More on Natural Selection

36-149 The Tree of Life

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Plan

1. Review
2. A Simple Selection Model
3. More on Natural Selection
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5. Books

Review: Natural Selection

Natural Selection is a process that influences which genotypes are propogated across generations.

Input: Variation in heritable characters.

Condition: Differential reproductive successive across different genotypes (in the given environment).

Output: The genotypes for the advantageous characters will increase in frequency across generations.

1. Those with greater reproductive success will produce more offspring.
2. Those offspring will tend to inherit the advantageous characters.
3. And thus in turn produce more offspring...

Common Misconceptions

- If Natural Selection is just "survival of the fittest," then it's all just a tautology. If the fittest are those who survive then the logic is circular.
- Natural Selection is a chance process. It's all purely random so nothing systematic can come of it.
- Natural Selection always improves organisms.
- We can't see Natural Selection operate, it only works over long periods of time.
- Natural Selection changes organisms by anticipating what they need to survive.
- If we see a feature on an organism, then that must be the most efficient feature possible.

A Simple Selection Model

- Now, we'll consider the same model with a small tweak.
- Use the Hardy-Weinberg assumptions with the addition that each genotype now survives with the following probabilities:

Genotype	Survival Probability
AA	1
Aa	1
aa	$1 - u$

  where u is a number between 0 and 1.

- What do you predict will happen to the allele frequencies in a population governed by this model?

Activity (cont'd)

Start with the set of cards you finished with last time.

Perform the same steps again with one exception: assume that homozygous recessive (aa) offspring do not survive.

Set aside any aa pairs you get.

After One Generation . . .

Think of the selection model as producing the next generation by random mating followed by elimination of the aa offspring.

After the random mating, we have (from earlier) that

$$D' = \text{Frequency of AA} = p^2$$
$$H' = \text{Frequency of Aa} = 2pq$$
$$R' = \text{Frequency of aa} = q^2.$$

Eliminating the aa's means reducing q^2 to $(1-u)q^2$.

$$\longrightarrow$$

$$D' = \text{Frequency of AA} = p^2$$
$$H' = \text{Frequency of Aa} = 2pq$$
$$R' = \text{Frequency of aa} = (1-u)q^2.$$

$$\swarrow$$

But these are relative frequencies: $D' + H' + R' = 1 - uq^2 < 1$.
Divide by the sum to get true proportions.

$$D' = \text{Frequency of AA} = \frac{p^2}{1-uq^2}$$
$$H' = \text{Frequency of Aa} = \frac{2pq}{1-uq^2}$$
$$R' = \text{Frequency of aa} = \frac{q^2-uq^2}{1-uq^2}.$$

After One Generation . . . (cont'd)

Because $p' = D' + H'/2$ and $q' = R' + H'/2$, it follows that

$$p' = \frac{p^2 + pq}{1 - uq^2} = \frac{p}{1 - uq^2}$$

$$q' = \frac{(1-u)q^2 + pq}{1 - uq^2} = \frac{q - uq^2}{1 - uq^2}$$

After each generation, the frequency of A has increased by a factor of $1/(1-uq^2)$, which is greater than 1 if $u > 0$.

Therefore, A will eventually be the only allele remaining.

Fitness

At the each generation in the selection model, the three genotypes have different rates of reproduction.

For example, in the first generation

Genotype	**Mate**	**Probability**	**Expected Offspring**		
			AA	Aa	aa
AA	AA	D	1	0	0
	Aa	H	0.5	0.5	0
	aa	R	0	1	0
Aa	AA	D	0.5	0.5	
	Aa	H	0.25	0.5	0.25
	aa	R	0	0.5	0.5
aa	AA	D	0	1	0
	Aa	H	0	0.5	0.5
	aa	R	0	0	1

On average, only $1 - u$ of the aa offspring survive from each mating.
All of the others survive.

Fitness (cont'd)

Weighting each possible mating by its probability, we get the expected number of offspring for each genotype.

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\begin{aligned}
\text{offspring(AA)} &= 1 \\
\text{offspring(Aa)} &= D + \frac{3}{4}H + \frac{1}{4}H(1-u) + \frac{1}{2}R + \frac{1}{2}R(1-s) \\
&= D + H + R - \frac{u}{2}(R + \frac{1}{2}H) \\
&= 1 - \frac{u}{2}q \\
\text{offspring(aa)} &= D + \frac{1}{2}H + \frac{1}{2}H(1-u) + R(1-u) \\
&= D + H + R - u(R + \frac{1}{2}H) \\
&= 1 - uq.
\end{aligned}
$$

Fitness (cont'd)

Natural selection indicates that those phenotypes/genotypes with greater *reproductive success* will propogate in the population.

The term *fitness* is used as a measure of reproductive success of each genotype relative to the others.

There are different ways to define it, but in this case take the fitness of a genotype as ratio of the expected number of offspring for that genotype over the comparable number for AA.

From the above calculations, we get

$$\begin{aligned} \text{fitness(AA)} &= 1 \\ \text{fitness(Aa)} &= 1 - \frac{u}{2}q \\ \text{fitness(aa)} &= 1 - uq. \end{aligned}$$

Eventually AA will take over the population!

Modes of Selection

- Stabilizing Selection

- Directional Selection

- Diversifying (or Disruptive) Selection

Walking along Fitness Landscapes

"Survival of the Fittest"

- Tautology? revisited
- What's "surviving"?
- Is and Ought

Activity

Setup

1. From the gene pool, select 10 cards, and then go back to your seat.
2. Count how many 'A' alleles you have and how many 'a' alleles you have and record these.
3. Combine the totals with your class mates (two by two, then four by four, etc.) to get a total count for the class.
4. Calculate the frequencies of A and a in the class and record these numbers.
5. Now *randomly* pair your alleles so that you have 5 gene pairs just like the genotypes of diploid organisms.
6. Count how many 'A' alleles you have and how many 'a' alleles you have and record these.
7. Combine the totals with your class mates (two by two, then four by four, etc.) to get a total count for the class.
8. Calculate the frequencies of each of these combinations in the class and record.

Activity

Mating: The First Generation

9. Spread your 10 cards out in one hand as if you were playing cards. Make sure that you are the only one who can see the alleles. Go around the room and find another person to trade with. Let that person take one of your alleles. You take one of their alleles. (Don't peek!) Now take your new allele and put it away so that you don't trade it again.
10. Trade 4 more alleles with 4 more people (only 1 trade with each person).
11. When you have only 5 alleles left in your hand and 5 new ones that are trades, sit down!
12. Again, pair up your 10 alleles randomly so that you have 5 pairs.
13. Count how many of each allele and each genotype you have in your hand;
14. Combine with classmates as bfore, calculate the frequencies of each of these combinations and record.

Mating: Rinse and Repeat. (It never gets old!)