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The Effects of Measurement Uncertainties and High Velocity Satellites on Mass Estimates of the Milky Way

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ABSTRACT

We have developed a Bayesian method for estimating the mass and cumulative mass profile of the Milky Way that uses the positions and velocities of Galactic satellites. A preliminary analysis using the kinematic data of globular clusters and dwarf galaxies, and assuming a simple Hernquist (1990) model, returned a mass estimate for the Milky Way that is in agreement with many other studies (Eadie, Harris, & Widrow 2015, hereafter EHW). However, this study also found that both high-velocity objects and measurement uncertainties have a strong influence on the estimated mass. Thus, we have developed a way to include measurement uncertainties by treating all of the satellite positions and velocities as parameters in a hierarchical Bayesian model (Eadie et al 2015b, 2015c). Using the same kinematic data and model assumptions as EHW but applying the hierarchical Bayesian method, the mass estimate for the Milky Way is significantly smaller. These results illustrate the importance of not only including measurement uncertainties in the analysis, but also obtaining precise, accurate velocity meaurements for high-velocity satellite objects around the Milky Way.

INTRODUCTION
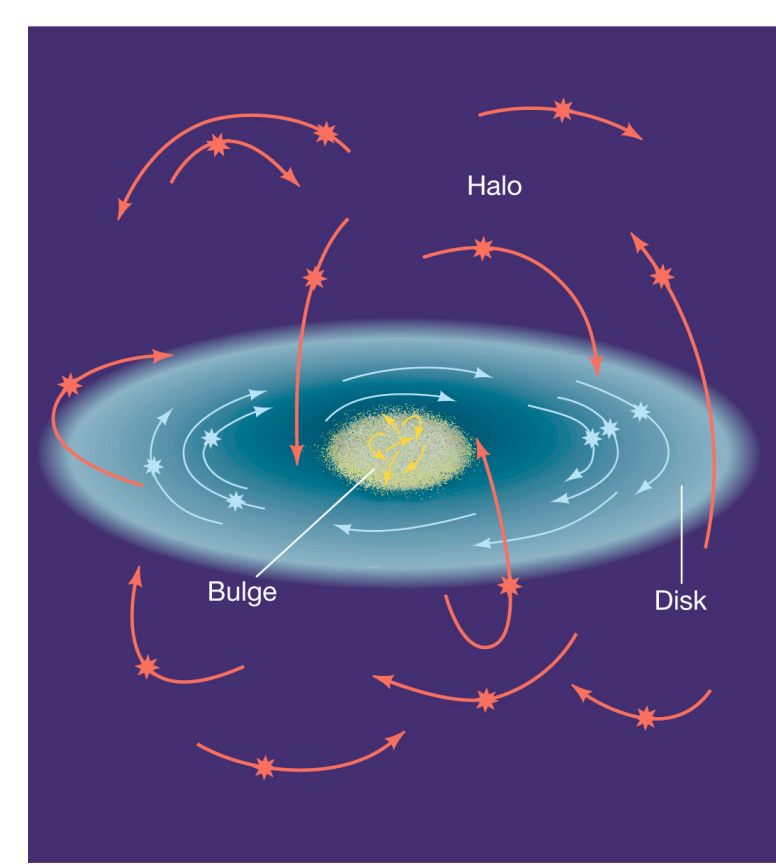
The Milky Way (MW) has many distant satellites, such as globular clusters (GCs) and dwarf galaxies (DGs), that are keys for estimating the mass of the Galaxy out to large radii. The kinematic properties of these satellites can be used to learn about the gravitational potential of the whole system, and thus the mass profile. Transforming this kind of data accurately to a mass profile $M(r)$, however, is difficult, because limited or incomplete data may substantially affect the analysis.

The geometry and mathematics of Galaxy models is simplest in a Galactocentric reference frame, where the speed v of an object is

$$v = \sqrt{v_r^2 + v_t^2}$$

However, we measure the line-of-sight velocity v_{los} and proper motion μ from our Heliocentric reference frame.
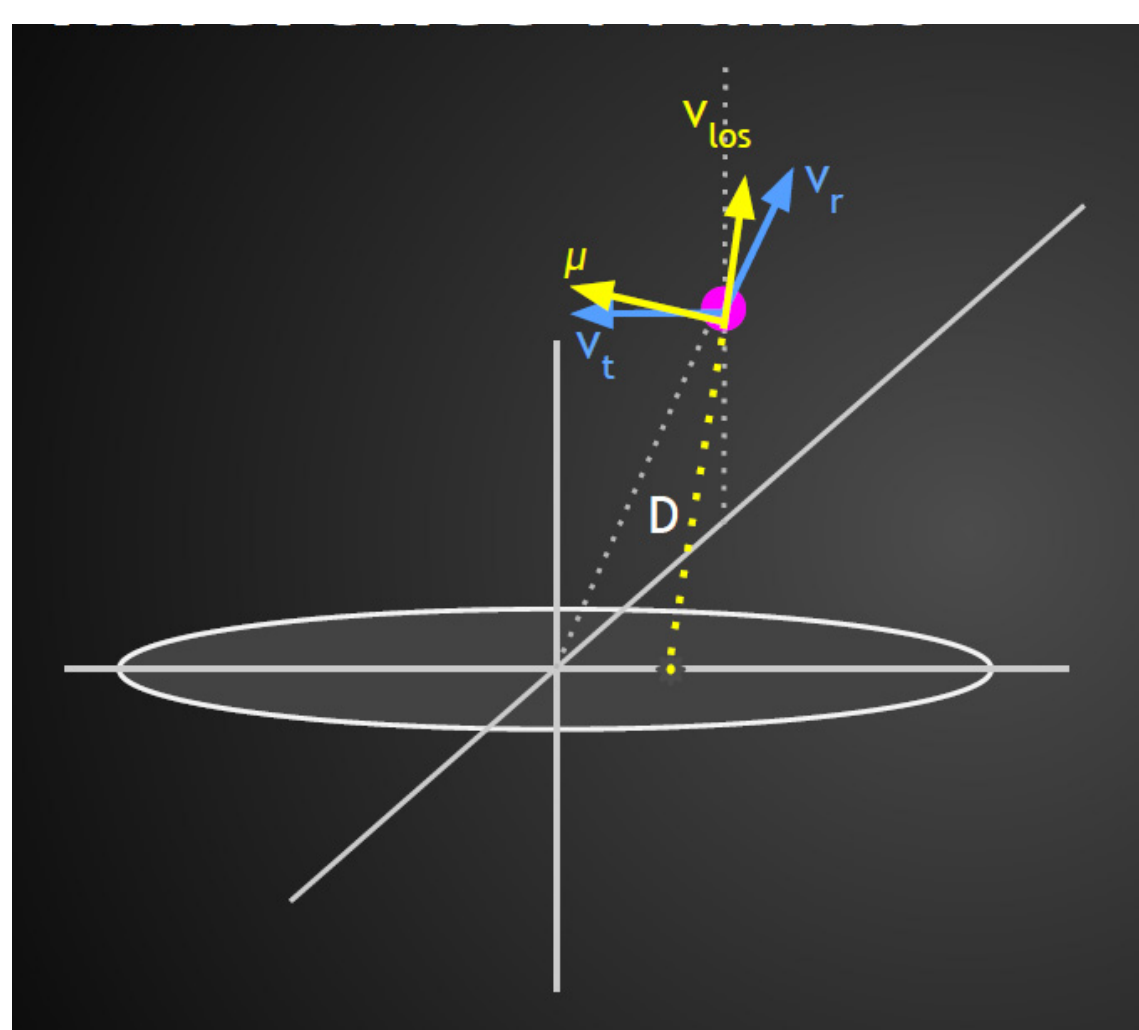
In many cases, tracers of the MW have only line-of-sight velocity measurements. Without a proper motion measurement, both v_r and v_t in the Galactocentric frame are unknown. We can make the approximation that v_r is roughly v_{los} for distant objects, but v_t is still unknown, and thus the transformation cannot be performed.
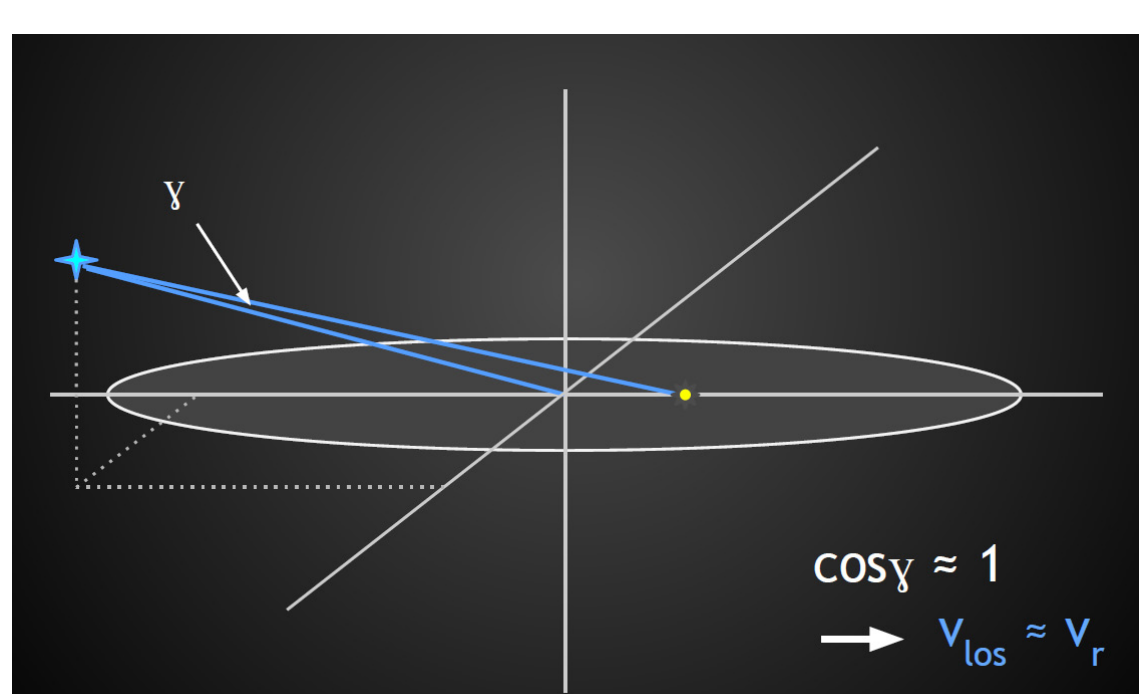
BAYESIAN INFERENCE

The Distribution Function (DF) $f(r,v)$ is a probability density function that gives the probability of finding a satellite with a position $\mathbf{r}$ and velocity $\mathbf{v}$ within an infinitesimal phase-space $d^3r d^3v$ (Binney & Tremaine 2008). Assuming satellites are independent, Bayes' rule reads

posterior distribution   Distribution Function   priors

$$p(\boldsymbol{\theta}|y) \propto \prod_{i=1}^{n} f(r_i, \mathbf{v}_i|\boldsymbol{\theta}) p(\boldsymbol{\theta})$$

where $\boldsymbol{\theta}$ is the vector of model parameters and y is the vector of kinematic data. In EHW, the unknown v_t components were treated as nuisance parameters, and the method was tested using both simulated and real data (GCs and DGs).

$$p(\boldsymbol{\theta}|y) \propto \prod_{i=1}^{n} f(r_i, v_{r,i})|\boldsymbol{\theta}, v_{t,i}) p(\boldsymbol{\theta}) p(v_{t,i})$$

unknown v_t's as parameters

EHW assumed a spherical Hernquist model and also assumed that the distribution of dark matter and satellites followed the same DF. Results from this study suggested that the method was quite feasible, but that the mass estimates could depend significantly on high-velocity objects and the measurement uncertainties.

Mass Estimates from Eadie et al (2015), ApJ 804:54

Model-σ^2	All Data		Without Pal 3	
	M_{tot} ($10^{12} M_\odot$)	Scale Radius (kpc)	M_{tot} ($10^{12} M_\odot$)	Scale Radius (kpc)
H-iso	1.55 ± 0.08	16.9 ± 2.3	1.36 ± 0.07	16.7 ± 2.2
H-OM	1.52 ± 0.08	16.7 ± 2.2	1.34 ± 0.06	16.4 ± 2.1
H-$\beta = 0.5$	1.47 ± 0.07	12.1 ± 1.9	1.31 ± 0.06	12.3 ± 1.9
Jaffe-iso	1.61 ± 0.09	47.7 ± 8.5	1.38 ± 0.06	48.9 ± 8.7

HIERARCHICAL BAYESIAN MODEL

A measurement of a quantity such as v_{los} is inherently uncertain, and this is precisely why measurement uncertainties are reported. The measurement uncertainties signify that we do not know the true value of a quantity. Therefore, we now treat the measurements $(r, v_{los}, \mu_\delta, \mu_\alpha cos\delta)$ as drawn from a distribution centered on the parameters $\vartheta = (r, v_{los}, \mu_\delta, \mu_\alpha cos\delta)$. We assume the data are normally distributed about their corresponding mean parameter values, with standard deviations equal to the measurement uncertainties $\Delta = (\Delta r, \Delta v_{los}, \Delta \mu_\delta, \Delta \mu_\alpha cos\delta)$. The likelihood is then,

$$\mathcal{L} = p(r|r, \Delta r) p(v_{los}|v_{los}, \Delta v_{los}) p(\mu_\delta|\mu_\delta, \Delta \mu_\delta) p(\mu_\alpha \cos\delta|\mu_\alpha \cos\delta, \Delta \mu_\alpha \cos\delta)$$

Thus, given N satellites with kinematic data y, and the known measurement uncertainties Δ, the posterior distribution for the model parameters θ is

Likelihood   Distribution Function   Hyperpriors

$$p(\boldsymbol{\theta}|\boldsymbol{y}, \Delta) \propto \prod_{i}^{N} \mathcal{L}(\boldsymbol{y_i}, \Delta_i|\vartheta_i) p(h(\vartheta_i)|\boldsymbol{\theta}) p(\boldsymbol{\theta}).$$

where the term $p(h(\vartheta)|\theta)$ is the DF for the Galaxy model given θ and $h(\vartheta)$ is the transformation from Heliocentric to Galactocentric coordinates, following Johnson & Soderblom (1987) and using updated values for the solar motion, local standard of rest, and J2000 epoch.

For a preliminary test of the hierarchical Bayesian model, we used the identical data set and Galaxy model from EHW. The mass profiles from before and after including measurement uncertainties are significantly different (Figure 1). High-velocity objects such as Pal 3 were previously shown to have significant leverage on the mass, but these objects also have very large uncertainties. Thus, when their uncertainties are taken into account, they have less influence on the mass estimate.
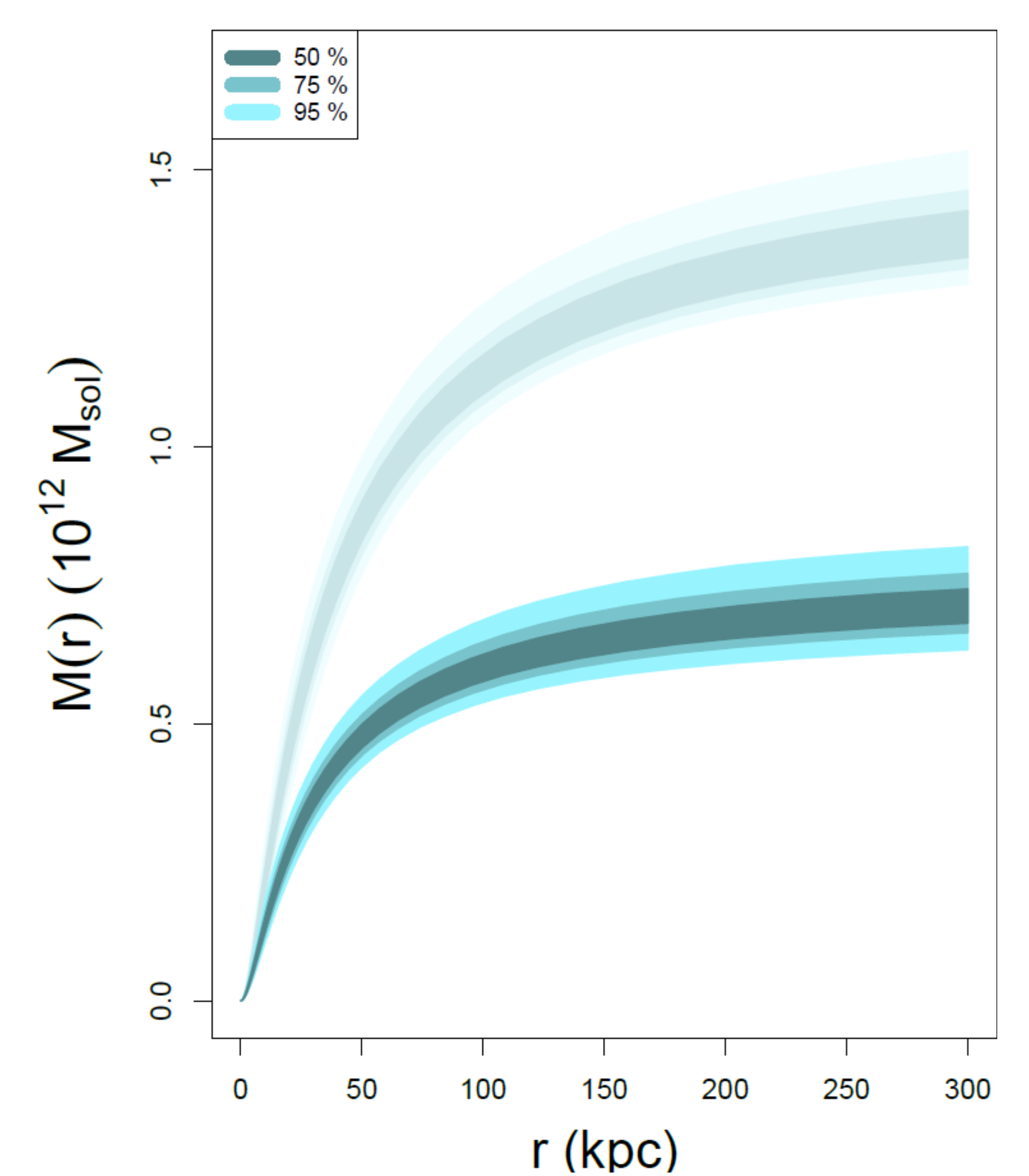

FIGURE 1. Milky Way cumulative mass profile, assuming an isotropic Hernquist model, before and after incorporating measurement uncertainties. The data include both GCs and DGs. (Eadie, Harris, & Springford 2015).

A MODEL WITH SEPARATE DISTRIBUTIONS FOR SATELLITES AND DARK MATTER

We now use a Galaxy model that two employs power-law profiles: one to describe the dark matter and one to describe the satellites. The spatial profiles of the two populations are different, in contrast to the Hernquist model. This model was first proposed by Evans et al (1997), and was later adopted by Deason et al (2011, 2012) in a maximum-likelihood analysis of blue horizontal branch stars to obtain a mass estimate of the Milky Way. Velocity anisotropy, parameter, β, is also incorporated into the model through the method introduced by Cuddeford (1991).

Satellite Density Profile
$$\rho_t \propto \frac{1}{r^\alpha}$$

Gravitational Potential
$$\Phi = \frac{\Phi_o}{r^\gamma}$$

Cumulative Mass Profile
$$M(r) = \frac{\gamma \Phi_o}{G} \left(\frac{r}{\text{kpc}}\right)^{1-\gamma}$$

Distribution Function for power-law model
$$f(\mathcal{E}, L) = \frac{L^{-2\beta} \mathcal{E}^{\frac{\beta(\gamma-2)}{\gamma} + \frac{\alpha}{\gamma} - \frac{3}{2}}}{\sqrt{8\pi^3} 2^{-2\beta} \Phi_o^{-\frac{2\beta}{\gamma} + \frac{\alpha}{\gamma}}} \frac{\Gamma\left(\frac{\alpha}{\gamma} - \frac{2\beta}{\gamma} + 1\right)}{\Gamma\left(\frac{\beta(\gamma-2)}{\gamma} + \frac{\alpha}{\gamma} - \frac{1}{2}\right)}$$

Because we assume different spatial distributions for the dark matter and satellites, it no longer seems appropriate to combine both GCs *and* DGs. Thus, we use a subsample of 89 out of the 157 Milky Way GCs listed in the catalog of Harris (1996, 2010). Most of these GCs have proper motion measurements, which are listed in the series of papers by Casetti-Dinescu (Dinescu et al. 2004, 2005, 2010, 2013), but 17 GCs in our subsample do not. Figure 2 shows the mass profiles that result when we do and do not use the hierarchical model that includes measurement uncertaintes. Similar to the results of the Hernquist model, the mass estimate is lower when measurement uncertainties are taken into account, albeit the difference is smaller than that seen in Eadie et al (2015b). Extrapolating out to the virial radius, the mass estimate for the Milky Way is $M_{vir} = 5.98 \times 10^{11} M_{sun}$, with 95% credible region $(4.40, 7.76) \times 10^{11}$ M_{sun}.
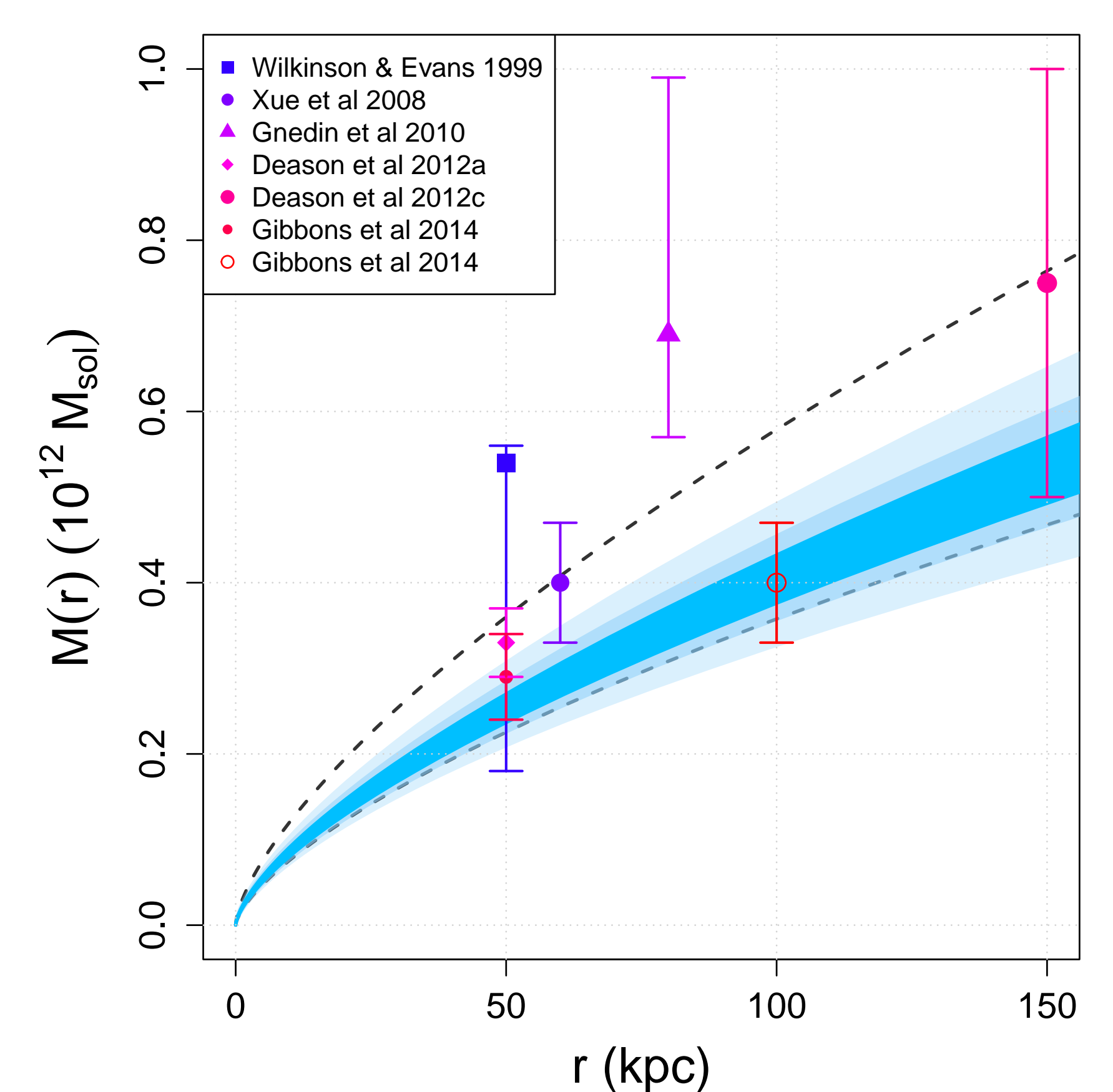

FIGURE 2. Comparison of the Milky Way cumulative mass profile, assuming the power-law model, before and after incorporating measurement uncertainties through the hierarchical Bayesian model (Eadie & Harris 2016, Eadie, Harris, & Springford 2016). The blue shaded areas represent the 50, 75, and 95% Bayesian credible regions when uncertainties are included, and the dashed black lines show the 95% credible region before incorporating measurement uncertainties. The points are mass estimates at specific radii, from other recent studies.

Virial Mass ($10^{11} M_{sun}$):   7.04   5.98
50% credible regions: (6.23, 7.76)   (5.39, 6.52)

CONCLUSIONS & FUTURE WORK

Our results show that the mass estimate of the Milky Way is sensitive to high-velocity objects and measurement uncertainties. Thus, our analysis and comparison to other studies strongly emphasizes the need for precise measurements of remote, virialized tracers (r > 30kpc) to place stronger constraints on the total mass of the Milky Way. The mass estimate we obtain using the power-law model is in agreement with recent studies that suggest a "lighter" Galaxy (e.g. Gibbons et al 2014, who used the Sagittarius stream to obtain a mass estimate for the Milky Way).

One advantage of the method presented here is that an estimate of the mass enclosed at any radius, along with uncertainties in that estimate, is obtained relatively easily. This feature makes comparing our results to other studies straightforward. Another advantage is that we use both complete and incomplete data, as opposed to common mass estimators in the literature which use one or the other (e.g. Bachall & Tremain 1981, Evans et al 2003; Watkins et al 2010).

The first data release from the GAIA mission will occur in summer 2016 (ESA 2016). The Tycho-GAIA astrometric solution (TGAS) could yield proper motion, parallax, and position measurements for 2.5 million Tycho-2 stars (Michalik et al. 2015). The analytical approach described here will be well-suited to this data. Furthermore, we intend to extend our method to other galaxies, where the full 3D space motions of tracers are never known.

ESA–D. Ducros, 2013
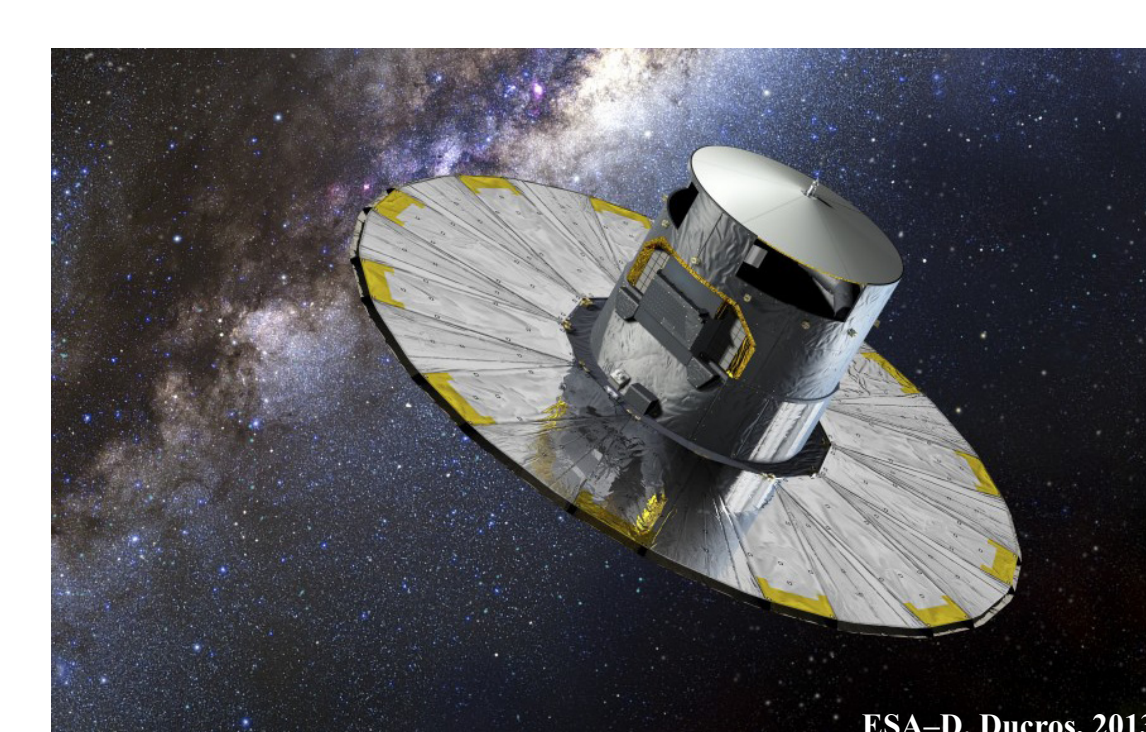
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