

1. Review: Directed Acyclic Graph (DAG) for the simplest mediator model:

$$T \rightarrow M \rightarrow Y$$

where T is treatment, M is the mediator and Y is the outcome.

2. Analysis usually starts by fitting two models:

$$\text{Model M: } M_i = \beta_{0M} + T_i\beta_1 + \epsilon_{i1}$$

$$\text{Model Y: } Y_i = \beta_{0Y} + T_i\beta_2 + M_i\theta + \epsilon_{i2}$$

3. Mediation is claimed according to the Causal Steps Approach if we have evidence that  $\beta_1 \neq 0$ ,  $\theta \neq 0$ , and  $\beta_2 = 0$  (complete mediation) or  $\beta_2 < \beta_1$  (partial mediation). These steps vary a bit from author to author.
4. Mediation is claimed in a more principled way if a test of  $H_0 : ab = 0$  is rejected where  $a$  is  $\beta_1$  above and  $b$  is  $\theta$  above. The Sobel test is based on the first order Taylor expansion of  $ab$  and the (shaky) assumption of a normal sampling distribution for  $ab$ .
5. Bootstrap methods do not require the assumption of a normal sampling distributions. They do have the minor disadvantages that usually no p-value is produced (only a CI) and the exact CI values vary somewhat each time the CI is recalculated. The bootstrap algorithm, which applies far beyond mediation, is
  - (a) Calculate the statistic of interest from the data
  - (b) Re-sample the data *with replacement* and recalculate the statistic
  - (c) Repeat step b, say, 1000 times.
  - (d) Use the average and 2.5 and 97.5 % quantiles of the resampled statistics as the mean and 95% CI.
6. Causality is a reasonable conclusion if T is randomized and if the errors  $\epsilon_{i1}$  and  $\epsilon_{i2}$  from above are uncorrelated.

Often this assumption can be made more reasonable by including pre-treatment covariates (X) in both models.

7. Breakout and Discussion

8. Multiple mediation

9. Moderated mediation