to reflecting distorted Brownian motion in D. We then apply it to the random placement of N hard discs in the unit sphere, the original application of Metropolis algorithm.

This is joint work with K. Burdzy and P. Diaconis.