$$y = \beta_0 + \varepsilon$$

$$X = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{n\times 1}$$

$$H_1 = X(X^TX)^{-1}X^T$$

$$X^TX = (1 \cdots 1)_{1\times n}\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}_{n\times 1} = n$$

$$(X^TX)^{-1} = \frac{1}{n}$$

$$\hat{y} = H_1 \cdot y = \begin{bmatrix} 1/n & \cdots & 1/n \\ \vdots & & \\ 1/n & \cdots & 1/n \end{bmatrix}\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} \bar{y} \\ \bar{y} \\ \vdots \\ \bar{y} \end{pmatrix}$$

$$\boxed{\text{Also, } \hat{\beta}_0 = \bar{y}}$$

$$H_1 = \left[\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}(1 \cdots 1)\right]_{n\times n} \cdot \frac{1}{n} = \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ \vdots & & & & 1 \\ 1 & \cdots & & & 1 \end{bmatrix}\frac{1}{n}$$

$$H_1 = \begin{bmatrix} 1/n & 1/n & 1/n & \cdots & 1/n \\ & \ddots & & & \\ 1/n & \cdots & & & 1/n \end{bmatrix}_{n\times n}$$

$$y = \beta_0 + \beta_1 X_1X_2X_3 + \varepsilon \qquad \text{(3-way interaction)}$$

H.P.: expand this model!

$$y = \beta_0 + \beta_7 X_1 + \beta_6 X_2 + \beta_5 X_3 + \beta_4 X_1X_3 + \beta_3 X_1X_2 + \beta_2 X_2X_3 + \underline{\beta_1 X_1X_2X_3} + \varepsilon$$