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Handout #8: Multi-way ANOVA

1. Two-way ANOVA interaction model (indices: i for factor A, j for factor B, k for subject):

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijk},\ \epsilon_{ijk} \sim N(0, \sigma^2)$$

2. Two-way ANOVA additive model (indices: i for factor A, j for factor B, k for subject):

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \epsilon_{ijk},\ \epsilon_{ijk} \sim N(0, \sigma^2)$$

3. Fitted values: By least squares or MLE, the fitted values are cell means for the ANOVA model, and the best set of "parallel" means for the additive model.

4. Two-way ANOVA table

Source	Df	SS	MS	F	p-value
Factor A	I_A-1	SS_A	$SS_A/(I_A-1)$	MS_A/MS_R	p-value
Factor B	I_B-1	SS_B	$SS_B/(I_B-1)$	MS_B/MS_R	p-value
Interaction	$(I_A-1)(I_B-1)$	$SS_{A:B}$	$SS_{A:B}/[(I_A-1)(I_B-1)]$	$MS_{A:B}/MS_R$	p-value
Residual	$\sum(n_{ij}-1)$	SS_R	$SS_R/\sum(n_{ij}-1)$		
Total	N-1	SS_T	$SS_T/(N-1)$		

Within Groups is synonym for Residual.

A + B (+ Interaction) = Between Groups (SS and df).

The I-1 (orthogonal) contrasts can be thought of as a decomposition of the factor SS and df, each with 1 df. Using $P(|W| < t_r) = P(W < F_{1,r})$, for a contrast we get $F = t^2$, $MS_C = t^2(MS_R)$, $SS_C = MS_C$, and these SS values sum to the factor SS.

5. Important difference between ANOVA and regression: Regression p-values refer to each coefficient conditioned on all other variables being in the model. In ANOVA, p-values refer to each factor conditioned on all *preceding* factors being in the model, so, if the model is not balanced, you will get different p-values for a factor depending on the order it is entered into the model.

6. Interpretation of main effects vs. interactions: Regression main effect p-values don't have useful meaning in the presence of an interaction. Regression main effect p-values change when interactions are added. ANOVA, which always adds interactions after the main effects, has main effect p-values that are the same as in the model without interactions.

7. Often one (or more factors) are “blocks”, which represent similar groups of subjects for some (non-quantifiable) reason. Because σ^2 represent the variance of each group of subjects who share the same levels of all explanatory variables including being in the same block, blocking reduces σ^2, which in turn raises power.

8. Breakout and Discussion