What's in a Song? Exploring Primary Drivers in the Perception of Music As Classical and Popular

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Abstract

What drives the perception of music as Classical, or Popular? We explore data and insights from Jimenez (2013), surrounding Instruments, Harmonic Motion, and Voice Leading to build hierarchical models to explore this question. We find that String and Piano instruments strongly associate with perceptions of music as Classical (+), while inversely associated with Popular. Further, the Pachelbel Hamonic Motion of I-V-VI in conjunction with with the contrary voice predicts perceptions of music as Classical (+), and inversely predicts Popular. Musicians also behave differently in general than non-musicians, rating music as less Classical and more Popular in general, but more Classical and less Popular when the song is in the Pachelbel Harmonic Motion. We conclude with a discussion of the limitations of the data and the framework, and recommendations of areas for future study.

1 Introduction

How do you hear classical, or popular music? With ears, of course, but what signals in the cochlear vibrations do we put to the name Classical or Popular? What auditory features do we identify as Classical, or Popular? Following Jimenez [Jimenez and Rossi, 2014], we seek to observe the role that different features of music, specifically Instrument, Hamonic Motion and Leading Voice, have in a layperson's perception of music as Classical, and as Popular.

The primary questions we seek to address are:

- Does Instrumentation, Harmonic Motion, or Voice Leading exert the strongest influence on ratings of music as Classical, or as Popular?
  - Is Instrument the strongest, as Jimenez hypothesized?
  - Among Harmonic Motion, does I-V-VI have the strongest association with classical ratings? And does this vary considering the subject's prior knowledge of this particular progression?

- Among Voice Leading, does Contrary Motion have the strongest positive association with Classical ratings, as previous research has suggested?

- Are there differences in the ways that musicians and non-musicians identify classical music?
- Are there differences in what drives rating classical and popular ratings?

2 Methods

Our data was collected by Ivan Jimenez and Vincent Rossi, for a designed experiment intendent to measure what factors, specifically Instrument, Harmonic Motion, and Voice Leading have on ratings of music as Popular and as Classical. The dataset consists of 36 musical stimuli rated by 70 listeners, recruited from undergraduates at the University of Pittsburgh. In addtion to the ratings of Classical and Popular and the three factors of interest, we have several variables which vary by individual. See table 1 for the detail breakout of all the variables.

We employ several methods in address of the primary research questions:

- To address the influence of the primary experimental variables - Instrument, Motion, and Voice, we build a multi-level random effects model, with compound error at the level of the individual (Subject).
  - First we build a simple observation level model using AIC and BIC optimization for the random effect variables.
  - Second, using stepAIC on a non-random effects model we identify the most valuable subject level covariates
  - Third, we manually update the random-effects model with the variables identified in the stepAIC procedure, maximizing AIC and BIC.
  - Fourth, we incorporate the Musician variable, and several interactions to evaluate how Musicianship may influence ratings
  - Fifth, we incorporate variables to deduce familiarity with Pachelbel's Canon, to see how these influence ratings of the Pachelbel motion
  - We repeat this process to generate models examining both Classical and Popular ratings.

We use the data analysis software, R [R Core Team, 2019], and several libraries, including: naniar [Tierney et al., 2019] for missingness EDA, Hmisc [Harrell, 2019] and corrplot [Wei and Simko, 2017] for correlation tables and graphics, various tidyverse [Wickham, 2017] functions for data manipulation, lme4 [Bates et al., 2015] for random effects models, and RLRsim [Scheipl et al., 2008] and LMERConvenienceFunctions [Tremblay et al., 2015] for random effect fitting functions.

Table 1: Original Variables [Jimenez and Rossi, 2014]

Variable name	Description
Classical	Rating as Classical (1-10) (not at all - very Classical sounding)
Popular	Rating as Popular (1-10) (not at all - very Popular sounding)
Subject	Subject id
Harmony	Harmonic Motion: I-V-vi, I-VI-V, I-V-IV, IV-I-V
Voice	Voice Leading: Contrary Motion, Parallel 3rds, Parallel 5ths
Instrument	Instrument: Electric Guitar, String Quartet, Piano
Selfdeclare	Are you a musician? (1-6) (1= not at all)
OMSI	Score on a test of musical knowledge
X16.minus.17	Auxiliary measure of listener ability to distinguish Classical vs Popular music
ConsInstr	how much did the listener concentrate on instrument (0-5, 0=not at all)
ConsNotes	how much did the listener concentrate on the notes (0-5, 0=not at all)
Instr.minus.Notes	difference between ConsInstr minus ConsNotes
PachListen	How familiar are you with Pachlebel's Canon (0-5, 0=not at all)
ClsListen	How much do you listen to classical music? (0-5, 0=not at all)
KnowRob	Have you heard Rob Paravonian's Pachelbel Rant? (0-5, 0=not at all)
KnowAxis	Have you heard Axis of Evil's Comedy bit on the 4 pachelbel chords in popular music? (0-5, 0=not at all)
X1990s2000s	How much do you listen to pop and rock from the 90's and 2000's? (0-5, 0=not at all)
X1990s2000s.minus.1960s1970s	Difference between prev variable and a similar variable referring to 60's and 70's pop and rock
CollegeMusic	Have you taken music classes in college (0=no, 1=yes)
NoClass	How many music classes have you taken?
APTheory	Did you take AP Music Theory class in High School?
Composing	Have you done any music composing? (0-5, 0=not at all)
PianoPlay	Do you play piano? (0-5, 0=not at all)
GuitarPlay	Do you play guitar? (0-5, 0=not at all)
X1stInstr	How proficient are you at your first musical instrument (0-5, 0=not at all)
X2ndInstr	How proficient are you at your second musical instrument (0-5, 0=not at all)

3 Results

3.1 Variable Transformations & Data Cleaning

In basic data analysis, we identified that there was substantial missingness in the X1stInstr and X2ndInstr variables. As there was not a clear imputation logic (no obvious patterns across variables), we decided to exclude these from our analysis. In addition, in exploring potentially valuable variable contributions and analysis, we need as full data as possible. To this end, in our model selection process we used mean imputation for our ordinal categorical variables, and mode imputation for the indicator variables. We believe this was prudent as there is not much missingness in most of our remaining variables, as seen in figure 1. However, to avoid over confidence in these effects, our final results are provided on the original data prior to imputation. (See pages 1 - 3 in Appendix)

In the two primary dependent variables of Classical and Popular, we found two types data errors. First, there are two observations with values of 19, which we pushed to be 10, as we think there was a 'fat finger' error, with the 9 and the 0 keys being right next to each other. The other error was ratings of 0. These are more difficult to deduce, and so we discard these data.

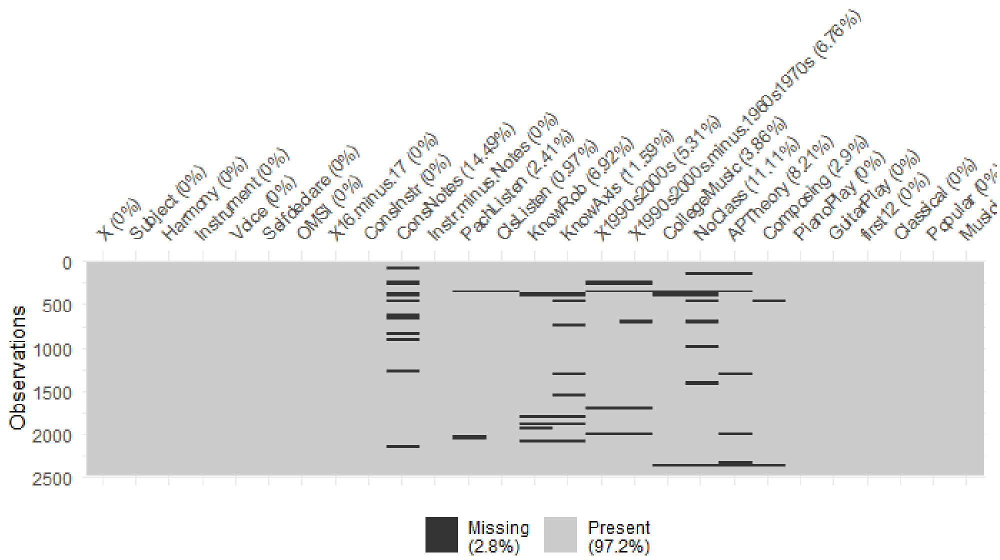

Figure 1: Missingness Plot

Prior to modeling, we sought a few variable transformations to faciltiate a meaningful and statistically efficient procedures. First, we used the natural logarthimic transformation of the OMSI variable (it is significantly right skewed, see appendix pg. 2). Second, we dichotimized the Selfdeclare variable to enable simple explanations of 'musicians' vs 'non-musicians', and test interactions for these groups. We attempted multiple cut offs

Table 2: New variables, interactions, and transformations

Variable Name	Construction	Reasoning
OMSI	$log_e(OMSI)$	Turn OMSI into a percentage effect - OMSI is right skewed
Musicianship	$Selfdeclare > 1$	The dichotimization of musicianship may be more meaningful than the constrained step size of the ordinal scale.

for the Musicianship variable, associating with different levels of Selfdeclare, but found the most informative cut off was between self declared musicianship of 0,1 vs 2,3,4,5.

3.2 Modeling Rating Music as Classical

First, in a simple linear model we identifed the statistically significant predictive importance of all three experimental factors in the model, with Harmonic motion of I-V-VI associating with higher classical ratings, both Piano and String instruments pointing to higher classical ratings, as well as a decrease in classical ratings when the Harmonic Motion was I-V-VI and the third parallel Voice leading. (See pages 3 - 6 in appendix)

With this confirmation of our direction, we updated the model in two directions to construct a final model - identifying random effects components, and identifying the fixed effects components.

In identifying the random effects, we proposed variation at the Subject level, hypothesizing that the 1) individuals have different baseline predlictions to rating music as Classical, and 2) individuals have different sensitivies to the universal effects that the influence that certain Hamonic Motions and Instruments have on perception of music as Classical. This subject level variation in both baseline and coefficient estimates we model as if it were randomly distributed around a universal value, with estimable variation. This random effects component becomes the subject level variation of our Multi-Level model.

In order to come up with our multi-level model random effects, we first confirmed the value in a random intercept, a repeated measures model, by performing an exact Log Ratio test, using exactLRT [Scheipl et al., 2008], confirming the validity of our approach (Page 7 in appendix). Then to check if there were additional random effects in the model, we used manual covariate searching, supplemented with the function fitLMER [Tremblay et al., 2015]. We found that random effects on the coefficients for Harmony and Instrument in addition to the intercept optimized AIC and BIC, and remained interpretable. (Pages 7 - 11 in appendix)

In finalizing the fixed effects, we performed stepAIC searching on a fixed effects model to highlight variables with explantory power, and then reduced the model once incorporating the random effects, to optimize AIC and BIC. (Page 12 in appendix) Adding the identified fixed effects into our multilevel model, we find that much of explanatory power is better explained via our random intercept and random effects model, which structure the variance in such a way that undermines the seeming signficiance of the subject level fixed effects. After working backwards from the full fixed effects from the stepAIC we only found that the fixed effects of ClsListen, Selfdeclare, OMSI, X1990s2000s and X1990s2000s.minus.1960s1970s to be explantory. Once working backwards to our limited model, we tested new random effect coefficients, but found that the original random effects of the intercept, Harmony and Instrument persisted. (Pages 12 - 21 in Appendix)

3.2.1 Effect of Musicianship on Classical Rating

To address our secondary point, we attempted to evaluate the effect of the subject being a Musician. We took the previous model and added the Musician covariate, as well as attempted interactions with the remaining relevant variables. We find that indeed it is influential, both on its own and in conjunction with the ClsListen and HarmonyIV-I-V. In addition, it could explain was so influential to the extent that we removed all but the ClsListen variable from our previous model.

Those that identify as musicians, on average, rate musical pieces they encounter as less Classical than their non-musician counterparts. In addition, Musicians rate the Harmonic Motion of I-V-VI significntly higher than their non-musician counterparts. Further, musicians who are more apt to listen to classical music on average rate all music as Classical.

In addition, we found that including the indicator parameter for Musician and its interactions in our model improves AIC and BIC over our previous model, so we include it in our final model.

See pages 22-32 in appendix for details

3.2.2 Effect of Harmonic Variation for those with knowledge of Pachelbel Motion

We attempted to incorporate the KnowAxis, KnowRob and PachListen into our otherwise final model to evaluate the effect of having explict prior knowledge of the popular Pachelbel Harmonic Motion of I-V-VI. To mediate the standard error inflation present when including multiple interaction terms with one variable, we worked with the pre-musician final model. We found that indeed knowledge of the Pachelebel Motion does explain some of their ratings, as they consistently rated music with the Pachelbel Motion higher than the other Harmonic Motions. Specifically, KnowRob (ordinal rating of knowledge of a particular commentary on the ubiquity of the Pachelbel Motion) and PachListen (ordinal rating of familiarity of Pachelbel's Canon in D) have a significant positive bump in Classical ratings when the music was in the Pachelbel Harmonic Motion. Familiarity with the Axis Pachelbel Harmonic Motion comedy skit however did not seem to have any effects.

When trying to combine these apparent effects with the Musician expanded model, however, the musician model alone is preferred. There is positive correlation among Musician and the three Pachelbel Motion variables, so we may be diluting the effects of each independently. This is especially true with the induced collinearity of Harmonic Motion when we include several interaction terms.

	Fixed Effects Model	Musician Model	KnowRob Model	KnowAxis Model	PachListen Model
(Intercept)	3.29***	5.36***	3.93***	3.75***	4.14***
	(0.75)	(0.44)	(0.30)	(0.31)	(0.76)
HarmonyI-V-IV	0.16	0.01	0.16	0.20	−0.15
	(0.15)	(0.22)	(0.16)	(0.16)	(0.39)
HarmonyI-V-VI	1.16***	−0.01	0.92***	1.22***	−0.41
	(0.21)	(0.34)	(0.21)	(0.23)	(0.69)
HarmonyIV-I-V	−0.15	−0.34	−0.14	−0.10	−0.31
	(0.15)	(0.22)	(0.16)	(0.16)	(0.39)
Voicepar3rd	−0.25	−0.25	−0.25	−0.25	−0.25
	(0.15)	(0.15)	(0.15)	(0.15)	(0.15)
Voicepar5th	−0.21	−0.21	−0.21	−0.21	−0.21
	(0.15)	(0.15)	(0.15)	(0.15)	(0.15)
Instrumentpiano	1.34***	1.34***	1.34***	1.34***	1.34***
	(0.17)	(0.17)	(0.17)	(0.17)	(0.17)
Instrumentstring	3.08***	3.08***	3.08***	3.08***	3.08***
	(0.23)	(0.23)	(0.23)	(0.23)	(0.23)
ClsListen	0.34**	−0.25	0.19*	0.20*	0.20*
	(0.11)	(0.23)	(0.09)	(0.09)	(0.09)
Selfdeclare	−0.33*				
	(0.17)				
OMSI	0.32*				
	(0.15)				
X1990s2000s.minus.1960s1970s	0.22*				
	(0.10)				
X1990s2000s	−0.23*				
	(0.11)				
HarmonyI-V-IV:Voicepar3rd	−0.37	−0.37	−0.37	−0.37	−0.37
	(0.21)	(0.21)	(0.21)	(0.21)	(0.21)
HarmonyI-V-VI:Voicepar3rd	−0.72***	−0.72***	−0.72***	−0.72***	−0.72***
	(0.21)	(0.21)	(0.21)	(0.21)	(0.21)
HarmonyIV-I-V:Voicepar3rd	0.53*	0.53*	0.53*	0.53*	0.53*
	(0.21)	(0.21)	(0.21)	(0.21)	(0.21)
HarmonyI-V-IV:Voicepar5th	−0.21	−0.21	−0.21	−0.21	−0.21
	(0.21)	(0.21)	(0.21)	(0.21)	(0.21)
HarmonyI-V-VI:Voicepar5th	−0.45*	−0.45*	−0.45*	−0.45*	−0.45*
	(0.21)	(0.21)	(0.21)	(0.21)	(0.21)
HarmonyIV-I-V:Voicepar5th	0.07	0.08	0.07	0.07	0.07
	(0.21)	(0.21)	(0.21)	(0.21)	(0.21)
Musician		−2.16***			
		(0.52)			
HarmonyI-V-IV:Musician		0.19			
		(0.21)			
HarmonyI-V-VI:Musician		1.52***			
		(0.35)			
HarmonyIV-I-V:Musician		0.24			
		(0.21)			
ClsListen:Musician		0.60*			
		(0.25)			
KnowRob			−0.06		
			(0.11)		
HarmonyI-V-IV:KnowRob			−0.01		
			(0.05)		
HarmonyI-V-VI:KnowRob			0.31***		
			(0.09)		
HarmonyIV-I-V:KnowRob			−0.02		
			(0.05)		
KnowAxis				0.11	
				(0.10)	
HarmonyI-V-IV:KnowAxis				−0.05	
				(0.05)	
HarmonyI-V-VI:KnowAxis				−0.07	
				(0.09)	
HarmonyIV-I-V:KnowAxis				−0.05	
				(0.05)	
PachListen					−0.06
					(0.16)
HarmonyI-V-IV:PachListen					0.07
					(0.08)
HarmonyI-V-VI:PachListen					0.35*
					(0.14)
HarmonyIV-I-V:PachListen					0.04
					(0.08)

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 3: Comparison of Classical Models

See pages 33 - 38 in Appendix for details.

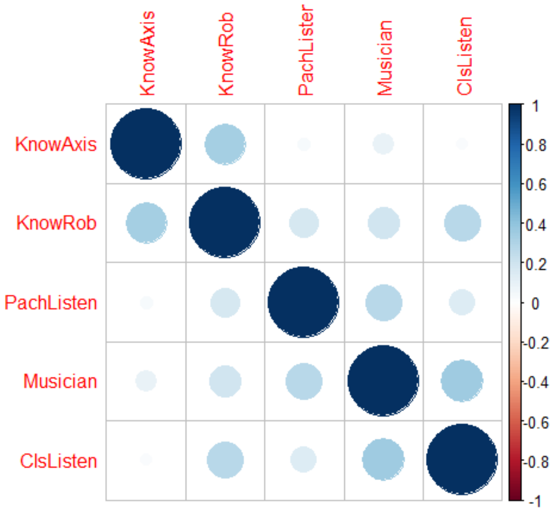

Figure 2: Correlation Plots of Potential Variables

3.3 Modelling Rating Music as Popular

For evaluating the effects of variables on Popular, we performed a highly similar process in model selection, and came up a slightly different model.

First we implemented an experimental design factors model with interactions. We found that no interaction terms appeared to add value to the model. Then we built in fixed effects with an exhaustive search of the space, finding that the same random effects components were valuable: the intercept, Harmony and Instrument. (Appendix pages 39-41) Then we used stepAIC to identify the most additional parameters to add to the model, initially including the KnowRob, PachListen, X1990s2000s, X1990s2000s.minus.X1970s1960s, GuitarPlay, Composing, ClsListen, APTheory and ConsNotes variables. (Page 41-42 in appendix)

However, when adding these to the random effects model, we find that some of these terms appear to drop out of significance with the random effects structured error. As these variables vary at the level of subject, by decomposing the error into a subject level component and an observation level component, we find that the variation that appeared to come from these variables is better explained by variation around an average coefficient effect. We find that with an exactLRT test between the simple linear model from stepAIC against the basic random effects model (only intercept) shows that the structuring the error component into two levels does provide additional explanatory power. Further, the full random effects found initially remain more valuable than simply the repeated measures model of the random intercept. (See pages 42-44 in appendix)

After working backwards from the full stepAIC with random effects model, we find an AIC/BIC optimum model which only includes the KnowRob fixed effect component.

	Classical	Popular
(Intercept)	5.34^{***}	5.54^{***}
	(0.44)	(0.32)
HarmonyI-V-IV	0.00	-0.32
	(0.23)	(0.20)
HarmonyI-V-VI	0.00	0.34
	(0.35)	(0.29)
HarmonyIV-I-V	-0.37	-0.11
	(0.22)	(0.22)
Voicepar3rd	-0.24	$0.15^{\cdot}$
	(0.15)	(0.08)
Voicepar5th	-0.21	0.17^{*}
	(0.15)	(0.08)
Instrumentpiano	1.35^{***}	-0.94^{***}
	(0.17)	(0.16)
Instrumentstring	3.11^{***}	-2.58^{***}
	(0.23)	(0.23)
ClsListen	-0.24	
	(0.23)	
Musician	-2.16^{***}	1.34^{***}
	(0.52)	(0.35)
HarmonyI-V-IV:Voicepar3rd	-0.38	
	(0.21)	
HarmonyI-V-VI:Voicepar3rd	-0.70^{***}	
	(0.21)	
HarmonyIV-I-V:Voicepar3rd	0.54^{*}	
	(0.21)	
HarmonyI-V-IV:Voicepar5th	-0.21	
	(0.21)	
HarmonyI-V-VI:Voicepar5th	-0.44^{*}	
	(0.21)	
HarmonyIV-I-V:Voicepar5th	0.10	
	(0.21)	
HarmonyI-V-IV:Musician	0.20	0.37
	(0.22)	(0.22)
HarmonyI-V-VI:Musician	1.48^{***}	-0.80^{*}
	(0.36)	(0.33)
HarmonyIV-I-V:Musician	0.26	-0.13
	(0.21)	(0.25)
ClsListen:Musician	0.59^{*}	
	(0.25)	

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$, $^{\cdot}p < .1$

Table 4: Classical and Popular Model Coefficients

Finally, to examine our hypothesis about Musicianship, we include this parameter and tested interactions with the existing variables present in the model to evaluate how a musician may behave differently than a non musician. We find that, just as in our Classical model, Musician is influential on its own, and in interaction with the Pachelbel Harmonic Motion. However it has the inverse relationship as in Classical, where Musicians on average rate what they hear as more Popular than non-musicians, but when they are presented with the Pachelbel I-V-VI motion, they rate the music as Less Popular. (Pages 45-48 in Appendix)

Our final model for Popular music was similar to, but a bit more sparse than the Classical model. We find that only the Musicianship variable is additionally informative than the multi level model with only the experimental factors. It seems that the random effects for the categorical data, which vary at the individual level, are sufficient in explaining the variance, against the subject level variables themselves.

Overall, the main takeaways mimic the Classical model, though the Voice and Harmonic Motion component did not have an interaction effect as it did in the Classical model, and the amount that an individual listened to Classical music also was not informative as to prediction of rating as Popular.

4 Discussion

From our models, we find that Instrument is indeed the most important predictor in determining rating of music as Classical and Popular, with String and Piano strongly associating with Classical, and the inverse for Popular ratings. In addition, we find the I-V-VI progression (the Pachelbel motion) not to influence listeners to rate music as more or less Classical in general, but to influence listeners to rate music as less Classical if the Leading Voice is not Contrary. The interaction between the Pachelbel Harmonic Motion and the non-contrary leading voice is very interesting. The contrary motion is, according to previous research (and hinted at in ours), more frequently associated with Classical music than the 3rds or 5ths. This supports the hypothesized dual effects of the I-V-VI harmonic progression, wherein it associated more with classical music, when in a classical music context, but outside of that context, it becomes inversely associated with Classical music ratings.

Voice on its own, however did not appear to have an obvious strong effect in rating music as Classical, though the coefficients on both non-Contrary motion was negative for Classical music, suggesting that contrary motion may be more associated with Classical music. Conversely, the leading Voice of a song being in Parallel 5ths and Parallel 3rds did significantly predict a song as being rated as more Popular, though to a fairly minor effect.

The musicianship of an individual was highly predictive of the individual rating music as non-Classical, and pro-Popular. It's not clear as to why this effect may be, but is worth further investigation in a specifically designed study. In addition, in both models we find that the Pachelbel motion to be influential for musicians - more Classical, and less Popular. Again the underlying cause of this effect is ambiguous, and is worthy of further investigation. Finally, in the classical model, we identifed the that the frequency an individual listened to classical music provided insight as to how much someone rates music as classical. This effect alone is negative (away from Classical) for those who do not identify as musicians, but those that do tend to have a higher baseline rating of music as Classical.

The familiarity of the individual with Pachelbel's Canon, and the ubiquity of Pachelbel's harmonic motion in general also appeared to influence how much an individual rated those particular chords as Classical. Both Familiarity with Pachelbels Canon itself as well as the Rob Pavorovian rant strongly predict an individual to rate a song as Classical, if it is in the I-V-VI progression.

A valuable addition to the literature may be in choosing a different duality of genre, such as Folk and Popular, or even Folk and Classical. 100 years ago these were the only two 'genres' in the United States, and the conception of Classical and Folk have relatively ossified, whereas Popular remains in flux.

We thank Jimenez and Rossi for their data and the opportunity to review it. Despite minor data errors and some missingness, there is a substantial takeaway analysis at hand. However, the extendability of our findings are necessarily limited, given the population and musical background of the listeners. The undergraduate population at the University of Pittsburgh is hardly an unbiased sample of the true world, or even United States populaton. In addition, the music is all in the Western canon, and nearly all vocalizations are in English. To be sure, this analysis is very Western centric (what is classical music if not Western classical music?), and our extrapolations are constrained as such. Simply an additional variable reflecting whether the subject was an international student may have been valuabe in our analysis.

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5 Covariance Tables

	Fixed Effects Model	Musician Model	KnowRob Model	KnowAxis Model	PachListen Model
AIC	9822.58	9808.95	9817.56	9828.71	9824.45
BIC	10061.12	10053.31	10056.10	10067.25	10062.99
Log Likelihood	-4870.29	-4862.47	-4867.78	-4873.35	-4871.23
Num. obs.	2485	2485	2485	2485	2485
Num. groups: Subject	70	70	70	70	70
Var: Subject (Intercept)	2.25	1.99	2.61	2.54	2.56
Var: Subject HarmonyI-V-IV	0.05	0.05	0.04	0.04	0.04
Var: Subject HarmonyI-V-VI	1.61	1.35	1.24	1.60	1.49
Var: Subject HarmonyIV-I-V	0.00	0.00	0.00	0.00	0.00
Var: Subject Instrumentpiano	1.65	1.65	1.65	1.65	1.65
Var: Subject Instrumentstring	3.37	3.37	3.37	3.37	3.37
Cov: Subject (Intercept) HarmonyI-V-IV	0.26	0.29	0.26	0.27	0.27
Cov: Subject (Intercept) HarmonyI-V-VI	-0.38	0.00	-0.38	-0.31	-0.27
Cov: Subject (Intercept) HarmonyIV-I-V	-0.02	0.03	-0.04	-0.03	-0.04
Cov: Subject (Intercept) Instrumentpiano	-0.67	-0.64	-0.81	-0.82	-0.78
Cov: Subject (Intercept) Instrumentstring	-1.41	-1.34	-1.64	-1.57	-1.57
Cov: Subject HarmonyI-V-IV HarmonyI-V-VI	0.07	0.05	0.06	0.06	0.05
Cov: Subject HarmonyI-V-IV HarmonyIV-I-V	0.00	0.01	0.00	-0.00	-0.00
Cov: Subject HarmonyI-V-IV Instrumentpiano	-0.20	-0.21	-0.20	-0.19	-0.21
Cov: Subject HarmonyI-V-IV Instrumentstring	-0.22	-0.25	-0.23	-0.23	-0.23
Cov: Subject HarmonyI-V-VI HarmonyIV-I-V	-0.00	-0.05	0.02	-0.01	-0.01
Cov: Subject HarmonyI-V-VI Instrumentpiano	-0.38	-0.51	-0.28	-0.36	-0.46
Cov: Subject HarmonyI-V-VI Instrumentstring	-0.97	-1.18	-0.70	-0.98	-1.01
Cov: Subject HarmonyIV-I-V Instrumentpiano	-0.01	-0.03	-0.02	0.00	-0.02
Cov: Subject HarmonyIV-I-V Instrumentstring	0.05	0.02	0.04	0.05	0.05
Cov: Subject Instrumentpiano Instrumentstring	1.54	1.54	1.54	1.54	1.54
Var: Residual	2.33	2.33	2.33	2.33	2.33

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 5: Variance and Covariance of Random Effects in Classical Models

	Classical	Popular
AIC	9703.52	9902.22
BIC	9947.47	10100.03
Log Likelihood	-4809.76	-4917.11
Num. obs.	2461	2485
Num. groups: Subject	69	70
Var: Subject (Intercept)	2.02	1.29
Var: Subject HarmonyI-V-IV	0.05	0.07
Var: Subject HarmonyI-V-VI	1.37	0.82
Var: Subject HarmonyIV-I-V	0.00	0.21
Var: Subject Instrumentpiano	1.67	1.38
Var: Subject Instrumentstring	3.38	3.32
Cov: Subject (Intercept) HarmonyI-V-IV	0.29	0.10
Cov: Subject (Intercept) HarmonyI-V-VI	-0.00	-0.05
Cov: Subject (Intercept) HarmonyIV-I-V	0.02	-0.14
Cov: Subject (Intercept) Instrumentpiano	-0.64	-0.21
Cov: Subject (Intercept) Instrumentstring	-1.34	-0.59
Cov: Subject HarmonyI-V-IV HarmonyI-V-VI	0.05	-0.06
Cov: Subject HarmonyI-V-IV HarmonyIV-I-V	0.01	-0.07
Cov: Subject HarmonyI-V-IV Instrumentpiano	-0.22	-0.08
Cov: Subject HarmonyI-V-IV Instrumentstring	-0.25	-0.19
Cov: Subject HarmonyI-V-VI HarmonyIV-I-V	-0.05	-0.14
Cov: Subject HarmonyI-V-VI Instrumentpiano	-0.51	-0.27
Cov: Subject HarmonyI-V-VI Instrumentstring	-1.20	-0.48
Cov: Subject HarmonyIV-I-V Instrumentpiano	-0.03	-0.08
Cov: Subject HarmonyIV-I-V Instrumentstring	0.02	-0.05
Cov: Subject Instrumentpiano Instrumentstring	1.55	1.55
Var: Residual	2.32	2.45

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 6: Variance and Covariance of Random Effects in Final Classical and Popular Models

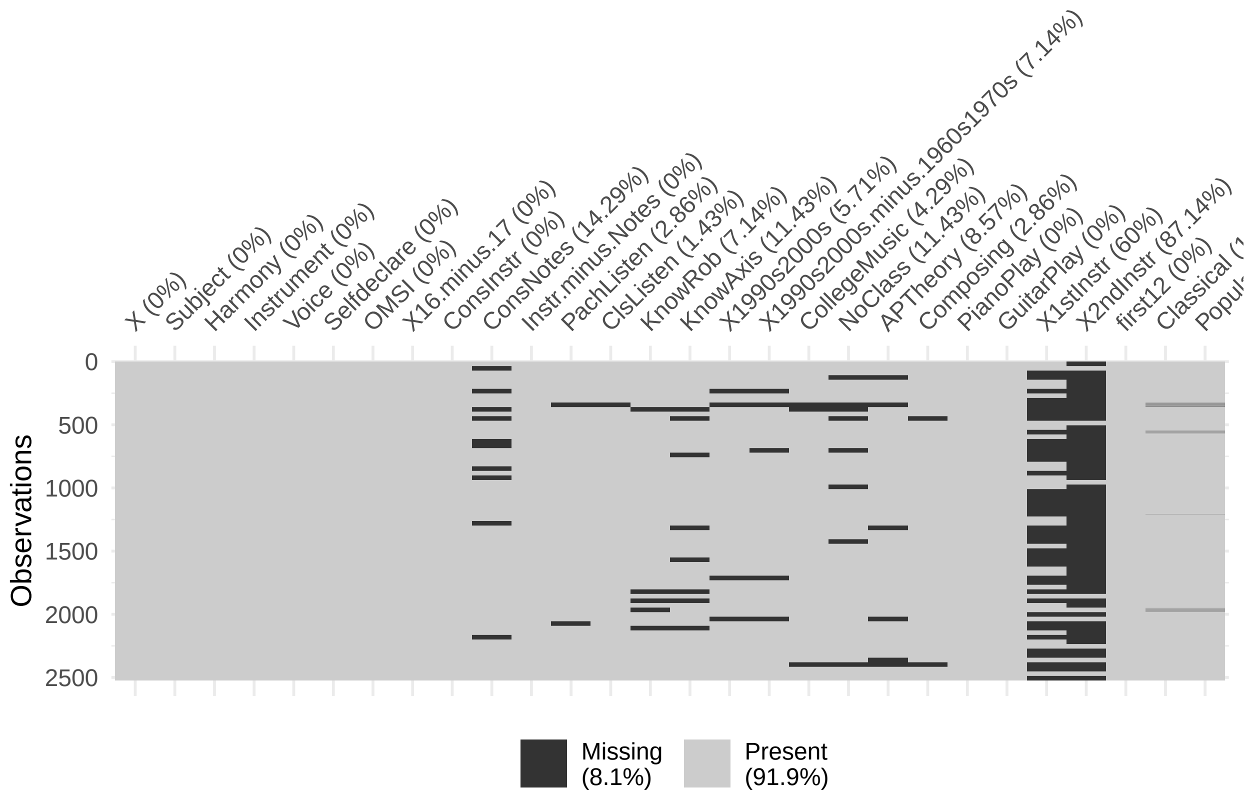
Code Appendix

andrew Follmann

12/08/2019

Data Overview

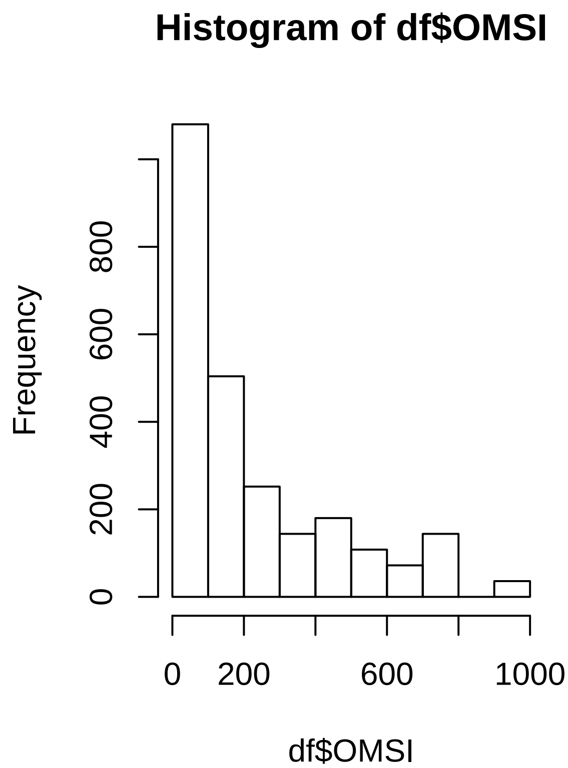
```r
vis_miss(df)
```

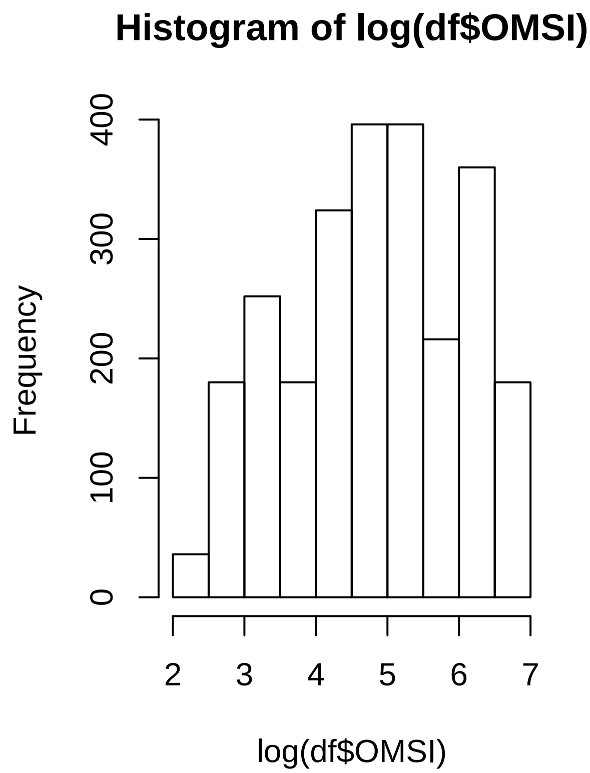


Variable Transformation

Histrogram of OMSI

```r
par(mfrow=c(1,2))
hist(df$OMSI)
hist(log(df$OMSI))
```





Data Cleaning

```
df <- subset(df, Classical>0, Popular>0)
df <- df[, !(names(df) %in% c('X1stInstr','X2ndInstr'))]
df$Classical <- ifelse(df$Classical == 19, 10, df$Classical)
df$Popular <- ifelse(df$Popular == 19, 10, df$Popular)
df$OMSI <- log(df$OMSI)
df$Musician <- (df$Selfdeclare > 1 ) + 0
```

Imputation

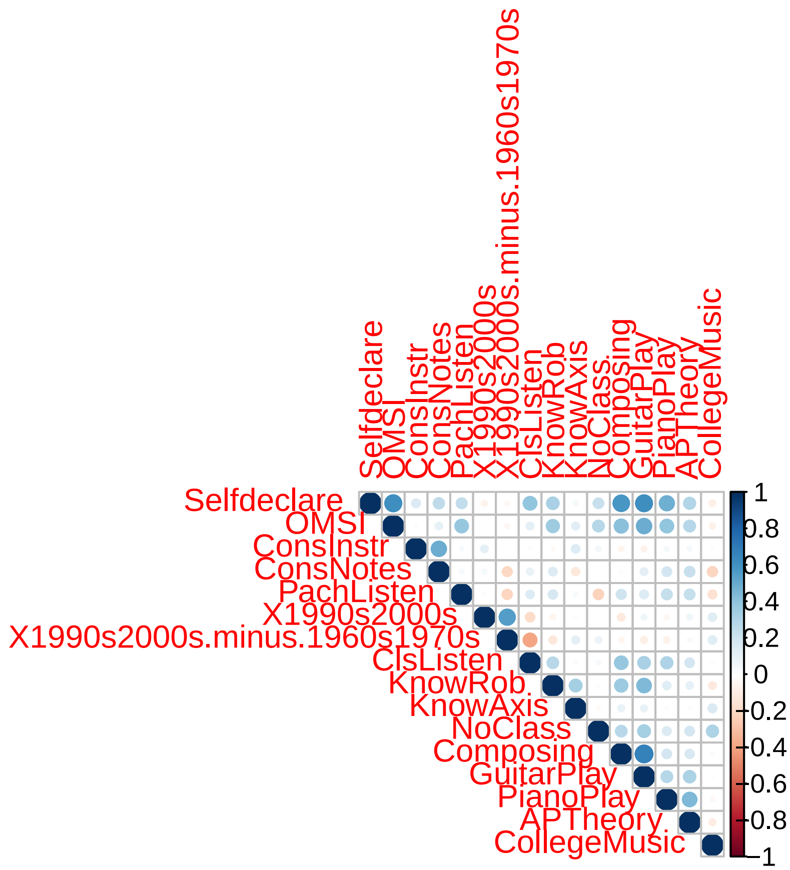
```
Mode <- function(x) {
  ux <- unique(x)
  ux[which.max(tabulate(match(x, ux)))]
}

df_orig <- df
# in our automatic techniques to find a model which maximizes
df[is.na(df$ConsNotes),'ConsNotes'] <- mean(df[!is.na(df$ConsNotes),'ConsNotes'])
df[is.na(df$PachListen),'PachListen'] <- mean(df[!is.na(df$PachListen),'PachListen'])
df[is.na(df$KnowRob),'KnowRob'] <- mean(df[!is.na(df$KnowRob),'KnowRob'])
df[is.na(df$KnowAxis),'KnowAxis'] <- mean(df[!is.na(df$KnowAxis),'KnowAxis'])
df[is.na(df$ClsListen),'ClsListen'] <- mean(df[!is.na(df$ClsListen),'ClsListen'])
df[is.na(df$NoClass),'NoClass'] <- mean(df[!is.na(df$NoClass),'NoClass'])
df[is.na(df$Composing),'Composing'] <- mean(df[!is.na(df$Composing),'Composing'])
df[is.na(df$X1990s2000s),'X1990s2000s'] <- mean(df[!is.na(df$X1990s2000s),'X1990s2000s'])
```

```r
df[is.na(df$X1990s2000s.minus.1960s1970s),'X1990s2000s.minus.1960s1970s'] <-
  mean(df[!is.na(df$X1990s2000s.minus.1960s1970s),'X1990s2000s.minus.1960s1970s'])
df[is.na(df$CollegeMusic),'CollegeMusic'] <- Mode(df[!is.na(df$CollegeMusic),'CollegeMusic'])
df[is.na(df$APTheory),'APTheory'] <- Mode(df[!is.na(df$APTheory),'APTheory'])
```

Correlations

```r
corrplot::corrplot(cor(as.matrix(df[,c(potential_num_vars, potential_factor_vars)])), type='upper')
```



Classical Model

Elementary Model

```r
lm_base <- lm(Classical ~ Harmony + Instrument + Voice, data=df)
lm_har_inst_int <- lm(Classical ~ Harmony * Instrument + Voice, data=df)
lm_har_voice_int <- lm(Classical ~ Harmony * Voice + Instrument, data=df)
lm_inst_voice_int <- lm(Classical ~ Harmony +  Instrument * Voice, data=df)
lm_full_int <- lm(Classical ~ Harmony * Instrument * Voice, data=df)
anova(lm_base, lm_har_inst_int,  lm_inst_voice_int,lm_har_voice_int,  lm_full_int)
```

```
## Analysis of Variance Table
##
## Model 1: Classical ~ Harmony + Instrument + Voice
## Model 2: Classical ~ Harmony * Instrument + Voice
## Model 3: Classical ~ Harmony + Instrument * Voice
## Model 4: Classical ~ Harmony * Voice + Instrument
```

```
## Model 5: Classical ~ Harmony * Instrument * Voice
##   Res.Df   RSS Df Sum of Sq      F    Pr(>F)
## 1   2477 12822
## 2   2471 12811  6    10.836 0.3498 0.9103051
## 3   2473 12813 -2    -2.166 0.2098 0.8107841
## 4   2471 12728  2    85.187 8.2504 0.0002685 ***
## 5   2449 12643 22    84.787 0.7465 0.7937114
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
#anova -> lm_har_voice_int
AIC(lm_base, lm_har_inst_int, lm_har_voice_int, lm_inst_voice_int, lm_full_int)
```

```
##                   df      AIC
## lm_base            9 11147.70
## lm_har_inst_int   15 11157.60
## lm_har_voice_int  15 11141.44
## lm_inst_voice_int 13 11154.02
## lm_full_int       37 11168.83
```

```r
BIC(lm_base, lm_har_inst_int, lm_har_voice_int, lm_inst_voice_int, lm_full_int)
```

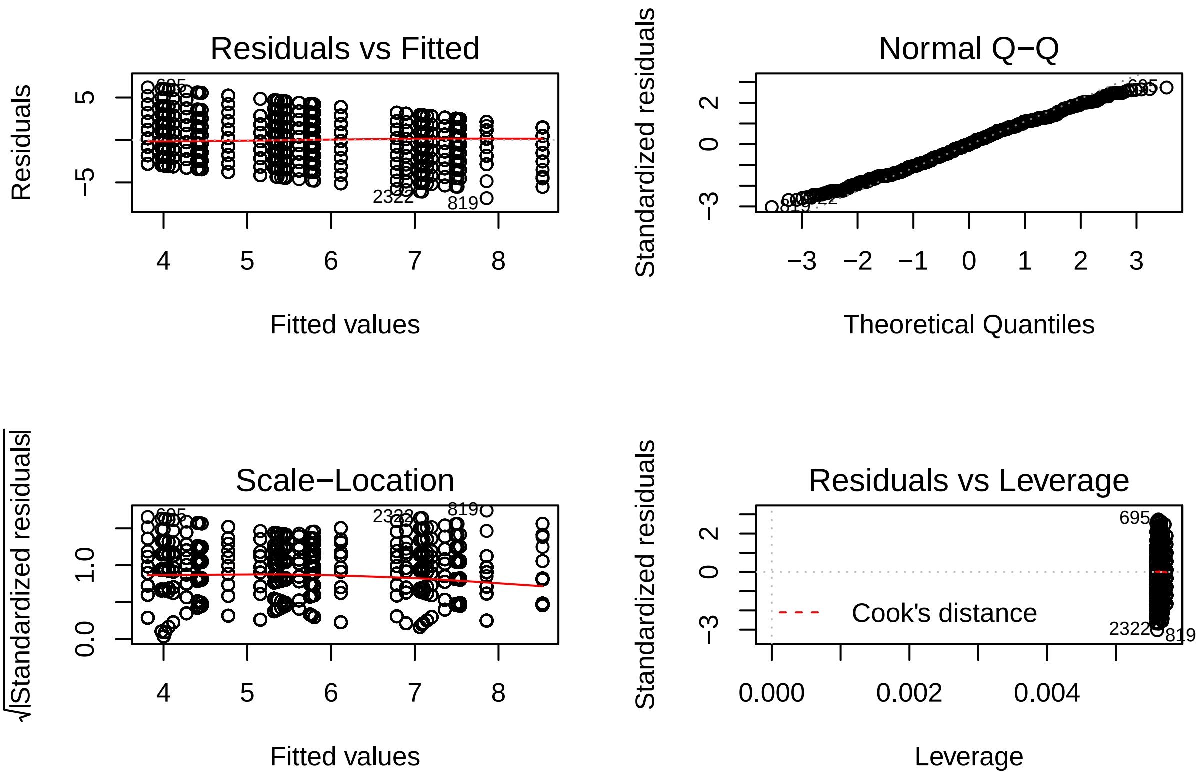
```
##                   df      BIC
## lm_base            9 11200.06
## lm_har_inst_int   15 11244.87
## lm_har_voice_int  15 11228.71
## lm_inst_voice_int 13 11229.65
## lm_full_int       37 11384.10
```

```r
lm_chosen <- lm_har_voice_int
```

Our AIC metrics supports the model with the voice and harmony interaction, while the BIC metric supports the base model.

Given the ease in interpreting the interaction model, and the AIC support I choose it.

```r
par(mfrow=c(2,2))
plot(lm_chosen)
```




```r
summary(lm_chosen)
```

```
##
## Call:
## lm(formula = Classical ~ Harmony * Voice + Instrument, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.858 -1.748 -0.019  1.653  6.189
##
## Coefficients:
##                             Estimate Std. Error t value Pr(>|t|)
## (Intercept)                  4.27412    0.16964  25.195  < 2e-16 ***
## HarmonyI-V-IV                0.15297    0.22282   0.686   0.4925
## HarmonyI-V-VI                1.16683    0.22282   5.237 1.77e-07 ***
## HarmonyIV-I-V               -0.15734    0.22228  -0.708   0.4791
## Voicepar3rd                 -0.25514    0.22228  -1.148   0.2511
## Voicepar5th                 -0.21305    0.22282  -0.956   0.3391
## Instrumentpiano              1.34427    0.11192  12.011  < 2e-16 ***
## Instrumentstring             3.08506    0.11121  27.740  < 2e-16 ***
## HarmonyI-V-IV:Voicepar3rd -0.36045    0.31493  -1.145   0.2525
## HarmonyI-V-VI:Voicepar3rd -0.72959    0.31531  -2.314   0.0208 *
## HarmonyIV-I-V:Voicepar3rd  0.54191    0.31532   1.719   0.0858 .
## HarmonyI-V-IV:Voicepar5th -0.21096    0.31550  -0.669   0.5038
## HarmonyI-V-VI:Voicepar5th -0.45486    0.31531  -1.443   0.1493
## HarmonyIV-I-V:Voicepar5th  0.07209    0.31474   0.229   0.8188
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.27 on 2471 degrees of freedom
## Multiple R-squared:  0.2587, Adjusted R-squared:  0.2548
## F-statistic: 66.33 on 13 and 2471 DF,  p-value: < 2.2e-16
```

In the models, it appears that the Harmonic motion from I - V - VI is associated with a higher classical rating than the base case of I - IV - V, affirming our main hypothesis. In addition, if the primary instrument is the piano, the person is more likely to associate the music as classical. The same is true for strings, to an even greater extent. These fixed effects of instrument have the most extreme effect. This is as we were expecting, that instrument would have the greatest influence.

The 3rd and 5th voice montion may suggest non-classical ratings, as they both have negative coefficients (in comparison to the contray motion base case). This effect was significant in the base-level model, but when including an interaction of Voice and harmony they maintain their sign but lose their significance. The interraction between the I-V-VI harmonic motion and the competing 3rds voice category is significantly negative, possibly substaniating the researcher's hypothesis that this harmonic progression can have different strong associations depending on context.

Random Effects of Experimental Design Variables

Repeated Effects Model

```r
lm_subject_re <- lmer(Classical ~ Harmony * Voice + Instrument + 1+ (1|Subject),
                      data=df, REML=F, control = lmerControl(optimizer = 'bobyqa'))
summary(lm_subject_re)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: Classical ~ Harmony * Voice + Instrument + 1 + (1 | Subject)
##    Data: df
## Control: lmerControl(optimizer = "bobyqa")
##
##      AIC      BIC   logLik deviance df.resid
##  10358.5  10451.6  -5163.2  10326.5     2469
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.9269 -0.6276 -0.0148  0.6482  3.9539
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  Subject  (Intercept) 1.664    1.290
##  Residual             3.442    1.855
## Number of obs: 2485, groups:  Subject, 70
##
## Fixed effects:
##                          Estimate Std. Error t value
## (Intercept)               4.27872    0.20737  20.634
## HarmonyI-V-IV             0.14888    0.18219   0.817
## HarmonyI-V-VI             1.16302    0.18218   6.384
## HarmonyIV-I-V            -0.15585    0.18171  -0.858
## Voicepar3rd              -0.25787    0.18173  -1.419
## Voicepar5th              -0.21522    0.18217  -1.181
## Instrumentpiano           1.34423    0.09160  14.675
## Instrumentstring          3.08259    0.09098  33.882
```

```
## HarmonyI-V-IV:Voicepar3rd -0.35215    0.25750  -1.368
## HarmonyI-V-VI:Voicepar3rd -0.72715    0.25778  -2.821
## HarmonyIV-I-V:Voicepar3rd  0.54205    0.25777   2.103
## HarmonyI-V-IV:Voicepar5th -0.21259    0.25792  -0.824
## HarmonyI-V-VI:Voicepar5th -0.44740    0.25780  -1.735
## HarmonyIV-I-V:Voicepar5th  0.06858    0.25731   0.267
```

```
##
## Correlation matrix not shown by default, as p = 14 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it
```

```r
exactLRT(lm_subject_re,lm_chosen)
```

```
## No restrictions on fixed effects. REML-based inference preferable.
```

```
##
##  simulated finite sample distribution of LRT. (p-value based on
##  10000 simulated values)
##
## data:
## LRT = 784.95, p-value < 2.2e-16
```

Using the Exact Log Ratio Test, we can see that our new model is significantly different from the OLS model, an improvement on our model without Random Effects.

All of the major effects identified remain, and increase in significance from our non-Random Effects model. The numerical point estimates are fairly stable, but the significance levels change a bit, with the String and Piano decreasing in significance (though still very significant). The Harmony and Voice variables, on the other hand, increase in significance. These effects on standard error reflect the restructuring of the variance into multiple components.

Full Random Effects Identification

```r
lm_subject_hvi_re <- lmer(
  Classical ~  1 + Harmony * Voice + Instrument +
    (1 + Harmony + Voice + Instrument|Subject), data=df,REML=F,
  control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```r
# boundary issue


lm_subject_vi_re <- lmer(
  Classical ~  1 + Harmony * Voice + Instrument +
    (1 +  Voice + Instrument|Subject), data=df,REML=F,
  control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```r
lm_subject_hi_re <- lmer(
  Classical ~  1 + Harmony * Voice + Instrument +
    (1 + Harmony + Instrument|Subject), data=df,REML=F,
  control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```r
lm_subject_hv_re <- lmer(
  Classical ~  1 + Harmony * Voice + Instrument +
    (1 + Harmony + Voice |Subject), data=df,REML=F,
  control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```r
lm_subject_h_re <- lmer(
  Classical ~  1 + Harmony * Voice + Instrument +
    (1 + Harmony  |Subject), data=df,REML=F,
  control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```r
lm_subject_v_re <- lmer(
  Classical ~  1 + Harmony * Voice + Instrument +
    (1 + Voice |Subject), data=df,REML=F,
  control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```r
lm_subject_i_re <- lmer(
  Classical ~  1 + Harmony * Voice + Instrument +
    (1 + Instrument |Subject), data=df,REML=F,
  control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))


# lm_subject_vi_re fails to converge
anova(lm_subject_re, lm_subject_hvi_re, lm_subject_vi_re, lm_subject_hi_re,
      lm_subject_hv_re, lm_subject_h_re, lm_subject_v_re, lm_subject_i_re)
```

```
## Data: df
## Models:
## lm_subject_re: Classical ~ Harmony * Voice + Instrument + 1 + (1 | Subject)
## lm_subject_v_re: Classical ~ 1 + Harmony * Voice + Instrument + (1 + Voice | Subject)
## lm_subject_i_re: Classical ~ 1 + Harmony * Voice + Instrument + (1 + Instrument |
## lm_subject_i_re:     Subject)
## lm_subject_h_re: Classical ~ 1 + Harmony * Voice + Instrument + (1 + Harmony |
## lm_subject_h_re:     Subject)
## lm_subject_vi_re: Classical ~ 1 + Harmony * Voice + Instrument + (1 + Voice + Instrument |
## lm_subject_vi_re:     Subject)
## lm_subject_hi_re: Classical ~ 1 + Harmony * Voice + Instrument + (1 + Harmony +
## lm_subject_hi_re:     Instrument | Subject)
## lm_subject_hv_re: Classical ~ 1 + Harmony * Voice + Instrument + (1 + Harmony +
## lm_subject_hv_re:     Voice | Subject)
## lm_subject_hvi_re: Classical ~ 1 + Harmony * Voice + Instrument + (1 + Harmony +
## lm_subject_hvi_re:     Voice + Instrument | Subject)
##                   Df     AIC   BIC  logLik deviance    Chisq Chi Df
## lm_subject_re     16 10358.5 10452 -5163.2  10326.5
## lm_subject_v_re   21 10368.4 10491 -5163.2  10326.4   0.0428      5
## lm_subject_i_re   21  9985.3 10107 -4971.6   9943.3 383.1919      0
## lm_subject_h_re   25 10268.3 10414 -5109.2  10218.3   0.0000      4
## lm_subject_vi_re  30  9993.8 10168 -4966.9   9933.8 284.5367      5
## lm_subject_hi_re  36  9824.9 10034 -4876.5   9752.9 180.8480      6
## lm_subject_hv_re  36 10282.7 10492 -5105.4  10210.7   0.0000      0
## lm_subject_hvi_re 51  9835.1 10132 -4866.6   9733.1 477.5782     15
```

```
##                    Pr(>Chisq)
## lm_subject_re
## lm_subject_v_re            1
## lm_subject_i_re       <2e-16 ***
## lm_subject_h_re            1
## lm_subject_vi_re      <2e-16 ***
## lm_subject_hi_re      <2e-16 ***
## lm_subject_hv_re           1
## lm_subject_hvi_re     <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The Manual tests suggest to use random effects for the slope of harmony, and instrument, but not necessarily voice.

Lets corroborate these with automatic methods.

```
re_autofit_aic <- fitLMER.fnc(lm_subject_re,ran.effects=c("(0 + Harmony|Subject)",
                                                          "(0 +  Instrument|Subject)",
                                                          '(0 +  Voice|Subject)',
                                                          '(0 + Instrument + Voice|Subject)',
                                                          '(0 + Harmony + Voice|Subject)'
), method="AIC", set.REML.FALSE=T)
re_autofit_bic <- fitLMER.fnc(lm_subject_re,ran.effects=c("(0 + Harmony|Subject)",
                                                          "(0 + Instrument|Subject)",
                                                          '(0 + Voice|Subject)',
                                                          '(0 + Instrument + Voice|Subject)',
                                                          '(0 + Harmony + Voice|Subject)'
),method="BIC", set.REML.FALSE=T)
anova(lm_subject_re, lm_subject_hi_re,lm_subject_hvi_re)
```

We tried to use the FitLMER function to confirm our manual approach, but the computation failed and we continue with our main effects. The manual methods were fairly exhaustive so we are confident with the selected random effects.

```
AIC(lm_subject_re, lm_subject_h_re, lm_subject_v_re, lm_subject_i_re,
    lm_subject_hv_re, lm_subject_hi_re,   lm_subject_hvi_re)
```

```
##                   df       AIC
## lm_subject_re     16 10358.490
## lm_subject_h_re   25 10268.332
## lm_subject_v_re   21 10368.447
## lm_subject_i_re   21  9985.255
## lm_subject_hv_re  36 10282.704
## lm_subject_hi_re  36  9824.948
## lm_subject_hvi_re 51  9835.126
```

```
BIC(lm_subject_re, lm_subject_h_re, lm_subject_v_re, lm_subject_i_re,
    lm_subject_hv_re, lm_subject_hi_re,   lm_subject_hvi_re)
```

```
##                   df      BIC
## lm_subject_re     16 10451.58
## lm_subject_h_re   25 10413.78
## lm_subject_v_re   21 10490.63
## lm_subject_i_re   21 10107.43
## lm_subject_hv_re  36 10492.15
## lm_subject_hi_re  36 10034.40
```

```
## lm_subject_hvi_re 51 10131.85
```

From our AIC and BIC tests of the MLE fit random effects models, we find that the Harmony and Instrument variables minimize the AIC and BIC, as shown before.

```
model_final_p2 <- lm_subject_hi_re
```

Now that our random effects have stablized, we can consider new fixed effect interactions.

```
lmer_check_fe_noint <- lmer(Classical ~  1 + Harmony + Voice + Instrument +
                              (1 + Harmony + Instrument|Subject), data=df,REML=F
                            ,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
lmer_check_fe_allint <- lmer(Classical ~  1 * Harmony * Instrument * Voice +
                               (1 + Harmony + Instrument|Subject), data=df,REML=F,
                             control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
anova(lm_subject_re,  lmer_check_fe_noint, lmer_check_fe_allint, model_final_p2)
```

```
## Data: df
## Models:
## lm_subject_re: Classical ~ Harmony * Voice + Instrument + 1 + (1 | Subject)
## lmer_check_fe_allint: Classical ~ 1 * Harmony * Instrument * Voice + (1 + Harmony +
## lmer_check_fe_allint:     Instrument | Subject)
## lmer_check_fe_noint: Classical ~ 1 + Harmony + Voice + Instrument + (1 + Harmony +
## lmer_check_fe_noint:     Instrument | Subject)
## model_final_p2: Classical ~ 1 + Harmony * Voice + Instrument + (1 + Harmony +
## model_final_p2:     Instrument | Subject)
##                      Df     AIC   BIC  logLik deviance   Chisq Chi Df
## lm_subject_re        16 10358.5 10452 -5163.2  10326.5
## lmer_check_fe_allint 23  9981.6 10116 -4967.8   9935.6 390.842      7
## lmer_check_fe_noint  30  9851.9 10026 -4896.0   9791.9 143.721      7
## model_final_p2       36  9824.9 10034 -4876.5   9752.9  38.978      6
##                      Pr(>Chisq)
## lm_subject_re
## lmer_check_fe_allint  < 2.2e-16 ***
## lmer_check_fe_noint   < 2.2e-16 ***
## model_final_p2        7.227e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Finally, in re-evaluating alternate interaction scenario, we are confirmed in our evaluation of our deduced model. The AIC and BIC are minimized simultaneously with out previsouly selected model, using the Harmony and Voice interaction, and the Harmony and Instrument Random Effects.

```
summary(model_final_p2)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: Classical ~ 1 + Harmony * Voice + Instrument + (1 + Harmony +
##     Instrument | Subject)
##    Data: df
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   9824.9  10034.4  -4876.5   9752.9     2449
##
## Scaled residuals:
```

```
##     Min      1Q  Median      3Q     Max
## -4.7616 -0.5770  0.0233  0.5754  4.1643
##
## Random effects:
##  Groups   Name             Variance Std.Dev. Corr
##  Subject  (Intercept)      2.529831 1.59054
##           HarmonyI-V-IV    0.040633 0.20158   0.82
##           HarmonyI-V-VI    1.608778 1.26838  -0.08  0.25
##           HarmonyIV-I-V    0.002713 0.05209  -0.19  0.19 -0.05
##           Instrumentpiano  1.654233 1.28617  -0.39 -0.77 -0.23 -0.19
##           Instrumentstring 3.367450 1.83506  -0.56 -0.61 -0.42  0.57  0.65
##  Residual                  2.335693 1.52830
## Number of obs: 2485, groups:  Subject, 70
##
## Fixed effects:
##                            Estimate Std. Error t value
## (Intercept)                 4.27849    0.22183  19.287
## HarmonyI-V-IV               0.15565    0.15205   1.024
## HarmonyI-V-VI               1.15935    0.21342   5.432
## HarmonyIV-I-V              -0.15361    0.14983  -1.025
## Voicepar3rd                -0.24714    0.14976  -1.650
## Voicepar5th                -0.21365    0.15012  -1.423
## Instrumentpiano             1.33775    0.17147   7.802
## Instrumentstring            3.07841    0.23183  13.279
## HarmonyI-V-IV:Voicepar3rd -0.36692    0.21222  -1.729
## HarmonyI-V-VI:Voicepar3rd -0.72249    0.21250  -3.400
## HarmonyIV-I-V:Voicepar3rd  0.53380    0.21241   2.513
## HarmonyI-V-IV:Voicepar5th -0.21088    0.21252  -0.992
## HarmonyI-V-VI:Voicepar5th -0.44801    0.21256  -2.108
## HarmonyIV-I-V:Voicepar5th  0.07357    0.21207   0.347

##
## Correlation matrix not shown by default, as p = 14 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it

## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

Our final model including only the experimental terms and their random effects is above.

It has much the same interpreation as the previous models. There is a positive effect of Instrument Piano or String (over guitar). There is also a significant positive effect for the Pachelbel harmony progression in the elementary case, but this effect inverts when paired with non-contrary voice motions.

The random effects variance are quite high, for the most part given their point estimates. This facilitates a pretty wide range for the set coefficient esitmates. All three of the statistically significant coefficients from the model have estimated variances greater than their point estimates. This implies that the draw that each subject coefficient estimate represents is from a relatiely wide distribution, such that the effective number of variables is behaving relatively like a general fixed effect model.

Additional Subject Level Fixed Effects

In our base level we will use the OLS linear model, without the random effects to efficiently heuristically search the parameter space using the stepAIC function. Then once we have identified a set of meaningful variables, we will manually backtrack through these models, until we find a model which is equal to, improves upon our model using only the experimental variables.

StepAIC without random effects to determine potentially valuable variables

```
lm_stepaic_base <- lm(Classical ~ Harmony*Voice + Instrument, data=df)

step_aic<-stepAIC(lm_stepaic_base, eval(paste('~ . +', paste(c(potential_num_vars, potential_factor_vars
                                        sep='')), trace=0)

names(coefficients(step_aic))
```

```
##  [1] "(Intercept)"                  "HarmonyI-V-IV"
##  [3] "HarmonyI-V-VI"                "HarmonyIV-I-V"
##  [5] "Voicepar3rd"                  "Voicepar5th"
##  [7] "Instrumentpiano"              "Instrumentstring"
##  [9] "ClsListen"                    "Selfdeclare"
## [11] "Composing"                    "OMSI"
## [13] "X1990s2000s.minus.1960s1970s" "X1990s2000s"
## [15] "APTheory"                     "ConsInstr"
## [17] "NoClass"                      "PianoPlay"
## [19] "KnowAxis"                     "HarmonyI-V-IV:Voicepar3rd"
## [21] "HarmonyI-V-VI:Voicepar3rd"    "HarmonyIV-I-V:Voicepar3rd"
## [23] "HarmonyI-V-IV:Voicepar5th"    "HarmonyI-V-VI:Voicepar5th"
## [25] "HarmonyIV-I-V:Voicepar5th"
```

Working backwards from saturated model

```
full_model_v1 <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                        Composing + X1990s2000s.minus.1960s1970s + X1990s2000s + PachListen +
                        ConsInstr +APTheory+NoClass+KnowAxis+PianoPlay +
                        (1 + Harmony +  Instrument | Subject), data=df, REML=F,
                      lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
full_model_v2 <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                        Composing + X1990s2000s.minus.1960s1970s + X1990s2000s + PachListen +
                        ConsInstr +APTheory+NoClass+PianoPlay + (1 + Harmony +  Instrument | Subject),
                      data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
full_model_v3 <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                        Composing + X1990s2000s.minus.1960s1970s + X1990s2000s + ConsInstr +
                        APTheory+NoClass+PianoPlay + (1 + Harmony +  Instrument | Subject),
                      data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
full_model_v4 <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                        Composing + X1990s2000s.minus.1960s1970s + X1990s2000s +
                        APTheory+NoClass+PianoPlay + (1 + Harmony +  Instrument | Subject),
                      data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
full_model_v5 <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                        Composing + X1990s2000s.minus.1960s1970s + X1990s2000s + APTheory+NoClass+
                        (1 + Harmony +  Instrument | Subject),
                      data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
full_model_v6 <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                        X1990s2000s.minus.1960s1970s + X1990s2000s + APTheory+NoClass+
                        (1 + Harmony +  Instrument | Subject),
                      data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
full_model_v7 <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                        X1990s2000s.minus.1960s1970s + X1990s2000s + APTheory+ (1 + Harmony +  Instrumen
                      data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
full_model_v8 <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                        X1990s2000s.minus.1960s1970s + X1990s2000s +  (1 + Harmony +  Instrument | Subje
                      data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
AIC(model_final_p2, full_model_v1, full_model_v2,full_model_v3, full_model_v4,
    full_model_v5, full_model_v6, full_model_v7, full_model_v8)
```

```
##                df      AIC
## model_final_p2 36 9824.948
## full_model_v1  48 9830.448
## full_model_v2  47 9828.538
## full_model_v3  46 9826.502
## full_model_v4  45 9824.883
## full_model_v5  44 9823.953
## full_model_v6  43 9823.081
## full_model_v7  42 9822.111
## full_model_v8  41 9822.577
```

```
BIC(model_final_p2, full_model_v1, full_model_v2,full_model_v3, full_model_v4,
    full_model_v5, full_model_v6, full_model_v7, full_model_v8)
```

```
##                df      BIC
## model_final_p2 36 10034.40
## full_model_v1  48 10109.71
## full_model_v2  47 10101.98
## full_model_v3  46 10094.13
## full_model_v4  45 10086.69
## full_model_v5  44 10079.95
## full_model_v6  43 10073.26
## full_model_v7  42 10066.47
## full_model_v8  41 10061.12
```

```
summary(full_model_v8)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula:
## Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare +
##     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
##     Harmony + Instrument | Subject)
##    Data: df
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
```

```
##
##      AIC      BIC   logLik deviance df.resid
##   9822.6  10061.1  -4870.3   9740.6     2444
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7469 -0.5780  0.0188  0.5728  4.0819
##
## Random effects:
##  Groups   Name             Variance Std.Dev. Corr
##  Subject  (Intercept)      2.254263 1.50142
##           HarmonyI-V-IV    0.049916 0.22342   0.79
##           HarmonyI-V-VI    1.607200 1.26775  -0.20  0.24
##           HarmonyIV-I-V    0.002686 0.05183  -0.22  0.19 -0.06
##           Instrumentpiano  1.654398 1.28623  -0.35 -0.69 -0.23 -0.20
##           Instrumentstring 3.369438 1.83560  -0.51 -0.54 -0.42  0.57  0.65
##  Residual                  2.334062 1.52776
## Number of obs: 2485, groups:  Subject, 70
##
## Fixed effects:
##                              Estimate Std. Error t value
## (Intercept)                   3.29032    0.74679   4.406
## HarmonyI-V-IV                 0.15574    0.15244   1.022
## HarmonyI-V-VI                 1.15941    0.21333   5.435
## HarmonyIV-I-V                -0.15391    0.14978  -1.028
## Voicepar3rd                  -0.24732    0.14971  -1.652
## Voicepar5th                  -0.21401    0.15006  -1.426
## Instrumentpiano               1.33786    0.17147   7.803
## Instrumentstring              3.07851    0.23189  13.276
## ClsListen                     0.33603    0.10593   3.172
## Selfdeclare                  -0.32714    0.16636  -1.966
## OMSI                          0.32433    0.15338   2.115
## X1990s2000s.minus.1960s1970s  0.21706    0.09782   2.219
## X1990s2000s                  -0.23223    0.10995  -2.112
## HarmonyI-V-IV:Voicepar3rd    -0.36677    0.21215  -1.729
## HarmonyI-V-VI:Voicepar3rd    -0.72227    0.21242  -3.400
## HarmonyIV-I-V:Voicepar3rd     0.53405    0.21233   2.515
## HarmonyI-V-IV:Voicepar5th    -0.21063    0.21245  -0.991
## HarmonyI-V-VI:Voicepar5th    -0.44808    0.21248  -2.109
## HarmonyIV-I-V:Voicepar5th     0.07434    0.21200   0.351

##
## Correlation matrix not shown by default, as p = 19 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it

## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
model_final_fixed_effect_p3_test <- full_model_v8
```

Reevaluate Random Effects

Now that we have settled into our Fixed effects model, lets see the optimal the random effects.

```r
re_p3_base <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                    X1990s2000s.minus.1960s1970s + X1990s2000s +  (1| Subject),
                  data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))

re_p3_i <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                  X1990s2000s.minus.1960s1970s + X1990s2000s +  (1 + Instrument| Subject),
                data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))

re_p3_v <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                  X1990s2000s.minus.1960s1970s + X1990s2000s +  (1 + Voice| Subject),
                data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```r
re_p3_h <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                  X1990s2000s.minus.1960s1970s + X1990s2000s +  (1 + Harmony| Subject) ,
                data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```r
re_p3_hi <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                   X1990s2000s.minus.1960s1970s + X1990s2000s +  (1 + Harmony + Instrument| Subject),
                 data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```r
re_p3_vi <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                   X1990s2000s.minus.1960s1970s + X1990s2000s +  (1 + Voice + Instrument| Subject),
                 data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```r
re_p3_vh <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                   X1990s2000s.minus.1960s1970s + X1990s2000s +  (1 + Voice + Harmony| Subject),
                 data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```r
re_p3_vhi <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                    X1990s2000s.minus.1960s1970s + X1990s2000s +  (1 + Voice + Harmony + Instrument| Sul
                  data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```r
re_p3_vxh <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                    X1990s2000s.minus.1960s1970s + X1990s2000s +  (1 + Voice * Harmony| Subject),
                  data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## Warning in optwrap(optimizer, devfun, getStart(start, rho$lower, rho$pp), :
## convergence code 1 from bobyqa: bobyqa -- maximum number of function
## evaluations exceeded

## boundary (singular) fit: see ?isSingular
```

```r
re_p3_vxhi <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                     X1990s2000s.minus.1960s1970s + X1990s2000s +  (1 + Instrument + Voice * Harmony| Su
                   data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## Warning in commonArgs(par, fn, control, environment()): maxfun < 10 *
```

```
## length(par)^2 is not recommended.
```

```
## boundary (singular) fit: see ?isSingular
```

```r
anova(full_model_v8, re_p3_v, re_p3_h, re_p3_i,
      re_p3_vh, re_p3_hi, re_p3_vi, re_p3_vhi, re_p3_vxh, re_p3_vxhi)
```

```
## Data: df
## Models:
## re_p3_v: Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare +
## re_p3_v:     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
## re_p3_v:     Voice | Subject)
## re_p3_i: Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare +
## re_p3_i:     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
## re_p3_i:     Instrument | Subject)
## re_p3_h: Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare +
## re_p3_h:     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
## re_p3_h:     Harmony | Subject)
## re_p3_vi: Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare +
## re_p3_vi:     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
## re_p3_vi:     Voice + Instrument | Subject)
## full_model_v8: Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare +
## full_model_v8:     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
## full_model_v8:     Harmony + Instrument | Subject)
## re_p3_vh: Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare +
## re_p3_vh:     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
## re_p3_vh:     Voice + Harmony | Subject)
## re_p3_hi: Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare +
## re_p3_hi:     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
## re_p3_hi:     Harmony + Instrument | Subject)
## re_p3_vhi: Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare +
## re_p3_vhi:     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
## re_p3_vhi:     Voice + Harmony + Instrument | Subject)
## re_p3_vxh: Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare +
## re_p3_vxh:     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
## re_p3_vxh:     Voice * Harmony | Subject)
## re_p3_vxhi: Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare +
## re_p3_vxhi:     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
## re_p3_vxhi:     Instrument + Voice * Harmony | Subject)
##                Df     AIC   BIC  logLik deviance   Chisq Chi Df Pr(>Chisq)
## re_p3_v        26 10369.1 10520 -5158.5  10317.1
## re_p3_i        26  9986.4 10138 -4967.2   9934.4 382.639      0     <2e-16
## re_p3_h        30 10266.8 10441 -5103.4  10206.8   0.000      4      1.000
## re_p3_vi       35  9994.7 10198 -4962.4   9924.7 282.053      5     <2e-16
## full_model_v8  41  9822.6 10061 -4870.3   9740.6 184.160      6     <2e-16
## re_p3_vh       41 10287.0 10526 -5102.5  10205.0   0.000      0      1.000
## re_p3_hi       41  9822.6 10061 -4870.3   9740.6 464.438      0     <2e-16
## re_p3_vhi      56  9832.9 10159 -4860.4   9720.9  19.693     15      0.184
## re_p3_vxh      98 10366.8 10937 -5085.4  10170.8   0.000     42      1.000
## re_p3_vxhi    125  9891.6 10619 -4820.8   9641.6 529.247     27     <2e-16
##
## re_p3_v
## re_p3_i       ***
## re_p3_h
## re_p3_vi      ***
```

```
## full_model_v8 ***
## re_p3_vh
## re_p3_hi      ***
## re_p3_vhi
## re_p3_vxh
## re_p3_vxhi    ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Trying out alternate random effects of our primary experimental factors, and the interactions present in our model, we find the harmony and instrument random effects to create our best model, as before.

```
re_p3_hi_self <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                        X1990s2000s.minus.1960s1970s + X1990s2000s +  (1 + Harmony + Instrument + Selfde
                      data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
re_p3_hi_cls <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare + OMSI +
                       X1990s2000s.minus.1960s1970s + X1990s2000s +  (1 + Harmony + Instrument + ClsLis
                     data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## Warning in optwrap(optimizer, devfun, getStart(start, rho$lower, rho$pp), :
## convergence code 1 from bobyqa: bobyqa -- maximum number of function
## evaluations exceeded
```

```
## boundary (singular) fit: see ?isSingular
```

```
# p3_autofit <- fitLMER.fnc(base_re_p3, ran.effects=c("(0 + Harmony|Subject)",
#                                                      "(0 +  Instrument|Subject)",
#                                                      '(0 +  Voice|Subject)',
#                                                      '(0 + Instrument + Voice|Subject)',
#                                                      '(0 + Harmony + Voice|Subject)',
#                                                      '(0 + Harmony + Instrument|Subject)',
#                                                      '(0 + Harmony + Voice + Instrument|Subject)',
#                                                      '(0 + Harmony * Voice|Subject)'
#                                                      ), method="AIC", set.REML.FALSE=T)

anova(model_final_fixed_effect_p3_test, re_p3_hi_self, re_p3_hi_cls)
```

```
## Data: df
## Models:
## model_final_fixed_effect_p3_test: Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare
## model_final_fixed_effect_p3_test:     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
## model_final_fixed_effect_p3_test:     Harmony + Instrument | Subject)
## re_p3_hi_self: Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare +
## re_p3_hi_self:     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
## re_p3_hi_self:     Harmony + Instrument + Selfdeclare | Subject)
## re_p3_hi_cls: Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare +
## re_p3_hi_cls:     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
## re_p3_hi_cls:     Harmony + Instrument + ClsListen | Subject)
##                                  Df    AIC   BIC  logLik deviance  Chisq
## model_final_fixed_effect_p3_test 41 9822.6 10061 -4870.3   9740.6
## re_p3_hi_self                    48 9830.0 10109 -4867.0   9734.0 6.6226
## re_p3_hi_cls                     48 9832.6 10112 -4868.3   9736.6 0.0000
##                                  Chi Df Pr(>Chisq)
## model_final_fixed_effect_p3_test
```

```
## re_p3_hi_self                         7     0.4692
## re_p3_hi_cls                          0     1.0000
```

Trying out alternate random effects using the ordinal variables associated with a higher background level of music knowledge, of Classical Listen and Self Declared musicianship, we find that the base random effects perform best.

When attempting to use the automatic random effect fit methods we continue to run into timeouts and errors, so we forgo them. We were fairly exhaustive in our search, so we are confident in the results.

Final Classical Model (prior to Musician and Pachelbel)

```
model_final_fixed_effect_p3 <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Selfdeclare +
                        X1990s2000s.minus.1960s1970s + X1990s2000s +  (1 + Harmony +  Instrument | Subj
                      data=df_orig, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(model_final_fixed_effect_p3)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula:
## Classical ~ Harmony * Voice + Instrument + ClsListen + Selfdeclare +
##     OMSI + X1990s2000s.minus.1960s1970s + X1990s2000s + (1 +
##     Harmony + Instrument | Subject)
##    Data: df_orig
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   9182.8   9418.5  -4550.4   9100.8     2276
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7380 -0.5768  0.0193  0.5741  4.0620
##
## Random effects:
##  Groups   Name             Variance Std.Dev. Corr
##  Subject  (Intercept)      1.89594  1.37693
##           HarmonyI-V-IV    0.05875  0.24238   0.77
##           HarmonyI-V-VI    1.62253  1.27379  -0.16  0.23
##           HarmonyIV-I-V    0.00532  0.07294  -0.06  0.57  0.34
##           Instrumentpiano  1.55128  1.24550  -0.29 -0.69 -0.23 -0.72
##           Instrumentstring 3.30897  1.81906  -0.52 -0.51 -0.42 -0.02  0.62
##  Residual                  2.36409  1.53756
## Number of obs: 2317, groups:  Subject, 65
##
## Fixed effects:
##                               Estimate Std. Error t value
## (Intercept)                    3.21303    0.74135   4.334
## HarmonyI-V-IV                  0.23633    0.15921   1.484
## HarmonyI-V-VI                  1.19049    0.22250   5.351
## HarmonyIV-I-V                 -0.11991    0.15660  -0.766
## Voicepar3rd                   -0.20027    0.15618  -1.282
## Voicepar5th                   -0.17402    0.15657  -1.111
## Instrumentpiano                1.44738    0.17347   8.344
## Instrumentstring               3.21186    0.23878  13.451
```

```
## ClsListen                      0.24280    0.10590   2.293
## Selfdeclare                   -0.30995    0.16093  -1.926
## OMSI                           0.28039    0.14785   1.896
## X1990s2000s.minus.1960s1970s   0.13632    0.09647   1.413
## X1990s2000s                   -0.11234    0.11326  -0.992
## HarmonyI-V-IV:Voicepar3rd     -0.53553    0.22111  -2.422
## HarmonyI-V-VI:Voicepar3rd     -0.75564    0.22138  -3.413
## HarmonyIV-I-V:Voicepar3rd      0.50653    0.22142   2.288
## HarmonyI-V-IV:Voicepar5th     -0.27940    0.22127  -1.263
## HarmonyI-V-VI:Voicepar5th     -0.52239    0.22168  -2.357
## HarmonyIV-I-V:Voicepar5th      0.08074    0.22118   0.365
```

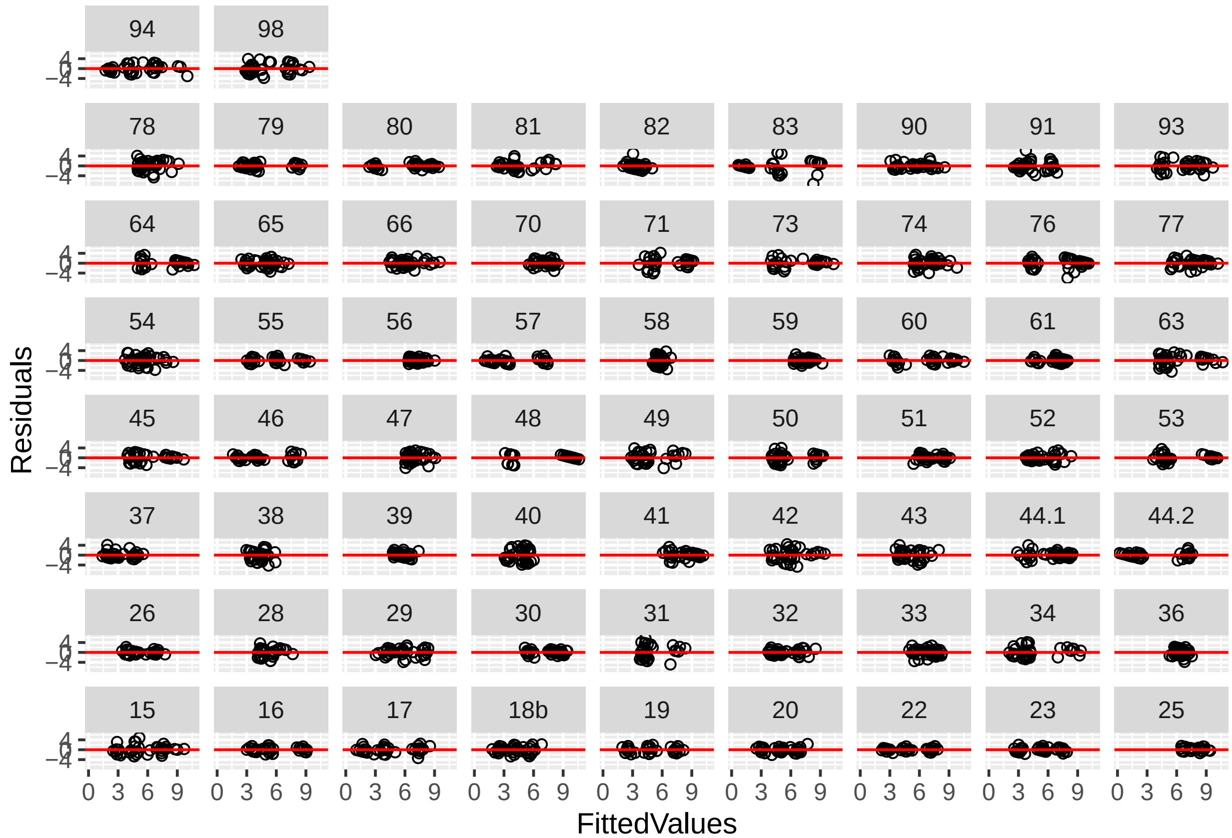
```
##
## Correlation matrix not shown by default, as p = 19 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it
```

```
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

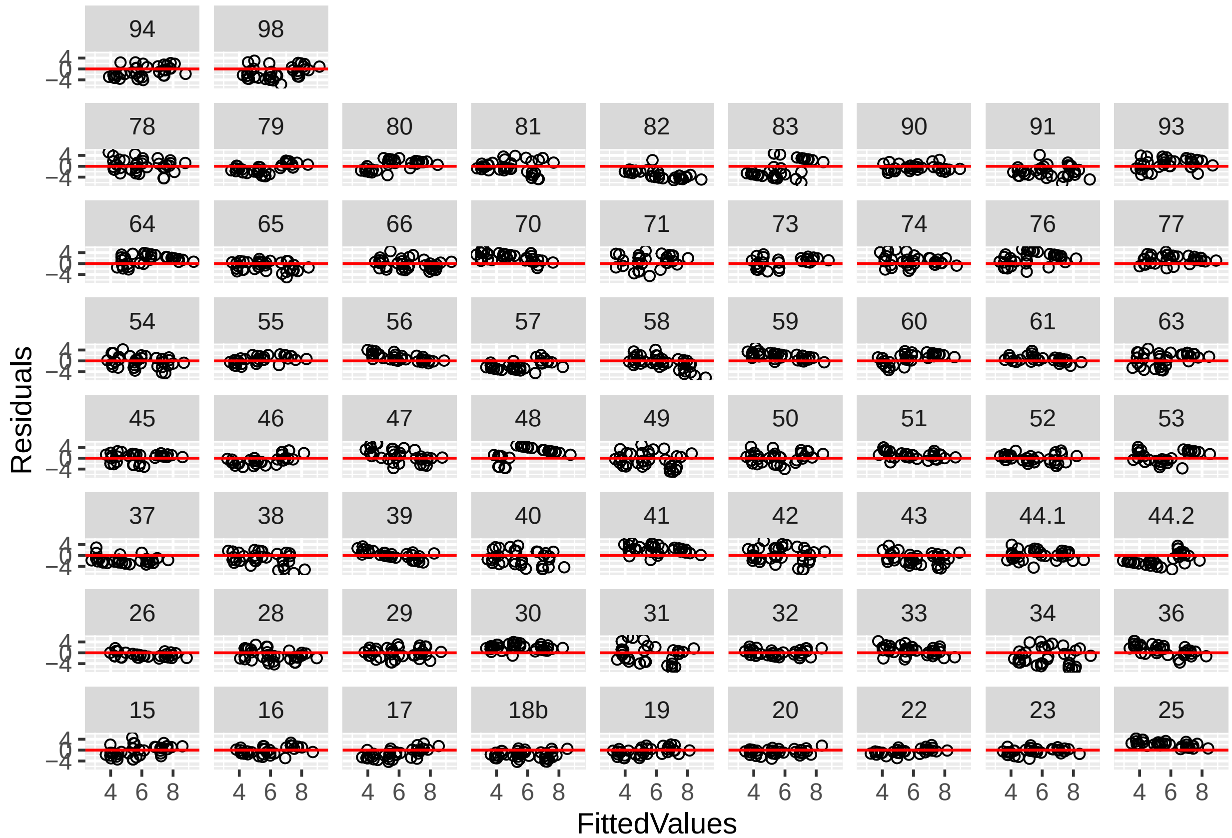
Residuals of Pre-Musician Classical Model

```
resid_marg <- data.frame(FittedValues=yhat.marg(model_final_fixed_effect_p3), Residuals=r.marg(model_fi
resid_cond <- data.frame(FittedValues=yhat.cond(model_final_fixed_effect_p3),
                         Residuals=r.cond(model_final_fixed_effect_p3), Subject= df_orig[!is.na(df_orig

ggplot(resid_cond, aes(x=FittedValues,y=Residuals)) +
  facet_wrap( ~ Subject, as.table=F) +   geom_point(pch=1) +
  geom_hline(yintercept=0, color='red')
```




```
ggplot(resid_marg, aes(x=FittedValues,y=Residuals)) +
  facet_wrap( ~ Subject, as.table=F) +   geom_point(pch=1) +
  geom_hline(yintercept=0, color='red')
```



Testing Musicianship in Classical Model

```
model3_musician_1 <- update(model_final_fixed_effect_p3_test, ~ . + Musician_1 - Selfdeclare) # best
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(model3_musician_1)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
##     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
##     Instrument | Subject) + Musician_1 + Harmony:Voice
##    Data: df
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   9823.3  10061.9  -4870.7   9741.3     2444
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7448 -0.5757  0.0207  0.5708  4.0603
##
## Random effects:
##  Groups   Name               Variance Std.Dev. Corr
##  Subject  (Intercept)        2.188351 1.47931
##           HarmonyI-V-IV      0.052486 0.22910   0.78
```

```
##           HarmonyI-V-VI    1.607541 1.26789  -0.21  0.24
##           HarmonyIV-I-V    0.002914 0.05398  -0.11  0.29 -0.05
##           Instrumentpiano  1.654235 1.28617  -0.32 -0.67 -0.23 -0.19
##           Instrumentstring 3.370236 1.83582  -0.49 -0.53 -0.42  0.55  0.65
##  Residual                  2.333802 1.52768
## Number of obs: 2485, groups:  Subject, 70
##
## Fixed effects:
##                             Estimate Std. Error t value
## (Intercept)                  3.72876    0.73771   5.055
## HarmonyI-V-IV                0.15574    0.15255   1.021
## HarmonyI-V-VI                1.15973    0.21333   5.436
## HarmonyIV-I-V               -0.15394    0.14978  -1.028
## Voicepar3rd                 -0.24730    0.14970  -1.652
## Voicepar5th                 -0.21405    0.15006  -1.426
## Instrumentpiano              1.33787    0.17146   7.803
## Instrumentstring             3.07846    0.23191  13.274
## ClsListen                    0.31086    0.10192   3.050
## OMSI                         0.17030    0.11995   1.420
## X1990s2000s.minus.1960s1970s  0.19563    0.09625   2.033
## X1990s2000s                 -0.20661    0.10966  -1.884
## Musician_1                  -0.64785    0.35766  -1.811
## HarmonyI-V-IV:Voicepar3rd   -0.36675    0.21214  -1.729
## HarmonyI-V-VI:Voicepar3rd   -0.72238    0.21241  -3.401
## HarmonyIV-I-V:Voicepar3rd    0.53414    0.21232   2.516
## HarmonyI-V-IV:Voicepar5th   -0.21065    0.21244  -0.992
## HarmonyI-V-VI:Voicepar5th   -0.44844    0.21247  -2.111
## HarmonyIV-I-V:Voicepar5th    0.07451    0.21198   0.351

##
## Correlation matrix not shown by default, as p = 19 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it

## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```r
model3_musician_1_h <- update(model3_musician_1, ~ . + Harmony*Musician_1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : unable to evaluate scaled gradient

## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge: degenerate Hessian with 1
## negative eigenvalues
```

```r
model3_musician_1_i <- update(model3_musician_1, ~ . + Instrument*Musician_1)
```

```
## boundary (singular) fit: see ?isSingular
```

```r
model3_musician_1_v <- update(model3_musician_1, ~ . + Voice*Musician_1)
model3_musician_1_hv <- update(model3_musician_1, ~ . + Harmony*Voice*Musician_1)
```

```
## boundary (singular) fit: see ?isSingular
```

```r
model3_musician_1_cl <- update(model3_musician_1, ~ . + ClsListen*Musician_1)
```

```
## boundary (singular) fit: see ?isSingular
```

```r
model3_musician_1_omsi <- update(model3_musician_1, ~ . + OMSI*Musician_1)
```

```
## boundary (singular) fit: see ?isSingular
```

```r
model3_musician_1_90 <- update(model3_musician_1, ~ . + X1990s2000s*Musician_1)
model3_musician_1_70 <- update(model3_musician_1, ~ . + X1990s2000s.minus.1960s1970s*Musician_1)
```

```
## boundary (singular) fit: see ?isSingular
```

```r
anova(model3_musician_1,  model3_musician_1_h,  model3_musician_1_v, model3_musician_1_hv,
      model3_musician_1_i, model3_musician_1_cl, model3_musician_1_omsi, model3_musician_1_90,
      model3_musician_1_70)
```

```
## Data: df
## Models:
## model3_musician_1: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1:     Instrument | Subject) + Musician_1 + Harmony:Voice
## model3_musician_1_cl: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1_cl:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1_cl:     Instrument | Subject) + Musician_1 + Harmony:Voice + ClsListen:Musician_1
## model3_musician_1_omsi: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1_omsi:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1_omsi:     Instrument | Subject) + Musician_1 + Harmony:Voice + OMSI:Musician_1
## model3_musician_1_90: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1_90:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1_90:     Instrument | Subject) + Musician_1 + Harmony:Voice + X1990s2000s:Musician_1
## model3_musician_1_70: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1_70:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1_70:     Instrument | Subject) + Musician_1 + Harmony:Voice + X1990s2000s.minus.1960
## model3_musician_1_v: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1_v:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1_v:     Instrument | Subject) + Musician_1 + Harmony:Voice + Voice:Musician_1
## model3_musician_1_i: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1_i:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1_i:     Instrument | Subject) + Musician_1 + Harmony:Voice + Instrument:Musician_1
## model3_musician_1_h: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1_h:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1_h:     Instrument | Subject) + Musician_1 + Harmony:Voice + Harmony:Musician_1
## model3_musician_1_hv: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1_hv:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1_hv:     Instrument | Subject) + Musician_1 + Harmony:Voice + Harmony:Musician_1 +
## model3_musician_1_hv:     Voice:Musician_1 + Harmony:Voice:Musician_1
##                        Df    AIC   BIC  logLik deviance  Chisq Chi Df
## model3_musician_1      41 9823.3 10062 -4870.7   9741.3
## model3_musician_1_cl   42 9820.5 10065 -4868.3   9736.5 4.7714      1
## model3_musician_1_omsi 42 9824.0 10068 -4870.0   9740.0 0.0000      0
## model3_musician_1_90   42 9824.4 10069 -4870.2   9740.4 0.0000      0
## model3_musician_1_70   42 9825.3 10070 -4870.6   9741.3 0.0000      0
## model3_musician_1_v    43 9826.5 10077 -4870.3   9740.5 0.7657      1
## model3_musician_1_i    43 9819.0 10069 -4866.5   9733.0 7.5625      0
## model3_musician_1_h    44 9813.0 10069 -4862.5   9725.0 8.0006      1
## model3_musician_1_hv   52 9821.3 10124 -4858.6   9717.3 7.6901      8
##                        Pr(>Chisq)
## model3_musician_1
```

```
## model3_musician_1_cl     0.028937 *
## model3_musician_1_omsi   1.000000
## model3_musician_1_90     1.000000
## model3_musician_1_70     1.000000
## model3_musician_1_v      0.381547
## model3_musician_1_i     < 2.2e-16 ***
## model3_musician_1_h      0.004676 **
## model3_musician_1_hv     0.464309
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
model3_musician_1_hi <- update(model3_musician_1, ~ . + Harmony*Musician_1 + Instrument*Musician_1)
```

```
## boundary (singular) fit: see ?isSingular
```

```r
model3_musician_1_hc <- update(model3_musician_1, ~ . + Harmony*Musician_1 + ClsListen*Musician_1)
model3_musician_1_hic <- update(model3_musician_1, ~ . + Harmony*Musician_1 + ClsListen*Musician_1 + Ins
```

```
## boundary (singular) fit: see ?isSingular
```

```r
anova(model3_musician_1,  model3_musician_1_h,  model3_musician_1_hc,  model3_musician_1_hi,
      model3_musician_1_hic, model3_musician_1_i, model3_musician_1_cl)
```

```
## Data: df
## Models:
## model3_musician_1: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1:     Instrument | Subject) + Musician_1 + Harmony:Voice
## model3_musician_1_cl: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1_cl:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1_cl:     Instrument | Subject) + Musician_1 + Harmony:Voice + ClsListen:Musician_1
## model3_musician_1_i: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1_i:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1_i:     Instrument | Subject) + Musician_1 + Harmony:Voice + Instrument:Musician_1
## model3_musician_1_h: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1_h:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1_h:     Instrument | Subject) + Musician_1 + Harmony:Voice + Harmony:Musician_1
## model3_musician_1_hc: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1_hc:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1_hc:     Instrument | Subject) + Musician_1 + Harmony:Voice + Harmony:Musician_1 +
## model3_musician_1_hc:     ClsListen:Musician_1
## model3_musician_1_hi: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1_hi:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1_hi:     Instrument | Subject) + Musician_1 + Harmony:Voice + Harmony:Musician_1 +
## model3_musician_1_hi:     Instrument:Musician_1
## model3_musician_1_hic: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model3_musician_1_hic:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model3_musician_1_hic:     Instrument | Subject) + Musician_1 + Harmony:Voice + Harmony:Musician_1 +
## model3_musician_1_hic:     ClsListen:Musician_1 + Instrument:Musician_1
##                       Df    AIC   BIC  logLik deviance  Chisq Chi Df
## model3_musician_1     41 9823.3 10062 -4870.7   9741.3
## model3_musician_1_cl  42 9820.5 10065 -4868.3   9736.5 4.7714      1
## model3_musician_1_i   43 9819.0 10069 -4866.5   9733.0 3.5929      1
## model3_musician_1_h   44 9813.0 10069 -4862.5   9725.0 8.0006      1
## model3_musician_1_hc  45 9810.2 10072 -4860.1   9720.2 4.7825      1
## model3_musician_1_hi  46 9814.6 10082 -4861.3   9722.6 0.0000      1
```

```
## model3_musician_1_hic 47 9811.8 10085 -4858.9   9717.8 4.7794      1
##                        Pr(>Chisq)
## model3_musician_1
## model3_musician_1_cl    0.028937 *
## model3_musician_1_i     0.058029 .
## model3_musician_1_h     0.004676 **
## model3_musician_1_hc    0.028750 *
## model3_musician_1_hi    1.000000
## model3_musician_1_hic   0.028802 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(model3_musician_1_hc)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
##     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
##     Instrument | Subject) + Musician_1 + Harmony:Voice + Harmony:Musician_1 +
##     ClsListen:Musician_1
##    Data: df
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   9810.2  10072.0  -4860.1   9720.2     2440
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7499 -0.5773  0.0179  0.5805  4.2873
##
## Random effects:
##  Groups   Name             Variance Std.Dev. Corr
##  Subject  (Intercept)      1.865485 1.36583
##           HarmonyI-V-IV    0.059070 0.24304   0.86
##           HarmonyI-V-VI    1.345580 1.15999  -0.08  0.18
##           HarmonyIV-I-V    0.004469 0.06685   0.47  0.56 -0.62
##           Instrumentpiano  1.652530 1.28551  -0.29 -0.68 -0.34 -0.38
##           Instrumentstring 3.371395 1.83614  -0.48 -0.56 -0.56  0.17  0.65
##  Residual                  2.331946 1.52707
## Number of obs: 2485, groups:  Subject, 70
##
## Fixed effects:
##                               Estimate Std. Error t value
## (Intercept)                   5.039763   0.781988   6.445
## HarmonyI-V-IV                 0.009733   0.225159   0.043
## HarmonyI-V-VI                -0.011220   0.338060  -0.033
## HarmonyIV-I-V                -0.336082   0.218795  -1.536
## Voicepar3rd                  -0.247718   0.149638  -1.655
## Voicepar5th                  -0.214575   0.149994  -1.431
## Instrumentpiano               1.338470   0.171369   7.810
## Instrumentstring              3.079388   0.231933  13.277
## ClsListen                    -0.148030   0.227975  -0.649
## OMSI                          0.144873   0.116489   1.244
## X1990s2000s.minus.1960s1970s  0.173217   0.093787   1.847
## X1990s2000s                  -0.198446   0.106183  -1.869
## Musician_1                   -2.222172   0.512972  -4.332
```

```
## HarmonyI-V-IV:Voicepar3rd    -0.367501   0.212049  -1.733
## HarmonyI-V-VI:Voicepar3rd    -0.722845   0.212319  -3.405
## HarmonyIV-I-V:Voicepar3rd     0.534469   0.212237   2.518
## HarmonyI-V-IV:Voicepar5th    -0.210123   0.212349  -0.990
## HarmonyI-V-VI:Voicepar5th    -0.446559   0.212383  -2.103
## HarmonyIV-I-V:Voicepar5th     0.075448   0.211899   0.356
## HarmonyI-V-IV:Musician_1      0.190211   0.213687   0.890
## HarmonyI-V-VI:Musician_1      1.516262   0.348740   4.348
## HarmonyIV-I-V:Musician_1      0.235604   0.207167   1.137
## ClsListen:Musician_1          0.541424   0.242535   2.232
```

```
##
## Correlation matrix not shown by default, as p = 23 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it
```

```r
model4_final <-  model3_musician_1_hc
model4_update_1 <- update(model4_final, ~ . - OMSI)
```

```
## boundary (singular) fit: see ?isSingular
```

```r
summary(model4_update_1)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula:
## Classical ~ Harmony + Voice + Instrument + ClsListen + X1990s2000s.minus.1960s1970s +
##     X1990s2000s + (1 + Harmony + Instrument | Subject) + Musician_1 +
##     Harmony:Voice + Harmony:Musician_1 + ClsListen:Musician_1
##    Data: df
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   9809.5  10065.5  -4860.8   9721.5     2441
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7550 -0.5798  0.0169  0.5856  4.2867
##
## Random effects:
##  Groups   Name             Variance Std.Dev. Corr
##  Subject  (Intercept)      1.876189 1.36974
##           HarmonyI-V-IV    0.060585 0.24614   0.87
##           HarmonyI-V-VI    1.346870 1.16055  -0.03  0.18
##           HarmonyIV-I-V    0.005874 0.07664   0.59  0.67 -0.53
##           Instrumentpiano  1.652409 1.28546  -0.31 -0.67 -0.34 -0.33
##           Instrumentstring 3.370939 1.83601  -0.50 -0.55 -0.56  0.15  0.65
##  Residual                  2.332064 1.52711
## Number of obs: 2485, groups:  Subject, 70
##
## Fixed effects:
##                               Estimate Std. Error t value
## (Intercept)                   5.689472   0.580020   9.809
## HarmonyI-V-IV                 0.009764   0.225372   0.043
## HarmonyI-V-VI                -0.011191   0.338198  -0.033
## HarmonyIV-I-V                -0.335961   0.219005  -1.534
## Voicepar3rd                  -0.247551   0.149642  -1.654
```

```
## Voicepar5th                  -0.214515   0.149998  -1.430
## Instrumentpiano               1.338343   0.171364   7.810
## Instrumentstring              3.079223   0.231920  13.277
## ClsListen                    -0.178552   0.229608  -0.778
## X1990s2000s.minus.1960s1970s  0.164085   0.094578   1.735
## X1990s2000s                  -0.190218   0.107123  -1.776
## Musician_1                   -2.155482   0.511197  -4.217
## HarmonyI-V-IV:Voicepar3rd    -0.367735   0.212053  -1.734
## HarmonyI-V-VI:Voicepar3rd    -0.723036   0.212324  -3.405
## HarmonyIV-I-V:Voicepar3rd     0.534346   0.212243   2.518
## HarmonyI-V-IV:Voicepar5th    -0.210181   0.212355  -0.990
## HarmonyI-V-VI:Voicepar5th    -0.446558   0.212389  -2.103
## HarmonyIV-I-V:Voicepar5th     0.075219   0.211905   0.355
## HarmonyI-V-IV:Musician_1      0.190325   0.213976   0.889
## HarmonyI-V-VI:Musician_1      1.516405   0.348917   4.346
## HarmonyIV-I-V:Musician_1      0.235747   0.207453   1.136
## ClsListen:Musician_1          0.567633   0.243705   2.329
##
## Correlation matrix not shown by default, as p = 22 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
model4_update_2 <- update(model4_final, ~ . - X1990s2000s - X1990s2000s.minus.1960s1970s - OMSI)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, : Model is nearly unide
##  - Rescale variables?
```

```
summary(model4_update_2)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula:
## Classical ~ Harmony + Voice + Instrument + ClsListen + (1 + Harmony +
##     Instrument | Subject) + Musician_1 + Harmony:Voice + Harmony:Musician_1 +
##     ClsListen:Musician_1
##    Data: df
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   9808.9  10053.3  -4862.5   9724.9     2443
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7612 -0.5745  0.0239  0.5845  4.2840
##
## Random effects:
##  Groups   Name            Variance Std.Dev. Corr
##  Subject  (Intercept)     1.990539 1.41086
##           HarmonyI-V-IV   0.052733 0.22964   0.88
##           HarmonyI-V-VI   1.345336 1.15989   0.00  0.18
##           HarmonyIV-I-V   0.003881 0.06229   0.29  0.43 -0.67
##           Instrumentpiano 1.651830 1.28524  -0.35 -0.72 -0.34 -0.41
##           Instrumentstring 3.370401 1.83587  -0.52 -0.59 -0.56  0.18  0.65
```

```
##  Residual                  2.332709 1.52732
## Number of obs: 2485, groups:  Subject, 70
##
## Fixed effects:
##                            Estimate Std. Error t value
## (Intercept)                5.355717   0.437142  12.252
## HarmonyI-V-IV              0.009852   0.224298   0.044
## HarmonyI-V-VI             -0.011656   0.338057  -0.034
## HarmonyIV-I-V             -0.336094   0.218746  -1.536
## Voicepar3rd               -0.247694   0.149663  -1.655
## Voicepar5th               -0.214497   0.150019  -1.430
## Instrumentpiano            1.338356   0.171346   7.811
## Instrumentstring           3.079428   0.231907  13.279
## ClsListen                 -0.254057   0.225754  -1.125
## Musician_1                -2.159452   0.519770  -4.155
## HarmonyI-V-IV:Voicepar3rd -0.367298   0.212083  -1.732
## HarmonyI-V-VI:Voicepar3rd -0.722931   0.212354  -3.404
## HarmonyIV-I-V:Voicepar3rd  0.534346   0.212272   2.517
## HarmonyI-V-IV:Voicepar5th -0.210092   0.212384  -0.989
## HarmonyI-V-VI:Voicepar5th -0.446693   0.212418  -2.103
## HarmonyIV-I-V:Voicepar5th  0.075468   0.211934   0.356
## HarmonyI-V-IV:Musician_1   0.189679   0.212491   0.893
## HarmonyI-V-VI:Musician_1   1.517020   0.348723   4.350
## HarmonyIV-I-V:Musician_1   0.235494   0.207084   1.137
## ClsListen:Musician_1       0.598235   0.247771   2.414
##
## Correlation matrix not shown by default, as p = 20 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it
## convergence code: 0
## Model is nearly unidentifiable: large eigenvalue ratio
##  - Rescale variables?
```

```
anova(model4_final, model4_update_1, model4_update_2)
```

```
## Data: df
## Models:
## model4_update_2: Classical ~ Harmony + Voice + Instrument + ClsListen + (1 + Harmony +
## model4_update_2:     Instrument | Subject) + Musician_1 + Harmony:Voice + Harmony:Musician_1 +
## model4_update_2:     ClsListen:Musician_1
## model4_update_1: Classical ~ Harmony + Voice + Instrument + ClsListen + X1990s2000s.minus.1960s1970s
## model4_update_1:     X1990s2000s + (1 + Harmony + Instrument | Subject) + Musician_1 +
## model4_update_1:     Harmony:Voice + Harmony:Musician_1 + ClsListen:Musician_1
## model4_final: Classical ~ Harmony + Voice + Instrument + ClsListen + OMSI +
## model4_final:     X1990s2000s.minus.1960s1970s + X1990s2000s + (1 + Harmony +
## model4_final:     Instrument | Subject) + Musician_1 + Harmony:Voice + Harmony:Musician_1 +
## model4_final:     ClsListen:Musician_1
##                 Df    AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model4_update_2 42 9808.9 10053 -4862.5   9724.9
## model4_update_1 44 9809.5 10066 -4860.8   9721.5 3.4394      2     0.1791
## model4_final    45 9810.2 10072 -4860.1   9720.2 1.3362      1     0.2477
```

```
final_model <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen + Musician + Musician*Harmony +
                      (1 + Harmony +  Instrument | Subject),
```

```r
                    data=df_orig, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5
```

```
## boundary (singular) fit: see ?isSingular
```

```r
summary(final_model)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula:
## Classical ~ Harmony * Voice + Instrument + ClsListen + Musician +
##     Musician * Harmony + Musician * ClsListen + (1 + Harmony +
##     Instrument | Subject)
##    Data: df_orig
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   9703.5   9947.5  -4809.8   9619.5     2419
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7821 -0.5738  0.0194  0.5829  4.2977
##
## Random effects:
##  Groups   Name             Variance Std.Dev. Corr
##  Subject  (Intercept)      2.017038 1.42022
##           HarmonyI-V-IV    0.053219 0.23069   0.88
##           HarmonyI-V-VI    1.367114 1.16924   0.00  0.18
##           HarmonyIV-I-V    0.003867 0.06218   0.25  0.41 -0.66
##           Instrumentpiano  1.674901 1.29418  -0.35 -0.72 -0.34 -0.43
##           Instrumentstring 3.376857 1.83762  -0.51 -0.58 -0.56  0.19  0.65
##  Residual                  2.319432 1.52297
## Number of obs: 2461, groups:  Subject, 69
##
## Fixed effects:
##                           Estimate Std. Error t value
## (Intercept)               5.336244   0.442273  12.066
## HarmonyI-V-IV             0.002251   0.227367   0.010
## HarmonyI-V-VI             0.001995   0.347599   0.006
## HarmonyIV-I-V            -0.372035   0.222708  -1.671
## Voicepar3rd              -0.242498   0.150117  -1.615
## Voicepar5th              -0.208119   0.150475  -1.383
## Instrumentpiano           1.346314   0.173266   7.770
## Instrumentstring          3.105698   0.233635  13.293
## ClsListen                -0.240551   0.231908  -1.037
## Musician                 -2.156302   0.524983  -4.107
## HarmonyI-V-IV:Voicepar3rd -0.381935   0.212518  -1.797
## HarmonyI-V-VI:Voicepar3rd -0.701036   0.212759  -3.295
## HarmonyIV-I-V:Voicepar3rd  0.540052   0.212803   2.538
## HarmonyI-V-IV:Voicepar5th -0.208802   0.212662  -0.982
## HarmonyI-V-VI:Voicepar5th -0.436771   0.213030  -2.050
## HarmonyIV-I-V:Voicepar5th  0.096339   0.212582   0.453
## HarmonyI-V-IV:Musician     0.200228   0.215568   0.929
## HarmonyI-V-VI:Musician     1.484885   0.357651   4.152
## HarmonyIV-I-V:Musician     0.262732   0.210240   1.250
## ClsListen:Musician         0.585571   0.253627   2.309
```

```
##
## Correlation matrix not shown by default, as p = 20 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it
```

```
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
final_classical_model <- final_model
```

summary of final classical model (Musician Included)

```
summary(final_model)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula:
## Classical ~ Harmony * Voice + Instrument + ClsListen + Musician +
##     Musician * Harmony + Musician * ClsListen + (1 + Harmony +
##     Instrument | Subject)
##    Data: df_orig
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   9703.5   9947.5  -4809.8   9619.5     2419
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7821 -0.5738  0.0194  0.5829  4.2977
##
## Random effects:
##  Groups   Name             Variance Std.Dev. Corr
##  Subject  (Intercept)      2.017038 1.42022
##           HarmonyI-V-IV    0.053219 0.23069   0.88
##           HarmonyI-V-VI    1.367114 1.16924   0.00  0.18
##           HarmonyIV-I-V    0.003867 0.06218   0.25  0.41 -0.66
##           Instrumentpiano  1.674901 1.29418  -0.35 -0.72 -0.34 -0.43
##           Instrumentstring 3.376857 1.83762  -0.51 -0.58 -0.56  0.19  0.65
##  Residual                  2.319432 1.52297
## Number of obs: 2461, groups:  Subject, 69
##
## Fixed effects:
##                            Estimate Std. Error t value
## (Intercept)                5.336244   0.442273  12.066
## HarmonyI-V-IV              0.002251   0.227367   0.010
## HarmonyI-V-VI              0.001995   0.347599   0.006
## HarmonyIV-I-V             -0.372035   0.222708  -1.671
## Voicepar3rd               -0.242498   0.150117  -1.615
## Voicepar5th               -0.208119   0.150475  -1.383
## Instrumentpiano            1.346314   0.173266   7.770
## Instrumentstring           3.105698   0.233635  13.293
## ClsListen                 -0.240551   0.231908  -1.037
## Musician                  -2.156302   0.524983  -4.107
## HarmonyI-V-IV:Voicepar3rd -0.381935   0.212518  -1.797
## HarmonyI-V-VI:Voicepar3rd -0.701036   0.212759  -3.295
## HarmonyIV-I-V:Voicepar3rd  0.540052   0.212803   2.538
```

```
## HarmonyI-V-IV:Voicepar5th -0.208802   0.212662  -0.982
## HarmonyI-V-VI:Voicepar5th -0.436771   0.213030  -2.050
## HarmonyIV-I-V:Voicepar5th  0.096339   0.212582   0.453
## HarmonyI-V-IV:Musician     0.200228   0.215568   0.929
## HarmonyI-V-VI:Musician     1.484885   0.357651   4.152
## HarmonyIV-I-V:Musician     0.262732   0.210240   1.250
## ClsListen:Musician         0.585571   0.253627   2.309
```

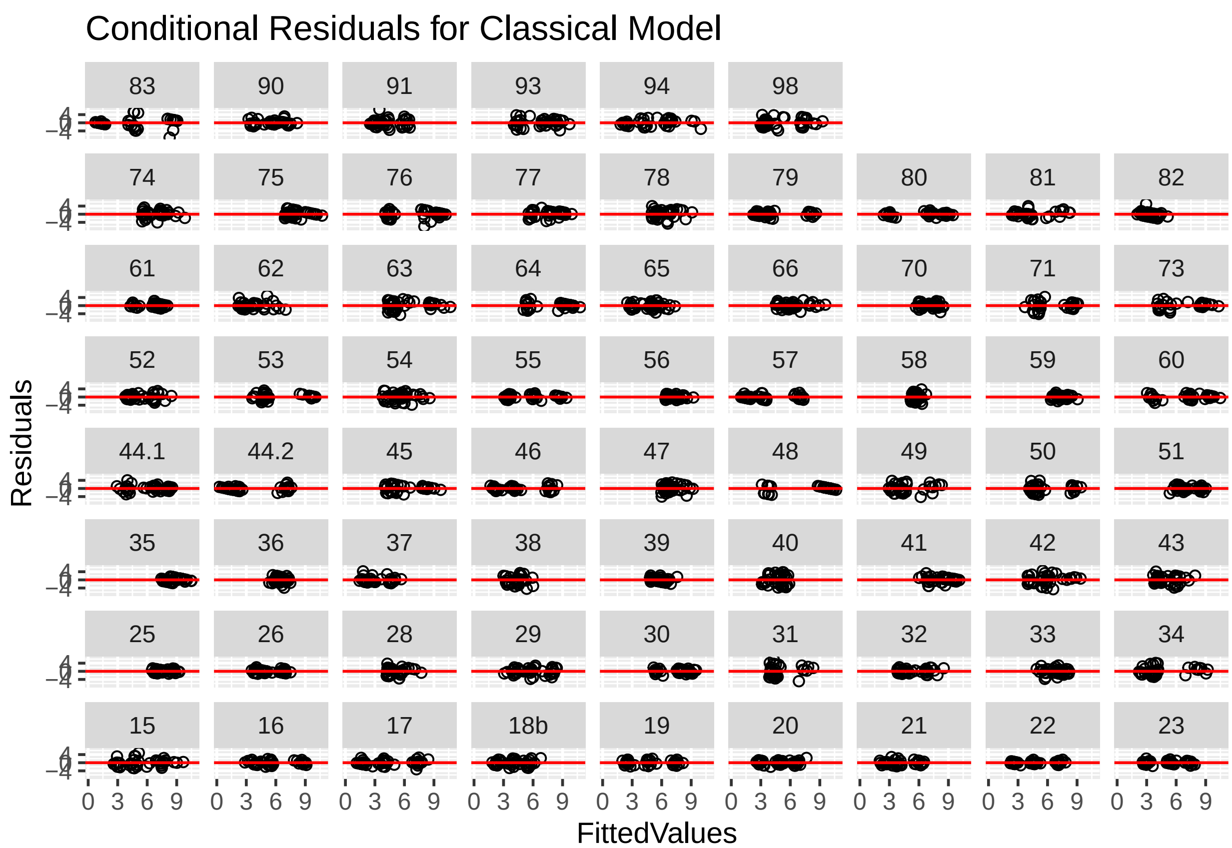
```
##
## Correlation matrix not shown by default, as p = 20 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it
```

```
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

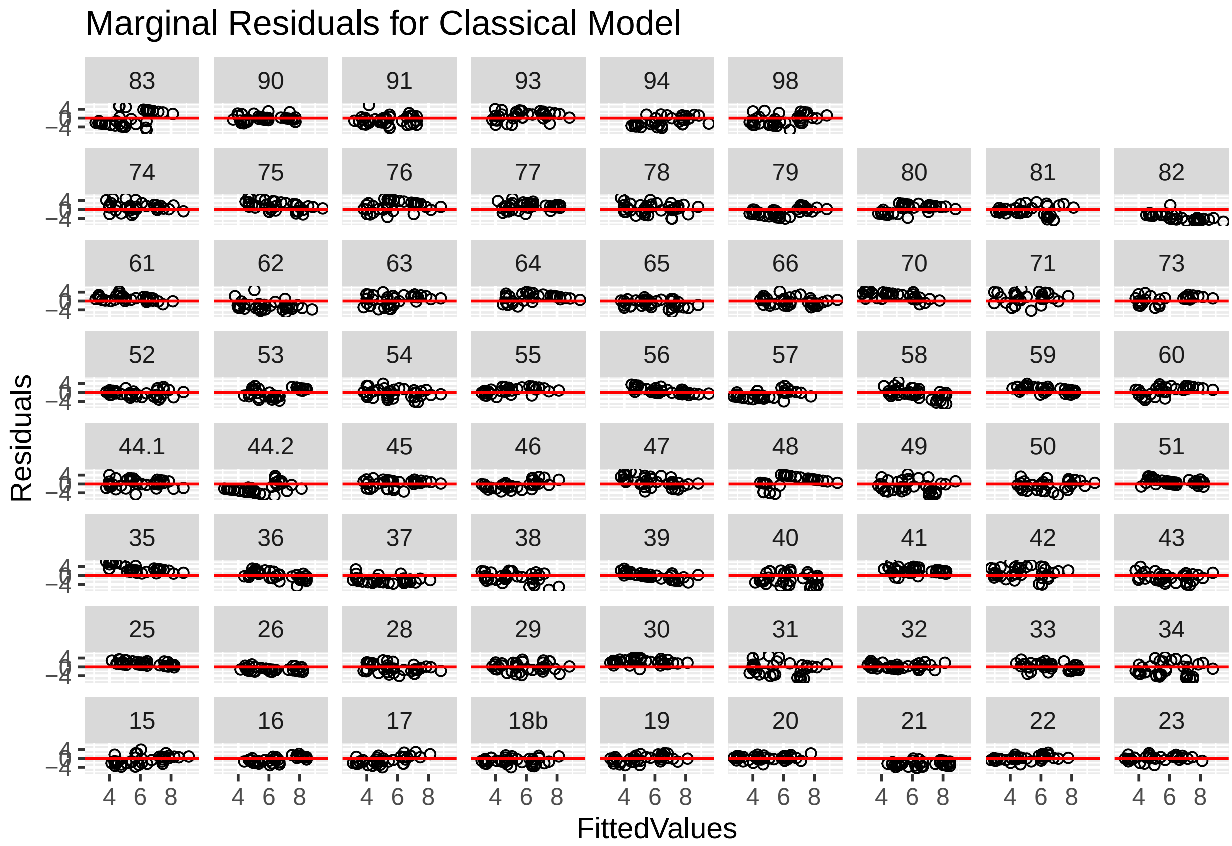
residuals of final classical model (Musician Included)

```r
resid_marg <- data.frame(FittedValues=yhat.marg(final_classical_model),
                         Residuals=r.marg(final_classical_model),
                         Subject= df_orig[!is.na(df_orig$ClsListen), 'Subject'])
resid_cond <- data.frame(FittedValues=yhat.cond(final_classical_model),
                         Residuals=r.cond(final_classical_model),
                         Subject= df_orig[!is.na(df_orig$ClsListen), 'Subject'])

ggplot(resid_cond, aes(x=FittedValues,y=Residuals)) +
  facet_wrap( ~ Subject, as.table=F) +   geom_point(pch=1) +
  geom_hline(yintercept=0, color='red')+ ggtitle('Conditional Residuals for Classical Model')
```




```
ggplot(resid_marg, aes(x=FittedValues,y=Residuals)) +
  facet_wrap( ~ Subject, as.table=F) +   geom_point(pch=1) +
  geom_hline(yintercept=0, color='red') + ggtitle('Marginal Residuals for Classical Model')
```



Comparing the dichotimzation we see that there is a difference in the significance of the musician indicator when we dichotimize at 1 v 2+, or 1-2 v 3+. In both cases the coefficient is negative, but when we dichotimize, setting responses of 2 and greater as Musicians, the magnitude more than twice as high, and the p-value significance jumps to the .05 level.

Testing knowledge of Pachelbel effects

```
final_model_test <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen + Musician + Musician*Harm
                      (1 + Harmony +  Instrument | Subject),
                      data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, : Model is nearly unider
##  - Rescale variables?
```

```
final_model_rob <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen + KnowRob*Harmony +
                      (1 + Harmony +  Instrument | Subject),
                      data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
final_model_rob_music <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Musician + KnowRob*I
                      (1 + Harmony +  Instrument | Subject),
                      data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
final_model_axis <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen + KnowAxis*Harmony +
                      (1 + Harmony +  Instrument | Subject),
```

```r
                        data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))


final_model_axis_music <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen +Musician + KnowAxis
                        (1 + Harmony +  Instrument | Subject),
                        data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```r
final_model_pach <- lmer(Classical ~ Harmony * Voice + Instrument + ClsListen + PachListen*Harmony +
                        (1 + Harmony +  Instrument | Subject),
                        data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))

final_model_pach_music <- lmer(Classical ~ Harmony * Voice + Instrument +ClsListen + Musician + PachList
                                    Musician*PachListen + (1 + Harmony +  Instrument | Subject),
                                  data=df, REML=F, lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun =
```

```
## boundary (singular) fit: see ?isSingular
```

```r
summary(final_model_rob)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: Classical ~ Harmony * Voice + Instrument + ClsListen + KnowRob *
##     Harmony + (1 + Harmony + Instrument | Subject)
##    Data: df
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   9817.6  10056.1  -4867.8   9735.6     2444
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7508 -0.5807  0.0186  0.5700  4.1052
##
## Random effects:
##  Groups   Name             Variance Std.Dev. Corr
##  Subject  (Intercept)      2.607308 1.61472
##           HarmonyI-V-IV    0.043771 0.20922   0.78
##           HarmonyI-V-VI    1.240641 1.11384  -0.21  0.25
##           HarmonyIV-I-V    0.002629 0.05128  -0.47  0.09  0.34
##           Instrumentpiano  1.654245 1.28617  -0.39 -0.75 -0.19 -0.28
##           Instrumentstring 3.368768 1.83542  -0.55 -0.60 -0.34  0.42  0.65
##  Residual                  2.334561 1.52793
## Number of obs: 2485, groups:  Subject, 70
##
## Fixed effects:
##                            Estimate Std. Error t value
## (Intercept)                3.925234   0.301830  13.005
## HarmonyI-V-IV              0.163037   0.157551   1.035
## HarmonyI-V-VI              0.918802   0.212847   4.317
## HarmonyIV-I-V             -0.140235   0.155149  -0.904
## Voicepar3rd               -0.247045   0.149727  -1.650
## Voicepar5th               -0.213805   0.150082  -1.425
## Instrumentpiano            1.337538   0.171463   7.801
## Instrumentstring           3.078133   0.231869  13.275
```

```
## ClsListen                   0.186502   0.094762   1.968
## KnowRob                    -0.064085   0.107314  -0.597
## HarmonyI-V-IV:Voicepar3rd -0.366670   0.212171  -1.728
## HarmonyI-V-VI:Voicepar3rd -0.722937   0.212439  -3.403
## HarmonyIV-I-V:Voicepar3rd  0.533575   0.212355   2.513
## HarmonyI-V-IV:Voicepar5th -0.210785   0.212471  -0.992
## HarmonyI-V-VI:Voicepar5th -0.449247   0.212500  -2.114
## HarmonyIV-I-V:Voicepar5th  0.074057   0.212021   0.349
## HarmonyI-V-IV:KnowRob     -0.009898   0.053639  -0.185
## HarmonyI-V-VI:KnowRob      0.314252   0.092470   3.398
## HarmonyIV-I-V:KnowRob     -0.017579   0.052421  -0.335

##
## Correlation matrix not shown by default, as p = 19 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it

## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```r
summary(final_model_axis)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula:
## Classical ~ Harmony * Voice + Instrument + ClsListen + KnowAxis *
##     Harmony + (1 + Harmony + Instrument | Subject)
##    Data: df
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   9828.7  10067.2  -4873.4   9746.7     2444
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7246 -0.5741  0.0256  0.5687  4.1102
##
## Random effects:
##  Groups   Name             Variance Std.Dev. Corr
##  Subject  (Intercept)      2.537817 1.59305
##           HarmonyI-V-IV    0.040789 0.20196   0.85
##           HarmonyI-V-VI    1.602571 1.26593  -0.15  0.23
##           HarmonyIV-I-V    0.001151 0.03393  -0.53 -0.29 -0.20
##           Instrumentpiano  1.653627 1.28593  -0.40 -0.72 -0.22  0.03
##           Instrumentstring 3.368301 1.83529  -0.54 -0.62 -0.42  0.76  0.65
##  Residual                  2.333493 1.52758
## Number of obs: 2485, groups:  Subject, 70
##
## Fixed effects:
##                           Estimate Std. Error t value
## (Intercept)                3.74654    0.30847  12.145
## HarmonyI-V-IV              0.19842    0.15850   1.252
## HarmonyI-V-VI              1.22423    0.22859   5.356
## HarmonyIV-I-V             -0.10483    0.15597  -0.672
## Voicepar3rd               -0.24753    0.14969  -1.654
## Voicepar5th               -0.21417    0.15005  -1.427
## Instrumentpiano            1.33795    0.17143   7.805
```

```
## Instrumentstring           3.07847    0.23185  13.278
## ClsListen                  0.20032    0.09152   2.189
## KnowAxis                   0.10968    0.09585   1.144
## HarmonyI-V-IV:Voicepar3rd -0.36667    0.21212  -1.729
## HarmonyI-V-VI:Voicepar3rd -0.72235    0.21239  -3.401
## HarmonyIV-I-V:Voicepar3rd  0.53457    0.21231   2.518
## HarmonyI-V-IV:Voicepar5th -0.21047    0.21242  -0.991
## HarmonyI-V-VI:Voicepar5th -0.44809    0.21246  -2.109
## HarmonyIV-I-V:Voicepar5th  0.07432    0.21197   0.351
## HarmonyI-V-IV:KnowAxis    -0.04690    0.04903  -0.956
## HarmonyI-V-VI:KnowAxis    -0.07120    0.09090  -0.783
## HarmonyIV-I-V:KnowAxis    -0.05398    0.04805  -1.123
```

```
##
## Correlation matrix not shown by default, as p = 19 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it
```

```r
summary(final_model_pach)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula:
## Classical ~ Harmony * Voice + Instrument + ClsListen + PachListen *
##     Harmony + (1 + Harmony + Instrument | Subject)
##    Data: df
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   9824.5  10063.0  -4871.2   9742.5     2444
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7647 -0.5751  0.0244  0.5718  4.1131
##
## Random effects:
##  Groups   Name             Variance Std.Dev. Corr
##  Subject  (Intercept)      2.558201 1.59944
##           HarmonyI-V-IV    0.042978 0.20731   0.80
##           HarmonyI-V-VI    1.487002 1.21943  -0.14  0.19
##           HarmonyIV-I-V    0.003046 0.05519  -0.44 -0.02 -0.22
##           Instrumentpiano  1.653678 1.28595  -0.38 -0.81 -0.29 -0.31
##           Instrumentstring 3.368148 1.83525  -0.54 -0.61 -0.45  0.49  0.65
##  Residual                  2.334491 1.52790
## Number of obs: 2485, groups:  Subject, 70
##
## Fixed effects:
##                           Estimate Std. Error t value
## (Intercept)                4.13556    0.76035   5.439
## HarmonyI-V-IV             -0.15370    0.39410  -0.390
## HarmonyI-V-VI             -0.41333    0.68505  -0.603
## HarmonyIV-I-V             -0.31254    0.38740  -0.807
## Voicepar3rd               -0.24765    0.14972  -1.654
## Voicepar5th               -0.21426    0.15008  -1.428
## Instrumentpiano            1.33790    0.17144   7.804
## Instrumentstring           3.07876    0.23185  13.279
```

```
## ClsListen                  0.19964    0.09222   2.165
## PachListen                -0.06389    0.16014  -0.399
## HarmonyI-V-IV:Voicepar3rd -0.36629    0.21217  -1.726
## HarmonyI-V-VI:Voicepar3rd -0.72240    0.21244  -3.401
## HarmonyIV-I-V:Voicepar3rd  0.53412    0.21235   2.515
## HarmonyI-V-IV:Voicepar5th -0.21022    0.21247  -0.989
## HarmonyI-V-VI:Voicepar5th -0.44819    0.21250  -2.109
## HarmonyIV-I-V:Voicepar5th  0.07460    0.21202   0.352
## HarmonyI-V-IV:PachListen   0.06849    0.08059   0.850
## HarmonyI-V-VI:PachListen   0.34853    0.14454   2.411
## HarmonyIV-I-V:PachListen   0.03512    0.07920   0.443
```

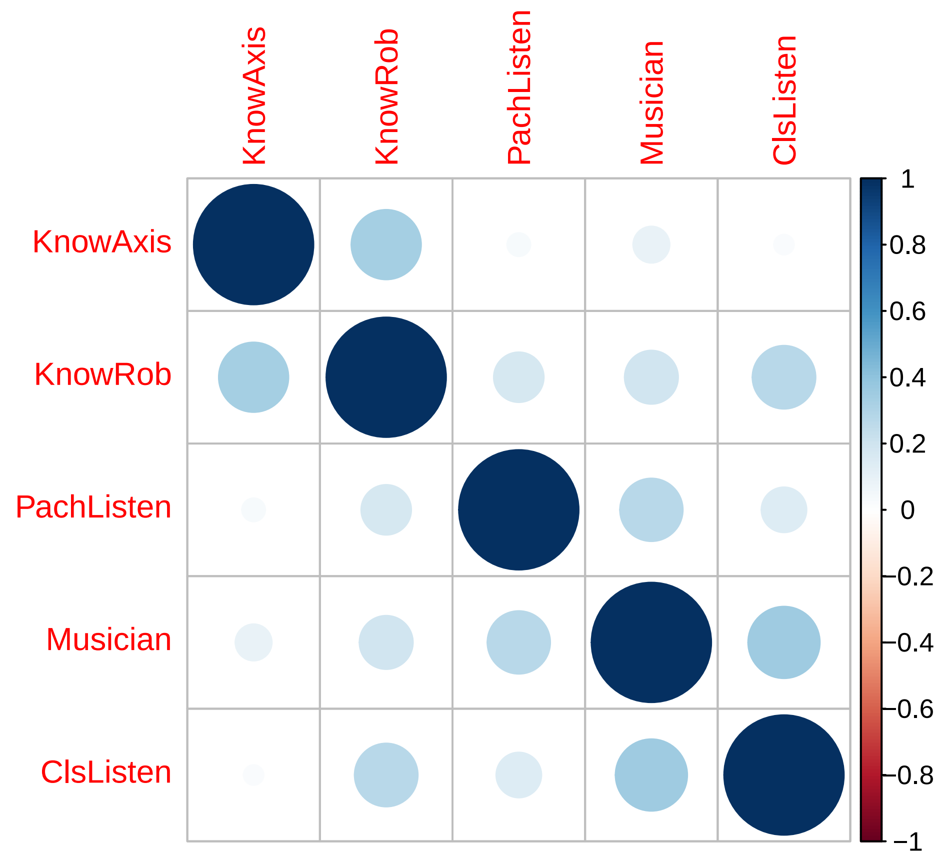
```
##
## Correlation matrix not shown by default, as p = 19 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it
```

```r
anova(final_model_test,final_model_axis,final_model_rob,final_model_pach,final_model_rob_music,final_mo
```

```
## Data: df
## Models:
## final_model_axis: Classical ~ Harmony * Voice + Instrument + ClsListen + KnowAxis *
## final_model_axis:     Harmony + (1 + Harmony + Instrument | Subject)
## final_model_rob: Classical ~ Harmony * Voice + Instrument + ClsListen + KnowRob *
## final_model_rob:     Harmony + (1 + Harmony + Instrument | Subject)
## final_model_pach: Classical ~ Harmony * Voice + Instrument + ClsListen + PachListen *
## final_model_pach:     Harmony + (1 + Harmony + Instrument | Subject)
## final_model_test: Classical ~ Harmony * Voice + Instrument + ClsListen + Musician +
## final_model_test:     Musician * Harmony + Musician * ClsListen + (1 + Harmony +
## final_model_test:     Instrument | Subject)
## final_model_rob_music: Classical ~ Harmony * Voice + Instrument + ClsListen + Musician +
## final_model_rob_music:     KnowRob * Harmony + Musician * KnowRob + (1 + Harmony + Instrument |
## final_model_rob_music:     Subject)
## final_model_axis_music: Classical ~ Harmony * Voice + Instrument + ClsListen + Musician +
## final_model_axis_music:     KnowAxis * Harmony + Musician * KnowAxis + (1 + Harmony +
## final_model_axis_music:     Instrument | Subject)
## final_model_pach_music: Classical ~ Harmony * Voice + Instrument + ClsListen + Musician +
## final_model_pach_music:     PachListen * Harmony + Musician * PachListen + (1 + Harmony +
## final_model_pach_music:     Instrument | Subject)
##                        Df    AIC   BIC  logLik deviance   Chisq Chi Df
## final_model_axis       41 9828.7 10067 -4873.4   9746.7
## final_model_rob        41 9817.6 10056 -4867.8   9735.6 11.1501      0
## final_model_pach       41 9824.5 10063 -4871.2   9742.5  0.0000      0
## final_model_test       42 9808.9 10053 -4862.5   9724.9 17.5025      1
## final_model_rob_music  43 9818.3 10068 -4866.1   9732.3  0.0000      1
## final_model_axis_music 43 9830.2 10080 -4872.1   9744.2  0.0000      0
## final_model_pach_music 43 9824.9 10075 -4869.5   9738.9  5.2387      0
##                        Pr(>Chisq)
## final_model_axis
## final_model_rob         < 2.2e-16 ***
## final_model_pach                1
## final_model_test        2.869e-05 ***
## final_model_rob_music           1
## final_model_axis_music          1
## final_model_pach_music  < 2.2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
corrplot::corrplot(cor(as.matrix(df[,c('KnowAxis', 'KnowRob', 'PachListen', 'Musician','ClsListen')])))
```



Popular Model

```
lm_popular_expansive <- lm(Popular ~ Instrument * Harmony * Voice,data=df)
model4_basic <- lm(Popular ~ Instrument + Harmony + Voice,data=df)
model4_aic <- stepAIC(lm_popular_expansive, ~., trace=0)
anova(model4_aic,model4_basic,lm_popular_expansive)
```

```
## Analysis of Variance Table
##
## Model 1: Popular ~ Instrument + Harmony
## Model 2: Popular ~ Instrument + Harmony + Voice
## Model 3: Popular ~ Instrument * Harmony * Voice
##   Res.Df   RSS Df Sum of Sq      F Pr(>F)
## 1   2479 12444
## 2   2477 12429  2    15.225 1.5170 0.2196
## 3   2449 12290 28   139.014 0.9893 0.4809
```

```
AIC(model4_aic,model4_basic,lm_popular_expansive)
```

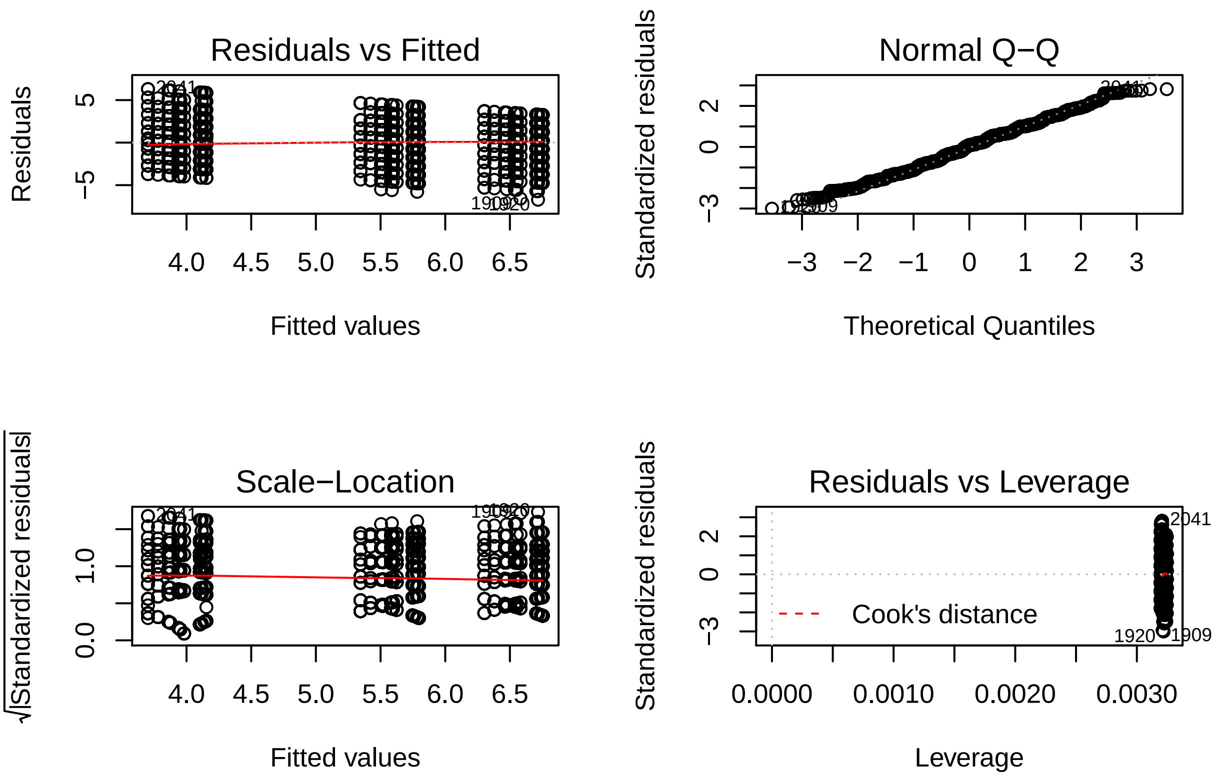
```
##                      df      AIC
## model4_aic            7 11069.42
## model4_basic          9 11070.38
```

```
## lm_popular_expansive 37 11098.43
```

Implementing a stepwise AIC procedure to find the model which minimizes Akaines' Information Coeficient suggests we only use the Instrument indicator variable. Our ANOVA F-test suggests that there is no discenible difference among any of the models, the AIC only, the all additive model, or the saturadted model. For the purposed of our project we will continue to with the addtivie experimental variable model

Looks like instrument remains is the main predictor, by far. Both string instruments and piano have string negative associations with the Popular rating. This is as our primary hypothesis would suggest, reflecting the inverse relationship that was experienced with Classical ratings. In addition, the Harmonic motion of I-V-VI is negatively signifciant against the baseline of I-IV-V. This similarly inverts the relationship we noted on ratings of Classical.

```
par(mfrow=c(2,2))
plot(model4_basic)
```



Our diagnostic plots are a bit curious but that is due to the bounded ordinal data which we are using. iAs our Fitted Values increase, the flat top of our residuals goes down. In addition, there aer bands of residuals at only a handful of fitted values. This is also due to the categorical nature of our data - we are bounded by discrete combinations of the binary variables. Other than these curious effects, our residuals don't exhibit heteroskedasticity or other biases, the data appears fairly normal, and there are no clearly problematic leverage points.

Random effects on Popular model

```
lm_pop_hvi_re <- lmer(Popular ~  1 + Harmony + Voice + Instrument + (1 + Harmony + Voice + Instrument|Su
                      data=df,REML=F,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun =
```

```
## boundary (singular) fit: see ?isSingular
```

```r
# boundary issue


lm_pop_vi_re <- lmer(Popular ~  1 + Harmony +Voice + Instrument + (1 +  Voice + Instrument|Subject),
                     data=df,REML=F,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 
```

```
## boundary (singular) fit: see ?isSingular
```

```r
lm_pop_hi_re <- lmer(Popular ~  1 + Harmony +Voice + Instrument + (1 + Harmony + Instrument|Subject),
                     data=df,REML=F,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 
```

```
## boundary (singular) fit: see ?isSingular
```

```r
lm_pop_hv_re <- lmer(Popular ~  1 + Harmony + Voice + Instrument + (1 + Harmony + Voice |Subject),
                     data=df,REML=F,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 
```

```
## boundary (singular) fit: see ?isSingular
```

```r
lm_pop_h_re <- lmer(Popular ~  1 + Harmony + Voice + Instrument + (1 + Harmony  |Subject),
                    data=df,REML=F,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1
```

```
## boundary (singular) fit: see ?isSingular
```

```r
lm_pop_v_re <- lmer(Popular ~  1 + Harmony + Voice + Instrument + (1 + Voice |Subject),
                    data=df,REML=F,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1
```

```
## boundary (singular) fit: see ?isSingular
```

```r
# boundary

lm_pop_i_re <- lmer(Popular ~  1 + Harmony + Voice + Instrument + (1 + Instrument |Subject),
                    data=df,REML=F,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1
lm_pop_re <- lmer(Popular ~  1 + Harmony + Voice + Instrument + (1 |Subject),
                  data=df,REML=F,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5
```

```r
anova(lm_pop_hvi_re, lm_pop_vi_re, lm_pop_hv_re, lm_pop_hi_re, lm_pop_h_re, lm_pop_i_re, lm_pop_v_re, l
```

```
## Data: df
## Models:
## lm_pop_re: Popular ~ 1 + Harmony + Voice + Instrument + (1 | Subject)
## lm_pop_i_re: Popular ~ 1 + Harmony + Voice + Instrument + (1 + Instrument |
## lm_pop_i_re:     Subject)
## lm_pop_v_re: Popular ~ 1 + Harmony + Voice + Instrument + (1 + Voice | Subject)
## lm_pop_h_re: Popular ~ 1 + Harmony + Voice + Instrument + (1 + Harmony | Subject)
## lm_pop_vi_re: Popular ~ 1 + Harmony + Voice + Instrument + (1 + Voice + Instrument |
## lm_pop_vi_re:     Subject)
## lm_pop_hv_re: Popular ~ 1 + Harmony + Voice + Instrument + (1 + Harmony + Voice |
## lm_pop_hv_re:     Subject)
## lm_pop_hi_re: Popular ~ 1 + Harmony + Voice + Instrument + (1 + Harmony + Instrument |
## lm_pop_hi_re:     Subject)
## lm_pop_hvi_re: Popular ~ 1 + Harmony + Voice + Instrument + (1 + Harmony + Voice +
## lm_pop_hvi_re:     Instrument | Subject)
##               Df     AIC   BIC  logLik deviance   Chisq Chi Df Pr(>Chisq)
## lm_pop_re     10 10340.4 10398 -5160.2  10320.4
## lm_pop_i_re   15 10004.9 10092 -4987.5   9974.9 345.461      5  < 2.2e-16
## lm_pop_v_re   15 10350.1 10437 -5160.1  10320.1   0.000      0     1.0000
## lm_pop_h_re   19 10293.3 10404 -5127.7  10255.3  64.766      4  2.882e-13
```

```
## lm_pop_vi_re  24 10018.2 10158 -4985.1   9970.2 285.162      5  < 2.2e-16
## lm_pop_hv_re  30 10308.0 10482 -5124.0  10248.0   0.000      6     1.0000
## lm_pop_hi_re  30  9914.4 10089 -4927.2   9854.4 393.588      0  < 2.2e-16
## lm_pop_hvi_re 45  9930.6 10192 -4920.3   9840.6  13.769     15     0.5431
##
## lm_pop_re
## lm_pop_i_re   ***
## lm_pop_v_re
## lm_pop_h_re   ***
## lm_pop_vi_re  ***
## lm_pop_hv_re
## lm_pop_hi_re  ***
## lm_pop_hvi_re
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

In our new random effects model where we have random effects on harmony and instrument. These are the same as in the classical data scenario, affirming thoughts regarding subject level variation in general along these dimensions. Different individuals are responsive to kinds of harmony and instruments, when evaluating music both for the dimensions of Popular and Classical.

```r
pop_model_re <- lm_pop_hi_re
```

Now that we have identified the Random Effects on the experimental variables, lets bring in the remaining variables and find our best model.

Fixed Effects selection for Popular model

Step AIC on fixed effects model

```r
step_aic_pop<-stepAIC(model4_basic, eval(paste('~ . +', paste(c(potential_num_vars, potential_factor_va
                                                              collapse=' + '), sep='')), trace=0)
summary(step_aic_pop)
```

```
##
## Call:
## lm(formula = Popular ~ Instrument + Harmony + KnowRob + Selfdeclare +
##     PachListen + X1990s2000s + GuitarPlay + Composing + X1990s2000s.minus.1960s1970s +
##     ClsListen + APTheory + ConsNotes, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.6273 -1.5808  0.1231  1.5338  6.0695
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)                    6.63605    0.27201  24.396  < 2e-16 ***
## Instrumentpiano               -0.94370    0.10769  -8.763  < 2e-16 ***
## Instrumentstring              -2.59522    0.10700 -24.254  < 2e-16 ***
## HarmonyI-V-IV                 -0.03412    0.12382  -0.276 0.782922
## HarmonyI-V-VI                 -0.27756    0.12387  -2.241 0.025131 *
## HarmonyIV-I-V                 -0.20258    0.12382  -1.636 0.101934
## KnowRob                        0.23374    0.03061   7.635 3.20e-14 ***
## Selfdeclare                    0.29269    0.05484   5.338 1.03e-07 ***
## PachListen                    -0.21572    0.04502  -4.792 1.75e-06 ***
## X1990s2000s                    0.16107    0.03864   4.169 3.17e-05 ***
```

```
## GuitarPlay                    -0.19257    0.04961  -3.881 0.000107 ***
## Composing                      0.17103    0.04652   3.677 0.000241 ***
## X1990s2000s.minus.1960s1970s -0.11906    0.03563  -3.341 0.000847 ***
## ClsListen                    -0.11190    0.03489  -3.207 0.001356 **
## APTheory                       0.24349    0.12225   1.992 0.046504 *
## ConsNotes                    -0.05312    0.02790  -1.904 0.057049 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.184 on 2469 degrees of freedom
## Multiple R-squared:  0.2338, Adjusted R-squared:  0.2291
## F-statistic: 50.22 on 15 and 2469 DF,  p-value: < 2.2e-16
```

Now we have an AIC optimized model in the simple linear model case. We now perform backwards variable selection on the identified Random Effects model, towards either the base model identified above or to an an AIC / BIC Optimized model.

Working backwards from saturated Popular model

```
pop_model_re_fixed_effect_1_base <- lmer(
  Popular ~ Voice+Instrument+Harmony+KnowRob+Selfdeclare+PachListen+X1990s2000s+ClsListen+Composing+
    GuitarPlay+X1990s2000s.minus.1960s1970s+ConsNotes+APTheory+OMSI + (1|Subject),
  REML=F,data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
exactLRT(pop_model_re_fixed_effect_1_base,step_aic_pop)
```

```
##
##  simulated finite sample distribution of LRT. (p-value based on
##  10000 simulated values)
##
## data:
## LRT = 609.62, p-value < 2.2e-16
```

Test that Original Random Effects perform better than simple repeated measures

```
pop_model_re_fixed_effect_1 <- lmer(
  Popular ~ Voice+Instrument+Harmony+KnowRob+Selfdeclare+PachListen+X1990s2000s+ClsListen+Composing+
    GuitarPlay+X1990s2000s.minus.1960s1970s+ConsNotes+APTheory+OMSI + (1 + Harmony + Instrument|Subject
  REML=F,data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
AIC(pop_model_re_fixed_effect_1_base, pop_model_re_fixed_effect_1)
```

```
##                                  df      AIC
## pop_model_re_fixed_effect_1_base 21 10349.74
## pop_model_re_fixed_effect_1      41  9923.23
```

Backtracing from full fixed effects in multi level model

```
pop_model_re_fixed_effect_2 <- lmer(
  Popular ~ Voice+Instrument+Harmony+KnowRob+Selfdeclare+PachListen+X1990s2000s+ClsListen+Composing+
    GuitarPlay+X1990s2000s.minus.1960s1970s+ConsNotes+OMSI + (1 + Harmony + Instrument|Subject),
  REML=F,data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))

summary(pop_model_re_fixed_effect_2)

pop_model_re_fixed_effect_3 <- lmer(
```

```r
  Popular ~ Voice+Instrument+Harmony+KnowRob+Selfdeclare+PachListen+X1990s2000s+ClsListen+Composing+
    X1990s2000s.minus.1960s1970s+ConsNotes+OMSI + (1 + Harmony + Instrument|Subject),
  REML=F,data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))

pop_model_re_fixed_effect_4 <- lmer(
  Popular ~ Voice+Instrument+Harmony+KnowRob+Selfdeclare+X1990s2000s+ClsListen+Composing+
    X1990s2000s.minus.1960s1970s+ConsNotes+OMSI + (1 + Harmony + Instrument|Subject),
  REML=F,data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))

pop_model_re_fixed_effect_5 <- lmer(
  Popular ~ Voice+Instrument+Harmony+KnowRob+Selfdeclare+X1990s2000s+ClsListen+Composing+
    ConsNotes+OMSI + (1 + Harmony + Instrument|Subject),
  REML=F,data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))

pop_model_re_fixed_effect_6 <- lmer(
  Popular ~ Voice+Instrument+Harmony+KnowRob+Selfdeclare+X1990s2000s+ClsListen+Composing+
    OMSI + (1 + Harmony + Instrument|Subject),
  REML=F,data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))

pop_model_re_fixed_effect_7 <- lmer(
  Popular ~ Voice+Instrument+Harmony+KnowRob+Selfdeclare+ClsListen+Composing+OMSI +
    (1 + Harmony + Instrument|Subject),
  REML=F,data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))

pop_model_re_fixed_effect_8 <- lmer(
  Popular ~ Voice+Instrument+Harmony+KnowRob+Selfdeclare+ClsListen+OMSI + (1 + Harmony + Instrument|Sub
  REML=F,data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))

pop_model_re_fixed_effect_9 <- lmer(
  Popular ~ Voice+Instrument+Harmony+KnowRob+Selfdeclare + OMSI + (1 + Harmony + Instrument|Subject),
  REML=F,data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))

pop_model_re_fixed_effect_10 <- lmer(
  Popular ~ Voice+Instrument+Harmony+KnowRob+Selfdeclare + (1 + Harmony + Instrument|Subject),
  REML=F,data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))

pop_model_re_fixed_effect_11 <- lmer(
  Popular ~ Voice+Instrument+Harmony+Selfdeclare + (1 + Harmony + Instrument|Subject),
  REML=F,data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))

pop_model_re_fixed_effect_13 <- lmer(
  Popular ~ Voice+Instrument+Harmony*KnowRob + (1 + Harmony + Instrument|Subject),REML=F,
  data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```r
pop_model_re_fixed_effect_12 <- lmer(
  Popular ~ Voice+Instrument+Harmony+KnowRob + (1 + Harmony + Instrument|Subject),REML=F,
  data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```r
# AIC(pop_model_re, pop_model_re_fixed_effect_1, pop_model_re_fixed_effect_2,
#     pop_model_re_fixed_effect_3, pop_model_re_fixed_effect_4, pop_model_re_fixed_effect_5,
#     pop_model_re_fixed_effect_6, pop_model_re_fixed_effect_7, pop_model_re_fixed_effect_8,
#     pop_model_re_fixed_effect_9, pop_model_re_fixed_effect_10, pop_model_re_fixed_effect_11,
```

```r
#     pop_model_re_fixed_effect_12,pop_model_re_fixed_effect_13 )
```

```r
summary(pop_model_re_fixed_effect_12)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula:
## Popular ~ Voice + Instrument + Harmony + KnowRob + (1 + Harmony +
##     Instrument | Subject)
##    Data: df
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   9913.0  10093.4  -4925.5   9851.0     2454
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9631 -0.5852  0.0123  0.5801  3.4655
##
## Random effects:
##  Groups   Name             Variance Std.Dev. Corr
##  Subject  (Intercept)      1.62587  1.2751
##           HarmonyI-V-IV    0.09516  0.3085    0.49
##           HarmonyI-V-VI    0.90407  0.9508   -0.15 -0.37
##           HarmonyIV-I-V    0.21089  0.4592   -0.26 -0.56 -0.29
##           Instrumentpiano  1.37840  1.1741   -0.26 -0.32 -0.17 -0.12
##           Instrumentstring 3.31604  1.8210   -0.38 -0.42 -0.21 -0.04  0.72
##  Residual                  2.44557  1.5638
## Number of obs: 2485, groups:  Subject, 70
##
## Fixed effects:
##                  Estimate Std. Error t value
## (Intercept)       6.44284    0.18742  34.377
## Voicepar3rd       0.14782    0.07693   1.922
## Voicepar5th       0.16938    0.07682   2.205
## Instrumentpiano  -0.94148    0.16038  -5.870
## Instrumentstring -2.58142    0.23081 -11.184
## HarmonyI-V-IV    -0.03681    0.09608  -0.383
## HarmonyI-V-VI    -0.27815    0.14422  -1.929
## HarmonyIV-I-V    -0.20315    0.10433  -1.947
## KnowRob           0.16177    0.08226   1.967
##
## Correlation of Fixed Effects:
##             (Intr) Vcpr3r Vcpr5t Instrmntp Instrmnts HI-V-I HI-V-V HIV-I-
## Voicepar3rd -0.204
## Voicepar5th -0.204  0.500
## Instrumntpn -0.285 -0.001 -0.001
## Instrmntstr -0.361 -0.001  0.000  0.679
## HrmnyI-V-IV -0.063 -0.002 -0.002 -0.107    -0.154
## HrmnyI-V-VI -0.238 -0.001 -0.002 -0.114    -0.159     0.171
## HrmnyIV-I-V -0.310  0.002 -0.002 -0.057    -0.020     0.278  0.142
## KnowRob     -0.338  0.000  0.000  0.001     0.001     0.000  0.000  0.000
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
pop_model_re_fixed_effect_final <- pop_model_re_fixed_effect_12
```

Our AIC minimized model is the same elementary base model as identified in the random effects identification stage, with the addition of the boolean of `KnowRob`, whether or not the the subject knows a video about the I V VI harmonic chord progression.

Testing Musicianship in finalized Multilevel Popular rankings model

```
pop_model3_musician_1 <- lmer(
  Popular ~ Musician_1 * (Voice+Instrument+Harmony+KnowRob) +  (1 + Harmony + Instrument|Subject),
  REML=F, data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
pop_model3_musician_1 <- lmer(
  Popular ~ Musician_1 * (Voice+Instrument+Harmony+KnowRob) +  (1 + Harmony + Instrument|Subject),
  REML=F, data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(pop_model3_musician_1)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula:
## Popular ~ Musician_1 * (Voice + Instrument + Harmony + KnowRob) +
##     (1 + Harmony + Instrument | Subject)
##    Data: df
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   9908.3  10141.0  -4914.1   9828.3     2445
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9808 -0.5876  0.0064  0.5750  3.3698
##
## Random effects:
##  Groups   Name             Variance Std.Dev. Corr
##  Subject  (Intercept)      1.27079  1.1273
##           HarmonyI-V-IV    0.06902  0.2627    0.33
##           HarmonyI-V-VI    0.81974  0.9054   -0.03 -0.25
##           HarmonyIV-I-V    0.20962  0.4578   -0.29 -0.60 -0.33
##           Instrumentpiano  1.31655  1.1474   -0.17 -0.25 -0.25 -0.15
##           Instrumentstring 3.21163  1.7921   -0.32 -0.39 -0.29 -0.06  0.71
##  Residual                  2.44544  1.5638
## Number of obs: 2485, groups:  Subject, 70
##
## Fixed effects:
##                             Estimate Std. Error t value
## (Intercept)                  5.54019    0.36088  15.352
## Musician_1                   1.24856    0.41179   3.032
## Voicepar3rd                  0.11892    0.16172   0.735
## Voicepar5th                  0.12594    0.16140   0.780
## Instrumentpiano             -0.49446    0.33028  -1.497
## Instrumentstring            -1.97118    0.47640  -4.138
```

```
## HarmonyI-V-IV                -0.35136    0.19772  -1.777
## HarmonyI-V-VI                 0.25398    0.29336   0.866
## HarmonyIV-I-V                -0.12456    0.21846  -0.570
## KnowRob                      -0.74648    0.79807  -0.935
## Musician_1:Voicepar3rd        0.03854    0.18385   0.210
## Musician_1:Voicepar5th        0.05561    0.18352   0.303
## Musician_1:Instrumentpiano  -0.58022    0.37582  -1.544
## Musician_1:Instrumentstring -0.79163    0.54226  -1.460
## Musician_1:HarmonyI-V-IV     0.40634    0.22482   1.807
## Musician_1:HarmonyI-V-VI    -0.68973    0.33381  -2.066
## Musician_1:HarmonyIV-I-V    -0.10214    0.24858  -0.411
## Musician_1:KnowRob           0.87137    0.80194   1.087
```

```
##
## Correlation matrix not shown by default, as p = 18 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it
```

```
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
pop_model3_musician_2 <- lmer(
  Popular ~ Voice+ Musician * (Instrument+Harmony+KnowRob) + (1 + Harmony + Instrument|Subject),
  REML=F, data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))

pop_model3_musician_3 <- lmer(
  Popular ~ Voice+ Musician * (Instrument+Harmony)  +KnowRob + (1 + Harmony + Instrument|Subject),
  REML=F,data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
pop_model3_musician_4 <- lmer(
  Popular ~ Voice+ Instrument+ Musician * Harmony  +KnowRob + (1 + Harmony + Instrument|Subject),
  REML=F, data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
pop_model3_musician_5 <- lmer(
  Popular ~ Voice+ Instrument+ Musician * Harmony  +KnowRob + (1 + Harmony + Instrument|Subject),
  REML=F, data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
pop_model3_musician_6 <- lmer(
  Popular ~ Voice+ Instrument+ Musician * Harmony  + (1 + Harmony + Instrument+Musician|Subject),
  REML=F, data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
pop_model3_musician_7 <- lmer(
  Popular ~ Voice+ Instrument+ Musician + Harmony  + (1 + Harmony + Instrument+Musician|Subject),
  REML=F, data=df,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
anova(pop_model3_musician_1, pop_model3_musician_2,pop_model3_musician_3,pop_model3_musician_7, pop_mode
```

```
## Data: df
## Models:
```

```
## pop_model3_musician_5: Popular ~ Voice + Instrument + Musician * Harmony + KnowRob +
## pop_model3_musician_5:     (1 + Harmony + Instrument | Subject)
## pop_model3_musician_3: Popular ~ Voice + Musician * (Instrument + Harmony) + KnowRob +
## pop_model3_musician_3:     (1 + Harmony + Instrument | Subject)
## pop_model3_musician_2: Popular ~ Voice + Musician * (Instrument + Harmony + KnowRob) +
## pop_model3_musician_2:     (1 + Harmony + Instrument | Subject)
## pop_model3_musician_7: Popular ~ Voice + Instrument + Musician + Harmony + (1 + Harmony +
## pop_model3_musician_7:     Instrument + Musician | Subject)
## pop_model3_musician_1: Popular ~ Musician_1 * (Voice + Instrument + Harmony + KnowRob) +
## pop_model3_musician_1:     (1 + Harmony + Instrument | Subject)
## pop_model3_musician_6: Popular ~ Voice + Instrument + Musician * Harmony + (1 + Harmony +
## pop_model3_musician_6:     Instrument + Musician | Subject)
##                        Df    AIC   BIC  logLik deviance  Chisq Chi Df
## pop_model3_musician_5 35 9906.2 10110 -4918.1   9836.2
## pop_model3_musician_3 37 9903.5 10119 -4914.8   9829.5 6.6591      2
## pop_model3_musician_2 38 9904.4 10126 -4914.2   9828.4 1.1285      1
## pop_model3_musician_7 38 9912.1 10133 -4918.0   9836.1 0.0000      0
## pop_model3_musician_1 40 9908.3 10141 -4914.1   9828.3 7.7734      2
## pop_model3_musician_6 41 9907.6 10146 -4912.8   9825.6 2.7254      1
##                        Pr(>Chisq)
## pop_model3_musician_5
## pop_model3_musician_3    0.03581 *
## pop_model3_musician_2    0.28809
## pop_model3_musician_7    1.00000
## pop_model3_musician_1    0.02051 *
## pop_model3_musician_6    0.09876 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Final Popular model

```
pop_model3_musician_final <- lmer(
  Popular ~ Voice + Instrument + Harmony * Musician + (1 + Harmony + Instrument|Subject),
  REML=F,data=df_orig,control = lmerControl(optimizer = 'bobyqa',optCtrl = list(maxfun = 1e5)))
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(pop_model3_musician_final)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula:
## Popular ~ Voice + Instrument + Harmony * Musician + (1 + Harmony +
##     Instrument | Subject)
##    Data: df_orig
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   9902.2  10100.0  -4917.1   9834.2     2451
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9816 -0.5847  0.0113  0.5796  3.3656
##
## Random effects:
##  Groups   Name             Variance Std.Dev. Corr
```

```
##  Subject  (Intercept)      1.29386  1.1375
##           HarmonyI-V-IV    0.06931  0.2633    0.32
##           HarmonyI-V-VI    0.82179  0.9065   -0.05 -0.25
##           HarmonyIV-I-V    0.21006  0.4583   -0.28 -0.59 -0.33
##           Instrumentpiano  1.37666  1.1733   -0.16 -0.26 -0.25 -0.15
##           Instrumentstring 3.32082  1.8223   -0.28 -0.39 -0.29 -0.06  0.73
##  Residual                  2.44545  1.5638
## Number of obs: 2485, groups:  Subject, 70
##
## Fixed effects:
##                        Estimate Std. Error t value
## (Intercept)             5.53596    0.31716  17.455
## Voicepar3rd             0.14868    0.07693   1.933
## Voicepar5th             0.16892    0.07682   2.199
## Instrumentpiano        -0.94232    0.16029  -5.879
## Instrumentstring       -2.58223    0.23096 -11.181
## HarmonyI-V-IV          -0.32238    0.19657  -1.640
## HarmonyI-V-VI           0.33932    0.28822   1.177
## HarmonyIV-I-V          -0.10640    0.21806  -0.488
## Musician                1.33809    0.35300   3.791
## HarmonyI-V-IV:Musician  0.36888    0.22312   1.653
## HarmonyI-V-VI:Musician -0.80043    0.32614  -2.454
## HarmonyIV-I-V:Musician -0.12559    0.24797  -0.506
##
## Correlation of Fixed Effects:
##             (Intr) Vcpr3r Vcpr5t Instrmntp Instrmnts HrI-V-IV HrI-V-VI
## Voicepar3rd -0.120
## Voicepar5th -0.120  0.500
## Instrumntpn -0.118 -0.001 -0.001
## Instrmntstr -0.157 -0.001  0.000  0.679
## HrmnyI-V-IV -0.208 -0.004 -0.001 -0.036    -0.060
## HrmnyI-V-VI -0.280 -0.001 -0.003 -0.083    -0.104     0.218
## HrmnyIV-I-V -0.387  0.002  0.000 -0.034    -0.015     0.295    0.131
## Musician    -0.859 -0.001  0.000  0.001     0.001     0.196    0.267
## HrmI-V-IV:M  0.191  0.004  0.000  0.000     0.001    -0.878   -0.187
## HrmI-V-VI:M  0.262  0.001  0.002  0.000     0.000    -0.188   -0.874
## HrmIV-I-V:M  0.343 -0.001 -0.002  0.001     0.001    -0.258   -0.113
##             HrIV-I-V Musicn HI-V-IV: HI-V-VI:
## Voicepar3rd
## Voicepar5th
## Instrumntpn
## Instrmntstr
## HrmnyI-V-IV
## HrmnyI-V-VI
## HrmnyIV-I-V
## Musician     0.350
## HrmI-V-IV:M -0.259   -0.224
## HrmI-V-VI:M -0.113   -0.306  0.214
## HrmIV-I-V:M -0.878   -0.399  0.295    0.129
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```