$Y \sim N(\mu, \Sigma)$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}_{n \times 1} \qquad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{k1} & \cdots & & a_{kn} \end{bmatrix}_{k \times n} \qquad \underbrace{\underset{k \times n}{A}\ \underset{n \times 1}{Y}} = \begin{bmatrix} a_{11}y_1 + a_{12}y_2 + \cdots a_{1n}y_n \\ a_{21}y_1 + \cdots y_{2n}y_n \\ \vdots \\ a_{k1}y_1 \cdots a_{kn}y_n \end{bmatrix}$$

$$E[AY] = \begin{bmatrix} E[a_{11}y_1 + \cdots a_{1n}y_n] \\ E[a_{21}y_1 + \cdots + a_{2n}y_n] \\ \vdots \\ E[a_{k1}y_1 + \cdots a_{kn}y_n] \end{bmatrix} = A E[Y] = A\mu$$

aside: If $Z = \begin{bmatrix} z_1 \\ \vdots \\ z_m \end{bmatrix}$ Then $Var(Z) = E[(Z - \mu_z)(Z - \mu_z)^T]$

$$= E\left[\begin{bmatrix} z_1 - \mu_{z1} \\ z_2 - \mu_{z2} \\ \vdots \\ z_m - \mu_{zm} \end{bmatrix} \begin{bmatrix} z_1 - \mu_{z1} & z_2 - \mu_{z2} & \cdots & z_m - \mu_{zm} \end{bmatrix} \right]$$

$$= \begin{bmatrix} Var(z_1) & Cov(z_1, z_2) & & \text{Cov's} \\ Cov(z_2, z_1) & Var(z_2) & & \\ & & \ddots & \\ \text{Cov's} & & & Var(z_n) \end{bmatrix}$$

$Var(Y) = \Sigma$

$$Var(AY) = E[A(y - \mu)[(y - \mu)^T A^T]]$$

$$= A E[(y - \mu)(y - \mu)^T] A^T$$

$$= A \Sigma A^T$$