

Exercises:

Note: there are several ways to attach each question below, e.g., some questions may have a Bayesian answer as well as a non-Bayesian answer. Except where explicitly noted, you will get full credit for choosing a single suitable method, and applying and interpreting it correctly, regardless of whether there is another method that also might work.

1. The three main experimental factors.
 - a. Examine the influence of the three main experimental factors (Instrument, Harmony & Voice) on classical ratings, using conventional linear models and/or variance models. Comment briefly on your findings, providing suitable brief evidence for each result. Hint: to determine whether harmony is important, for example, one might compare the fit of a model with harmony in it, to one without Harmony. To determine how particular kinds of how particular kinds of harmony affect ratings, one might begin by looking at fixed effects estimates in a suitable model. Etc.

```
> summary(lm.1<-lm(Classical~harmony+voice-1))
```

Call:

```
lm(formula = Classical ~ harmony + voice - 1)
```

Residuals:

Min	1Q	Median	3Q	Max
-7.204	-1.830	0.545	2.876	13.384

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
harmony	1.08032	0.04317	25.02	<2e-16 ***
voice	1.29436	0.05472	23.66	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.155 on 2491 degrees of freedom
(27 observations deleted due to missingness)

Multiple R-squared: 0.7545, Adjusted R-squared: 0.7543

F-statistic: 3828 on 2 and 2491 DF, p-value: < 2.2e-16

```
> summary(lm.2<-lm(Classical~harmony+instrument-1))
```

Call:

```
lm(formula = Classical ~ harmony + instrument - 1)
```

Residuals:

Min	1Q	Median	3Q	Max
-7.4884	-1.4884	0.1688	1.8461	10.5116

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
harmony	0.49664	0.03318	14.97	<2e-16 ***
instrument	2.16729	0.04198	51.62	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.427 on 2491 degrees of freedom
(27 observations deleted due to missingness)

Multiple R-squared: 0.8547, Adjusted R-squared: 0.8546

F-statistic: 7329 on 2 and 2491 DF, p-value: < 2.2e-16

```
> summary(lm.3<-lm(Classical~harmony+instrument+voice-1))
```

Call:

```
lm(formula = Classical ~ harmony + instrument + voice - 1)
```

Residuals:

Min	1Q	Median	3Q	Max
-7.5353	-1.5069	0.0506	1.8575	11.0222

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
harmony	0.40345	0.03670	10.993	< 2e-16 ***
instrument	2.02842	0.04815	42.127	< 2e-16 ***
voice	0.27876	0.04827	5.775	8.65e-09 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.411 on 2490 degrees of freedom
(27 observations deleted due to missingness)

Multiple R-squared: 0.8567, Adjusted R-squared: 0.8565

F-statistic: 4960 on 3 and 2490 DF, p-value: < 2.2e-16

The instrument factor is largest factor on classical rating (coefficients 2.02842); 0.40345 in harmony, and 0.26876 in the voice factor.

Voice factors are relatively less important than the others. I compare the fit of model (lm.3) with all three factors in it, to one without Voice (lm.2). Comparing the fit of the model with all factors in it, to one without Voice, its R-squared value is very slightly larger than the one without Voice. On the other hand, Instrument factor is highly important than the others. I compare the fit of model (lm.3) with all three factors in it, to one without Instrument factor (lm.1), R-squared value highly improves.

To determine how particular kinds of Voice affect ratings, we look at the particular types of voice. The coefficients for all voice stimuli are significant, with having different coefficients. Same as hypotheses, 'Contray Motion' highly affects classical ratings (with 2.66900 coefficients) than the other stimuli in voice.

```
> summary(lm.4a<-lm(Classical~harmony+instrument+factor(voice)-1))
```

Call:

```
lm(formula = Classical ~ harmony + instrument + factor(voice) -
1)
```

Residuals:

```
Min 1Q Median 3Q Max
-6.3792 -1.7176 -0.0516 1.7588 11.2512
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
harmony	0.09481	0.04150	2.285	0.0224 *
instrument	1.56686	0.05668	27.642	<2e-16 ***
factor(voice)1	2.66900	0.17351	15.383	<2e-16 ***
factor(voice)2	2.25622	0.17338	13.013	<2e-16 ***
factor(voice)3	2.29944	0.17359	13.246	<2e-16 ***

```
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 2.318 on 2488 degrees of freedom
(27 observations deleted due to missingness)

Multiple R-squared: 0.8676, Adjusted R-squared: 0.8673

F-statistic: 3259 on 5 and 2488 DF, p-value: < 2.2e-16

To determine how particular kinds of harmonic progression affect ratings, we look at the particular types of harmony. The coefficients for all harmony stimuli are statistically significant, with having different coefficients. Same as hypotheses, 'I-V-vi' highly affects classical ratings (with 3.58916 coefficients) than the other stimuli in harmonic.

```
> summary(lm.4b)
```

Call:

```
lm(formula = Classical ~ factor(harmony) + instrument + voice -
1)
```

Residuals:

```
Min 1Q Median 3Q Max
-6.733 -1.767 0.016 1.800 11.615
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
factor(harmony)1	2.81904	0.18389	15.330	< 2e-16	***
factor(harmony)2	2.78790	0.18423	15.133	< 2e-16	***
factor(harmony)3	3.58916	0.18428	19.477	< 2e-16	***
factor(harmony)4	2.86997	0.18410	15.590	< 2e-16	***
instrument	1.56673	0.05623	27.862	< 2e-16	***
voice	-0.18532	0.05640	-3.286	0.00103	**

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.3 on 2487 degrees of freedom
(27 observations deleted due to missingness)

Multiple R-squared: 0.8697, Adjusted R-squared: 0.8694

F-statistic: 2767 on 6 and 2487 DF, p-value: < 2.2e-16

To determine how particular kinds of instrument affect ratings, we look at the particular types of instrument. The coefficients for instrument stimuli are statistically significant, with having different coefficients. Note that the difference of coefficients are big; the difference of string (coefficient 7.54050) and guitar (4.40750) are 3.133. This implies that people are inclined to call music played by a string quartet "classical".

```
> summary(lm.4c<-lm(Classical~harmony+factor(Instrument)+voice-1))
```

Call:

```
lm(formula = Classical ~ harmony + factor(Instrument) + voice -  
1)
```

Residuals:

Min	1Q	Median	3Q	Max
-6.3675	-1.6416	-0.0818	1.6819	11.2632

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
harmony	0.09524	0.04151	2.294	0.02186	*
factor(Instrument)guitar	4.40750	0.17353	25.399	< 2e-16	***
factor(Instrument)piano	5.78022	0.17391	33.236	< 2e-16	***
factor(Instrument)string	7.54050	0.17342	43.482	< 2e-16	***
voice	-0.18466	0.05687	-3.247	0.00118	**

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.319 on 2488 degrees of freedom
(27 observations deleted due to missingness)

Multiple R-squared: 0.8675, Adjusted R-squared: 0.8672

F-statistic: 3257 on 5 and 2488 DF, p-value: < 2.2e-16

- b. Since we have approximately 36 ratings from each participant, we can fit a random intercept for each participant if we wish. Such a model is called a “repeated measures” model.
- i. Carefully write this model in mathematical terms as a hierarchical linear model.

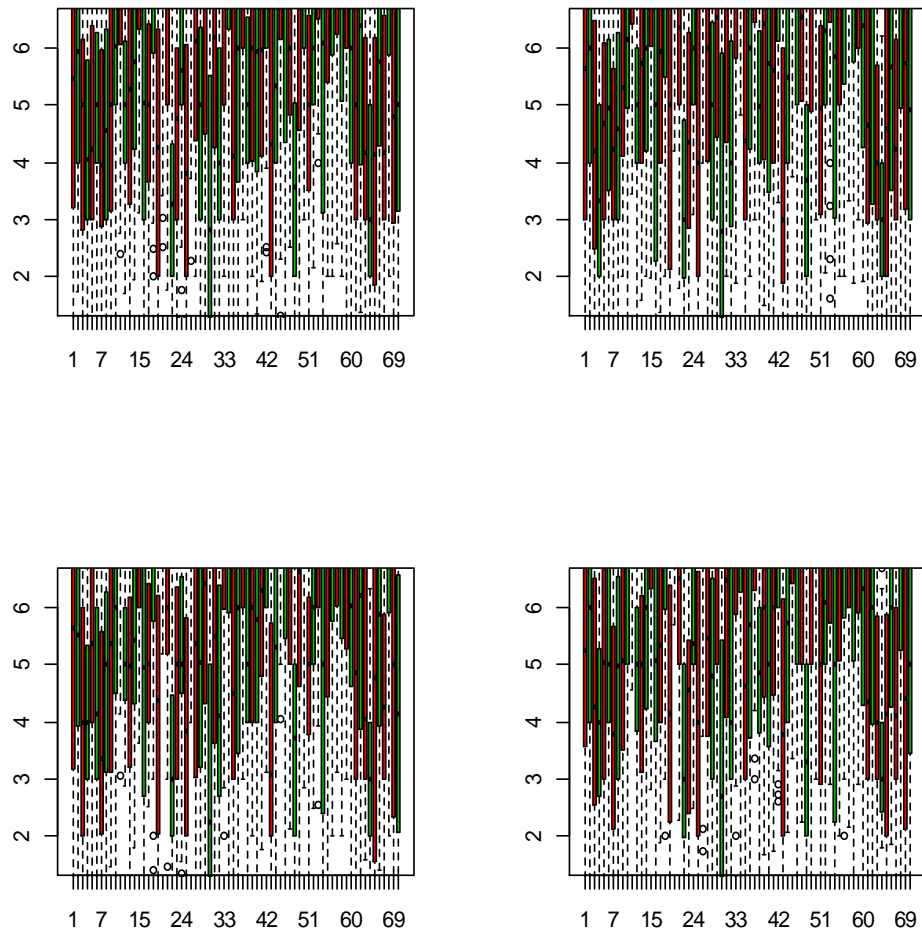
$$y_i = \alpha_{0[j]} + \alpha_1 x_{1i} + \alpha_2 x_{2i} + \alpha_3 x_{3i} + \epsilon_i, \epsilon_i \sim \text{iid } N(0, \sigma^2)$$

$$\alpha_{0[j]} = \beta_0 + \eta_{0[j]} \sim \text{iid } N(0, \tau_0^2)$$

- ii. Use at least two different methods to test whether the random intercept is needed in the model. Is the random effect needed? Justify your answer with evidence from your models.

Simulation Check

To check whether each subject should be random effect, I look at the spread in the simulated data, vs spread of the original data “classical”, especially between groups defined by “subject”



The box-plots show that the data (red) is less spread out within subjects than predictions (green) from the fixed effects would suggest. The variability in classical rating between subjects is smaller the expected from data simulated from the rube.lm.5.new.fit model ("newclassic"). Thus, we keep Random-effects

```
#summary(lm.5<-lm(Classical~harmony+instrument+voice+fsubject))
rube.lm.5 <- "model {
  # LEVEL 01
  for (i in 1:N) {
    Classical[i] ~ dnorm(mu[i],sig2inv)
    mu[i] <- a0[subject[i]] + a1*harmony[i]+a2*instrument[i]+a3*voice[i]
  }
  # LEVEL 02
  for (j in 1:J){
    a0[j] ~ dnorm(0,0.0001)
```

```

    }
    a1 ~ dnorm(0,0.0001)
    a2 ~ dnorm(0,0.0001)
    a3 ~ dnorm(0,0.0001)

    sig2inv <- pow(sig,-2)
    sig ~ dunif(0,100)

  }"

data.list<-list(Classical=Classical, harmony=harmony,
instrument=instrument,
voice=voice, subject=subject, N=length(Classical),
J=length(unique(subject)))

rube.lm.5.inits<-function(){
  list (a0=rnorm(J),a1=rnorm(1),a2=rnorm(1),a3=rnorm(1),
        sig=runif(1,0,10))
}

rube(rube.lm.5, data.list,rube.lm.5.inits)

rube.lm.5.fit <-rube(rube.lm.5, data.list,rube.lm.5.inits,
                    parameters.to.save=c("a0","a1","a2","a3",
                    "sig"),
                    n.chains=3)
rube.lm.5.fit
#windows()
#p3(rube.lm.5.fit)

# now we generate some fake data sets (fake 'classical's')...

#attach(rube.lm.5.fit$sims.list)
a0.hat <- apply(rube.lm.5.fit$sims.list$a0,2,mean)
a1.hat <- mean(rube.lm.5.fit$sims.list$a1)
a2.hat <- mean(rube.lm.5.fit$sims.list$a2)
a3.hat <- mean(rube.lm.5.fit$sims.list$a3)
sig.hat <- mean(rube.lm.5.fit$sims.list$sig)
detach()

rube.lm.5.new <- "model {
  # LEVEL 01
  for (i in 1:N) {
    Classical[i] ~ dnorm(mu[i],sig2inv)
    mu[i] <- a0[subject[i]] + a1*harmony[i]+a2*instrument[i]+a3*voice[i]
  }
  # LEVEL 02
  for (j in 1:J){

```

```

a0[j] ~ dnorm(0,0.0001)
}
a1 ~ dnorm(0,0.0001)
a2 ~ dnorm(0,0.0001)
a3 ~ dnorm(0,0.0001)

sig2inv <- pow(sig,-2)
sig ~ dunif(0,100)

for (i in 1:N){
newClassical[i]~dnorm(mu[i], sig2inv)
}

}"

rube.lm.5.new.inits<-function(){
  list (a0=a0.hat,a1=a1.hat,a2=a2.hat,a3=a3.hat,
        sig=sig.hat)
}

rube(rube.lm.5.new, data.list,rube.lm.5.new.inits)
rube.lm.5.new.fit <-rube(rube.lm.5.new, data.list,rube.lm.5.new.inits,
                        parameters.to.save=c("newClassical"),
                        n.iter=400,
                        n.chains=1)
newclassic <- rube.lm.5.new.fit$sims.list$newClassical
dim(newclassic)
n.sims <- rube.lm.5.new.fit$n.keep
n<-nrow(ratings.data)

boxes <- function(i,col=2:3,ylim=c(1.5,6.5)) {
  tmp <- c(t(cbind(Classical,newclassic[i,])))
  boxplot(split(tmp,rep(subject,rep(2,n))),col=col,ylim=ylim)
}

par(mfrow=c(2,2))
samp <- sample(1:n.sims,4)
for (i in samp) boxes(i)

```

Directly testing random effects

```

library(RLRSim)
summary(lmer.2<-
lmer(Classical~harmony+instrument+voice+(1|fsubject)))
> exactRLRT(lmer.2)

```


simulated finite sample distribution of RLRT.

(p-value based on 10000 simulated values)

data:

RLRT = 738.9519, p-value < 2.2e-16

We have significant p-value, so we keep the random intercept for each subject (participant).

- iii. Re-examine the influence of the three main experimental factors (Instrument, Harmony & Voice) on Classical rating, using the repeated-measures model with the random intercept for participants.

```
#summary(lmer.2<-lmer(Classical~harmony+instrument+voice+(1|fsubject)))
rube.lmer.2 <- "model {
  # LEVEL 01
  for (i in 1:N) {
    Classical[i] ~ dnorm(mu[i],sig2inv)
    mu[i] <- a0[subject[i]] + b1*harmony[i]+b2*instrument[i]+b3*voice[i]
  }
  # LEVEL 02
  for (j in 1:J){
    a0[j] ~ dnorm(b0,tau2inv)
  }
  b0 ~ dnorm(0,0.0001)
  b1 ~ dnorm(0,0.0001)
  b2 ~ dnorm(0,0.0001)
  b3 ~ dnorm(0,0.0001)

  tau2inv <- pow(tau,-2)
  tau ~ dunif(0,100)

  sig2inv <- pow(sig,-2)
  sig ~ dunif(0,100)
}"

data.list<-list(Classical=Classical, harmony=harmony, instrument=instrument,
voice=voice, subject=subject, N=length(Classical), J=length(unique(subject)))

rube.lmer.2.inits<-function(){
  list (b0=rnorm(1),b1=rnorm(1),b2=rnorm(1),b3=rnorm(1),
        a0=rnorm(J),tau=runif(1,0,10),sig=runif(1,0,10))
}

rube(rube.lmer.2, data.list,rube.lmer.2.inits)
rube.lmer.2.fit <-rube(rube.lmer.2, data.list,rube.lmer.2.inits,
                      parameters.to.save=c("b0","b1","b2","b3",
                      "sig","tau"),
```

```

n.chains=3)
rube.lmer.2.fit

> (DIC <- c(rube.lm.5.fit$DIC,rube.lmer.2.fit$DIC))
[1] 10406.60 10404.59
> # check AIC and BIC...
> lm.3<-lm(Classical~harmony+instrument+voice)
> lm.5<-lm(Classical~harmony+instrument+voice+fsubject)
> lmer.2<-lmer(Classical~harmony+instrument+voice+(1|subject))
>
> (AIC <- c(AIC(lm.3),AIC(lm.5),AIC(lmer.2)))
[1] 11277.94 10408.57 10557.61
>
> (BIC <- c(BIC(lm.3),BIC(lm.5),BIC(lmer.2)))
[1] 11307.05 10839.34 10592.54

```

	lm.3	lm.5	lmer.2
AIC	11277.94	10408.57	10557.61
BIC	11307.05	10839.34	10592.54
DIC		10406.60	10404.59

AIC prefer lm.5, BIC and DIC prefer lmer.2

```

> rube.lmer.2.fit
Rube Results:
      mean  sd   2.5%   25%   50%   75%   97.5% Rhat n.eff
b0    2.77e+00 0.2291 2.32e+00 2.62e+00  2.768  2.930 3.24e+00 1.00 1000
b1    9.59e-02 0.0349 2.48e-02 7.35e-02  0.097  0.117 1.64e-01 1.00 1000
b2    1.57e+00 0.0480 1.47e+00 1.53e+00  1.566  1.599 1.66e+00 1.01 260
b3   -1.85e-01 0.0485 -2.80e-01 -2.19e-01 -0.185 -0.151 -9.35e-02 1.00 1000
deviance 1.03e+04 12.2658 1.03e+04 1.03e+04 10334.905 10343.194 1.04e+04 1.00 1000
sig    1.92e+00 0.0288 1.87e+00 1.90e+00  1.923  1.943 1.98e+00 1.00 750
tau    1.33e+00 0.1210 1.11e+00 1.25e+00  1.325  1.403 1.58e+00 1.01 320

DIC = 10410.83

```

After controlling the random intercept for participants, instrument is the largest influence on classical ratings ("b2" the coefficient of instrument is 1.57), harmony influence is next influential ("b1" the coefficient of harmony is 0.0959) and voice is negative influence on classical ratings ("b3" the coefficient of voice is -1.86). The estimated standard deviation of participants and residual are 1.92 and 1.33.

- c. The random intercept in a repeated measures model can account for "personal biases" in ratings: perhaps person A is more inclined to rate everything as classical, and person B is more inclined to rate everything as popular. This can be accounted for by the random intercept.

Alternatively, perhaps personal biases vary with the type of instrument type of harmony, and/ or type of voice leading. For example, perhaps people vary in the degree to which they are inclined to call music played by a string quartet "classical". This suggests, e.g., a random effect of the form (1 | Subject: Instrument): a random draw is made from a single normal distribution, for each person/instrument combination. One could argue for a similar random effect for each person/harmony combination, and for each person/voice leading combination.

- i. Determine whether a model with all three new random effect terms (but not the original single random intercept) is better or worse than each of the models in problem 1a and 1b. Provide suitable evidence to justify your answer.

```
#problem 1a
lm.4b<-lm(Classical~factor(harmony)+instrument+voice-1))
#problem 1b
lmer.2<-lmer(Classical~harmony+instrument+voice+(1|fsubject)))
# now
lmer.3<-
lmer(Classical~(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)
)
```

To compare a fixed-effects model (problem 1a) and mixed models (1b and now), First, I fit the WinBugs version of lm.4b and lmer.3. For the Winbug version of lmer.2, I reported it above question 1b.

```
# lm.4b<-lm(Classical~factor(harmony)+instrument+voice-1))

rube.lm.4b <- "model {
# LEVEL 01
for (i in 1:N) {
  Classical[i] ~ dnorm(mu[i],sig2inv)
  mu[i] <- a0[harmony[i]] + a1*voice[i]+a2*instrument[i]
}
# LEVEL 02
for (j in 1:J){
  a0[j] ~ dnorm(0,0.0001)
}
a1 ~ dnorm(0,0.0001)
a2 ~ dnorm(0,0.0001)

sig2inv <- pow(sig,-2)
sig ~ dunif(0,100)

}"
```

```

data.list<-list(Classical=Classical, harmony=harmony,
instrument=instrument,
voice=voice, N=length(Classical), J=length(unique(harmony)))
J=length(unique(harmony))
rube.lm.4b.inits<-function(){
  list (a0=rnorm(J),a1=rnorm(1),a2=rnorm(1),
        sig=runif(1,0,10))
}

rube(rube.lm.4b, data.list,rube.lm.4b.inits)

rube.lm.4b.fit <-rube(rube.lm.4b, data.list,rube.lm.4b.inits,
                    parameters.to.save=c("a0","a1","a2",
                    "sig"),
                    n.chains=3)
rube.lm.4b.fit

```

Check the fit results of WinBugs version (rube.lm.4b.fit) of lm.4 and lm.4

<pre> > display(lm.4b) lm(formula = Classical ~ voice + instrument + factor(harmony) - 1) coef.est coef.se voice -0.19 0.06 instrument 1.57 0.06 factor(harmony)1 2.82 0.18 factor(harmony)2 2.79 0.18 factor(harmony)3 3.59 0.18 factor(harmony)4 2.87 0.18 --- n = 2493, k = 6 residual sd = 2.30, R-Squared = 0.87 </pre>	<pre> > c(rube.lm.4b.fit\$median\$a1, + rube.lm.4b.fit\$median\$a2, + rube.lm.4b.fit\$median\$a0[1:4]) [1] -0.1872814 1.5675592 2.8263618 2.7889199 3.5845661 2.8706569 </pre>
----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

```

rube.lmer.3 <- "model {
# LEVEL 01
for (i in 1:N) {
  Classical[i] ~ dnorm(mu[i],sig2inv)
  mu[i] <- a0[subject[i],harmony[i]]
+a1[subject[i],instrument[i]]+a2[subject[i],voice[i]]
}

# LEVEL 02
for (j in 1:n.subject){
  for (k in 1:n.harmony){
    a0[j,k] ~ dnorm(b0,tau02inv)}

```

```

    for (l in 1:n.instrument){
      a1[j,l] ~ dnorm(b1,tau12inv)}

    for (m in 1:n.voice){
      a2[j,m] ~ dnorm(b2,tau22inv)}
  }

  b0 ~ dnorm(0,0.0001)
  b1 ~ dnorm(0,0.0001)
  b2 ~ dnorm(0,0.0001)

  tau02inv <- pow(tau0,-2)
  tau0 ~ dunif(0,100)

  tau12inv <- pow(tau1,-2)
  tau1 ~ dunif(0,100)

  tau22inv <- pow(tau2,-2)
  tau2 ~ dunif(0,100)

  sig2inv <- pow(sig,-2)
  sig ~ dunif(0,100)
}"

rube.lmer.3.data.list <-list(Classical=Classical, harmony=harmony,
instrument=instrument,
voice=voice, subject=subject, N=length(Classical),
n.subject=length(unique(subject)),n.instrument=length(unique(instrument)),
n.voice=length(unique(voice)),n.harmony=length(unique(harmony)))

n.subject=length(unique(subject))
n.instrument=length(unique(instrument))
n.voice=length(unique(voice))
n.harmony=length(unique(harmony))
JK=n.subject*n.harmony
JL=n.subject*n.instrument
JM=n.subject*n.voice

rube.lmer.3.inits<-function(){
  list (b0=rnorm(1),b1=rnorm(1),b2=rnorm(1),

a0=matrix(rep(rnorm(JK)),n.subject,n.harmony),a1=matrix(rep(rnorm(JL)),n.subject,n.instrument),
      a2=matrix(rep(rnorm(JM)),n.subject,n.voice),
      tau0=runif(1,0,10),tau1=runif(1,0,10),tau2=runif(1,0,10),sig=runif(1,0,10
))
}

```

```
rube(rube.lmer.3, rube.lmer.3.data.list,rube.lmer.3.inits)
rube.lmer.3.fit <-rube(rube.lmer.3, rube.lmer.3.data.list,rube.lmer.3.inits,
  parameters.to.save=c("b0","b1","b2","a0","a1","a2",
    "sig","tau0","tau1","tau2"),
  n.chains=3)
rube.lmer.3.fit
```

Check the fit results of WinBugs version (rube.lmer.3.fit) of lmer.3 and lmer.3

```
> display(lmer.3)
lmer(formula = Classical ~ (1 |
fsubject:harmony) + (1 |
fsubject:voice) +
  (1 | fsubject:instrument))
coef.est coef.se
  5.78   0.15

Error terms:
Groups      Name      Std.Dev.
fsubject:harmony (Intercept) 0.73
fsubject:instrument (Intercept) 1.99
fsubject:voice (Intercept) 0.26
Residual                1.56

AIC = 10222.8, DIC = 10208.8
deviance = 10210.8
```

```
> c(rube.lmer.3.fit$median$tau0,
rube.lmer.3.fit$median$tau1,
rube.lmer.3.fit$median$tau2)
[1] 0.7308901
1.9948963
0.2496442

> rube.lmer.3.fit$median$sig
[1] 1.566827
```

Checking model fits

```
> (DIC <- c(rube.lm.4b.fit$DIC,rube.lmer.2.fit$DIC,rube.lmer.3.fit$DIC))
[1] 11236.91 10410.83 10016.67
> (AIC <- c(AIC(lm.4b),AIC(lmer.2),AIC(lmer.3)))
[1] 11235.76 10557.61 10222.76
> (BIC <- c(BIC(lm.4b),BIC(lmer.2),BIC(lmer.3)))
[1] 11276.51 10592.54 10251.87
```

	lm.4	lmer.2	lmer.3
AIC	11235.76	10557.61	10222.76
BIC	11276.51	10592.54	10251.87
DIC	11236.91	10410.83	10016.67

So AIC, BIC, and DIC prefer lmer.3. Thus, lmer.3 model with all three new random effect terms (but not the original single random intercept- lmer.2) is better than each of the models in problem 1a and 1b.

- ii. Re-examine the influence of three main experimental factors (Instrument, Harmony & Voice) on Classical ratings, using the model with all three new

random effect terms in it. Comment briefly on your findings, providing suitable brief evidence for each result. In addition, comment on the sizes of the three estimated variance components, with respect to each other and with respect to the estimated residual variance.

Now I include fixed effects of three main experimental factors with all three random effect terms in it.

```
> c(rube.lmer.4.fit$median$a3, rube.lmer.4.fit$median$a4,
rube.lmer.4.fit$median$a5)
[1] 0.2280234 2.5316650 -0.1122975
```

After controlling three random effects, the instrument factor is largest influential factor on classical rating (coefficients 2.5316650); 0.2280234 in harmony, and -0.1122975 in the voice factor.

```
> c(rube.lmer.4.fit$median$sig^2, rube.lmer.4.fit$median$tau0^2,
rube.lmer.4.fit$median$tau1^2, rube.lmer.4.fit$median$tau2^2)
[1] 2.44814086 0.58440494 2.94060483 0.02488935
```

subject:instrument (Intercept) has largest variance component(2.94060483) than the other two random effects, and residual variance. On the other hand, subject:voice (Intercept) has smallest variance component (0.02488935- lowest personal bias for voice) than the other random effects.

subject:voice (Intercept) variance is 0.58440494 and residual variance component is 2.44814086

For checking, I also attach the results of lmer version

```
> summary(lmer.4<-
lmer(Classical~harmony+voice+instrument+(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1))

Random effects:
Groups      Name      Variance Std.Dev.
fsubject:harmony (Intercept) 0.58317 0.7637
fsubject:instrument (Intercept) 2.94010 1.7147
fsubject:voice (Intercept) 0.04343 0.2084
Residual      2.44373 1.5632
Number of obs: 2493, groups: fsubject:harmony, 280; fsubject:instrument, 210; fsubject:voice, 210

Fixed effects:
      Estimate Std. Error t value
```

harmony	0.22868	0.04649	4.918
voice	-0.10916	0.04103	-2.660
instrument	2.52716	0.08297	30.458

	lmer.3	lmer.4
AIC	10222.76	10173.6
BIC	10251.87	10214.33
DIC	10016.67	9941.147

AIC, BIC and DIC all prefer lmer.4.

```
#####3
rube.lmer.4 <- "model {
# LEVEL 01
for (i in 1:N) {
  Classical[i] ~ dnorm(mu[i],sig2inv)
  mu[i] <- a0[subject[i],harmony[i]]
+a1[subject[i],instrument[i]]+a2[subject[i],voice[i]]+a3*harmony[i]+a4*instrume
nt[i]+a5*voice[i]
}

# LEVEL 02
for (j in 1:n.subject){
  for (k in 1:n.harmony){
    a0[j,k] ~ dnorm(0,tau02inv)}

  for (l in 1:n.instrument){
    a1[j,l] ~ dnorm(0,tau12inv)}

  for (m in 1:n.voice){
    a2[j,m] ~ dnorm(0,tau22inv)}
}

# b0 ~ dnorm(0,0.0001)
# b1 ~ dnorm(0,0.0001)
# b2 ~ dnorm(0,0.0001)

a3 ~ dnorm(0,0.0001)
a4 ~ dnorm(0,0.0001)
a5 ~ dnorm(0,0.0001)

tau02inv <- pow(tau0,-2)
tau0 ~ dunif(0,100)

tau12inv <- pow(tau1,-2)
tau1 ~ dunif(0,100)
```



```

tau22inv <- pow(tau2,-2)
tau2 ~ dunif(0,100)

sig2inv <- pow(sig,-2)
sig ~ dunif(0,100)
}"

rube.lmer.4.data.list <-list(Classical=Classical, harmony=harmony,
instrument=instrument,
voice=voice, subject=subject, N=length(Classical),
n.subject=length(unique(subject)),n.instrument=length(unique(instrument)),
n.voice=length(unique(voice)),n.harmony=length(unique(harmony)))

n.subject=length(unique(subject))
n.instrument=length(unique(instrument))
n.voice=length(unique(voice))
n.harmony=length(unique(harmony))
JK=n.subject*n.harmony
JL=n.subject*n.instrument
JM=n.subject*n.voice

rube.lmer.4.inits<-function(){
  list (a3=rnorm(1),a4=rnorm(1),a5=rnorm(1),

a0=matrix(rep(rnorm(JK)),n.subject,n.harmony),a1=matrix(rep(rnorm(JL)),n.subject,n.instrument),
      a2=matrix(rep(rnorm(JM)),n.subject,n.voice),
      tau0=runif(1,0,10),tau1=runif(1,0,10),tau2=runif(1,0,10),sig=runif(1,0,10
))
}

rube(rube.lmer.4, rube.lmer.4.data.list,rube.lmer.4.inits)
rube.lmer.4.fit <-rube(rube.lmer.4, rube.lmer.4.data.list,rube.lmer.4.inits,
      parameters.to.save=c("a0","a1","a2","a3","a4","a5",
      "sig","tau0","tau1","tau2"),
      n.chains=3)
rube.lmer.4.fit

c(rube.lmer.4.fit$median$a3, rube.lmer.4.fit$median$a4,
rube.lmer.4.fit$median$a5)
c(rube.lmer.4.fit$median$sig^2, rube.lmer.4.fit$median$tau0^2,
rube.lmer.4.fit$median$tau1^2,rube.lmer.4.fit$median$tau2^2)
rube.lmer.4.fit$median$tau2^2

```

- iii. Carefully write this model in the mathematical terms as a hierarchical linear model.

$$Y_i = \alpha_{j[i],k[i]} + \alpha_{j[i],l[i]} + \alpha_{j[i],k[i]} + \alpha_1 X_{k[i]} + \alpha_2 X_{l[i]} + \alpha_3 X_{m[i]} + \epsilon_i ,$$

$$\epsilon_i \sim \text{iid } N(0, \sigma^2)$$

$$\alpha_{j[i],k[i]} \sim \text{iid } N(0, \tau_0^2)$$

$$\alpha_{j[i],l[i]} \sim \text{iid } N(0, \tau_1^2)$$

$$\alpha_{j[i],m[i]} \sim \text{iid } N(0, \tau_2^2)$$

Because they are design variables in the experiment, the three experimental factors, Instrument, Harmony, and Voice, should be included in all models for the remainder of this homework, regardless of what you found about their influence or lack of influence on ratings.

2. Individual covariates. For this problem, begin with your best model from problem 1.
- a. Determine which individual covariates should be added to the model as fixed effects. Show a suitable summary of your work, and list the final set of variables that you would include in the model. Hint: Some covariates that are actually factor variables are coded as numeric. Be careful to treat them as factors!

To examine researcher's main hypotheses, I include "KnowRob" variables (In a standard model, KnowRob variables is in statistically significant level.)

```
> summary(test<-lm(Classical~harmony+instrument+voice+KnowRob+CollegeMusic))
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	2.47036	0.22815	10.828	< 2e-16 ***
harmony	0.10168	0.04358	2.333	0.01973 *
instrument	1.59208	0.05963	26.698	< 2e-16 ***
voice	-0.15383	0.05971	-2.576	0.01005 *
KnowRob	0.08520	0.02840	3.000	0.00273 **
CollegeMusic	0.06281	0.11787	0.533	0.59417

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.318 on 2255 degrees of freedom
(7 observations deleted due to missingness)
Multiple R-squared: 0.2456, Adjusted R-squared: 0.2439
F-statistic: 146.8 on 5 and 2255 DF, p-value: < 2.2e-16

Additionally, I consider CollegeMusic variables and compare the following models with best model from problem 1. Although in a standard model level, the CollegeMusic variables is not important, in terms of model fit, I include CollegeMusic variables.

```
Imer.4<-
lmer(Classical~harmony+voice+instrument+(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1)
Imer.5<-
lmer(Classical~KnowRob+harmony+voice+instrument+(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1)
Imer.6<-
lmer(Classical~KnowRob+CollegeMusic+harmony+voice+instrument+(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1)

(AIC <- c(AIC(Imer.4),AIC(Imer.5),AIC(Imer.6)))
(BIC <- c(BIC(Imer.4),BIC(Imer.5),BIC(Imer.6)))
(DIC <- c(DIC(Imer.4),DIC(Imer.5),DIC(Imer.6)))
```

	Imer.4	Imer.5	Imer.6
AIC	10173.6	9470.856	9181.633
BIC	10214.33	9516.854	9233.145
DIC	9941.147	9423.3	9130.3

AIC, BIC, DIC prefer Imer.6. Thus, I use KnowRob and CollegeMusic as my final set of variables.

- b. Once the fixed effects are settled, go back and check to see whether there should be any change in the random effects. Provide suitable evidence to justify your answer.

I tried several other models

```
display(Imer.6<-
lmer(Classical~KnowRob+CollegeMusic+harmony+voice+instrument+(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1))
display(Imer.7<-
lmer(Classical~KnowRob+CollegeMusic+harmony+voice+instrument+(1+KnowRob|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1))
display(Imer.8<-
lmer(Classical~KnowRob+CollegeMusic+harmony+voice+instrument+(1|fsubject:harmony)+(1+KnowRob|fsubject:voice)+(1|fsubject:instrument)-1))
display(Imer.9<-
lmer(Classical~KnowRob+CollegeMusic+harmony+voice+instrument+(1|fsubject:harmony)+(1|fsubject:voice)+(1+KnowRob|fsubject:instrument)-1))

display(Imer.10 <-
```

```

Imer(Classical~KnowRob+CollegeMusic+harmony+voice+instrument+(1+KnowRob+CollegeMusic|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1))
display(Imer.11 <-
Imer(Classical~KnowRob+harmony+voice+instrument+(1+KnowRob|fsubject:harmony)+(1|fsubject:voice)+(1+CollegeMusic|fsubject:instrument)-1))
display(Imer.12 <-
Imer(Classical~KnowRob+harmony+voice+instrument+(1+KnowRob+CollegeMusic|fsubject:harmony)+(1|fsubject:voice)+(1+CollegeMusic|fsubject:instrument)-1))
display(Imer.13 <-
Imer(Classical~KnowRob+CollegeMusic+harmony+voice+instrument+(1+KnowRob+CollegeMusic|fsubject:harmony)+(1|fsubject:instrument)-1))

```

I keep fixed effect unchanged and compare some of changes in random effects
Then, I look at AIC, BIC and DICs.

	Imer.6	Imer.7	Imer.9	Imer.10	Imer.11	Imer.12	Imer.13
AIC	9181.633	9171.257	9184.491	9176.724	9187.600	9226.5	9176.5
BIC	9233.145	9234.216	9247.451	9256.854	9256.283	9312.374	9250.924
DIC	9130.3	9116.2	9129.2	9115.6	9132.5	9097	9117.1

Among these fit models, there is no unique fitted model that AIC, BIC DIC all prefer.

Additional finding is that the subject:voice random terms effect seems minor.

	Imer.7	Imer.7b
AIC	9171.257	9171.05
BIC	9234.216	9228.285
DIC	9116.2	9117.7

```
display(Imer.7b<-
```

```

Imer(Classical~KnowRob+CollegeMusic+harmony+voice+instrument+(1+KnowRob|fsubject:harmony)+(1|fsubject:instrument)-1))

```

For this part, I choose Imer.7 based on AIC.

- c. Briefly interpret the effect of each variable kept in the final model, on Classical ratings.

```
display(Imer.7<-
```

```

Imer(Classical~KnowRob+CollegeMusic+harmony+voice+instrument+(1+KnowRob|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1))

```

```

> display(Imer.7<-
Imer(Classical~KnowRob+CollegeMusic+harmony+voice+instrument+(1+KnowRob|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1))
coef.est coef.se
KnowRob    0.16  0.08
CollegeMusic 1.05  0.26
harmony     0.17  0.05
voice      -0.11  0.04

```

instrument 2.17 0.11

Error terms:

Groups	Name	Std.Dev.	Corr
fsubject:harmony	(Intercept)	0.66	
KnowRob	0.14	0.85	
fsubject:instrument	(Intercept)	1.63	
fsubject:voice	(Intercept)	0.20	
Residual		1.55	
AIC = 9171.3, DIC = 9116.2			
deviance = 9132.7			

> coef(lmer.7

\$`fsubject:harmony`

	(Intercept)	KnowRob	CollegeMusic	harmony	voice	instrument
15:1	-3.332790e-01	0.103209449	1.045879	0.1684322	-0.1071864	2.173042
15:2	-5.752323e-01	0.059623288	1.045879	0.1684322	-0.1071864	2.173042
98:3	6.990707e-01	0.289179902	1.045879	0.1684322	-0.1071864	2.173042
98:4	-5.063218e-01	0.072037025	1.045879	0.1684322	-0.1071864	2.173042

\$`fsubject:instrument`

	(Intercept)	KnowRob	CollegeMusic	harmony	voice	instrument
15:1	0.7880831117	0.1632473	1.045879	0.1684322	-0.1071864	2.173042
15:2	0.3944807953	0.1632473	1.045879	0.1684322	-0.1071864	2.173042
98:2	-1.9704048648	0.1632473	1.045879	0.1684322	-0.1071864	2.173042
98:3	0.1946205440	0.1632473	1.045879	0.1684322	-0.1071864	2.173042

\$`fsubject:voice`

	(Intercept)	KnowRob	CollegeMusic	harmony	voice	instrument
15:1	0.1295211021	0.1632473	1.045879	0.1684322	-0.1071864	2.173042
15:2	-0.0971058051	0.1632473	1.045879	0.1684322	-0.1071864	2.173042
98:1	0.0932214959	0.1632473	1.045879	0.1684322	-0.1071864	2.173042
98:2	-0.0520496216	0.1632473	1.045879	0.1684322	-0.1071864	2.173042

> summary(lmer.7)

Random effects:

Groups	Name	Variance	Std.Dev.	Corr
fsubject:harmony	(Intercept)	0.43261	0.6577	
	KnowRob	0.01936	0.1391	0.85
fsubject:instrument	(Intercept)	2.66606	1.6328	
fsubject:voice	(Intercept)	0.03875	0.1969	
Residual		2.39289	1.5469	

Fixed effects:

	Estimate	Std. Error	t value
KnowRob	0.16325	0.08301	1.967
CollegeMusic	1.04588	0.26469	3.951
harmony	0.16843	0.04790	3.516
voice	-0.10719	0.04272	-2.509

instrument	2.17304	0.11169	19.457
------------	---------	---------	--------

lmer(Classical~KnowRob+CollegeMusic+harmony+voice+instrument+(1+KnowRob|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1))

The coefficients for KnowRob variables within subject:harmony simuli varies. The correlation between the random intercept and KnowRob is 0.85. For subject:voice and instrument, the coefficients for KnowRob are constant.

After controlling the subject and experimental factor combined random effects, the fixed coefficients of three main factors are harmony (0.16843), voice(-0.10719) and instrument(2.17304) and the coefficients of knowRob and CollegeMusic are 0.16325 and 1.04588.

The subject:instrument (Intercept) errors have estimated standard deviation 1.6328.

The subject:harmony(intercept) errors have estimated standard deviation 0.6577.

The subject:voice(intercept) errors have estimated standard deviation 0.1969

3. Musicians vs. Non-musicians. One of the secondary hypotheses is that people who self-identity as musicians may be influenced by things that do not influence non-musicians. Dichotomize "Self declare" ("are you a musician?") so that about half the participants are categorized as self-declared musicians, and half not.

```
#Musicians vs. Non-musicians#  
n <- dim(ratings.data)[1]  
musician <- rep (NA, n)  
for (i in 1:n){  
  if (Selfdeclare[i]>=4) musician[i] <- 1  
  else {musician[i]<-0}  
}
```

Examine and report on any interactions between the dichotomized musician variables and other predictors in the model. Provide suitable evidence for, and comment on, your results.

I compare the following models

display(lmer.7<-

lmer(Classical~KnowRob+CollegeMusic+harmony+voice+instrument+(1+KnowRob|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1))

display(lmer.14<-

lmer(Classical~KnowRob+CollegeMusic+musician+harmony+voice+instrument+(1+KnowRob|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1))

display(lmer.15<-

lmer(Classical~KnowRob*musician+CollegeMusic+harmony+voice+instrument+(1+KnowRob|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1))

```
display(lmer.16<-
lmer(Classical~KnowRob+CollegeMusic+harmony*musician+voice+instrument+(1+Know
Rob|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1))
display(lmer.17<-
lmer(Classical~KnowRob+CollegeMusic+harmony+voice+instrument*musician+(1+Know
Rob|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)-1))
```

	lmer.7	lmer.14	lmer.15	lmer.16	lmer.17
AIC	9171.3	9173.4	9173.8	9176.5	9164.2
BIC	9234.216	9242.078	9248.212	9250.949	9238.561
DIC	9116.2	9115.7	9110.8	9112.2	9103.2

Just including musician variables does not improve model fit (from comparison lmer.7 and lmer.14) .
But from lmer.17, the interaction between musician and instrument is worth to fit the model.

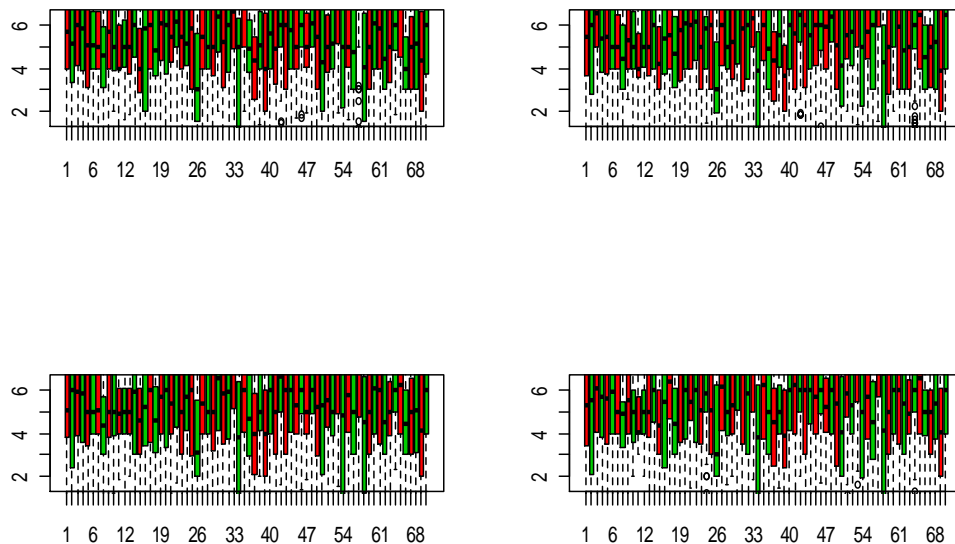
4. Classical vs. Popular. Please re-examine the data in terms of the "Popular" ratings, instead of the "Classical" ratings, using similar hierarchical linear models. Provide brief answers to the following questions:
 - a. Comment on the influence of Instrument, Harmony & Voice on popular ratings, providing suitable brief evidence for each result.

The influence of Voice on popular ratings look larger than the others (for classical ratings, the influence of Instrument was large)

```
> display(lm.p.1<-lm(Popular~harmony+instrument+voice-1))
lm(formula = Popular ~ harmony + instrument + voice - 1)
      coef.est coef.se
harmony  0.81   0.04
instrument 0.03   0.06
voice    1.42   0.06
---
n = 2493, k = 3
residual sd = 2.95, R-Squared = 0.75
```

Now, I look at the winbug version of the following lm.p.1, lmer.p.2, and lmer.p.3 models

```
lm.p.1<-lm(Popular~harmony+instrument+voice)
lmer.p.2<-lmer(Popular~harmony+instrument+voice+(1|fsubject))
lmer.p.3<-
lmer(Popular~(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument))
```



Data (red) is less spread out within subjects than predictions (green) from the fixed effects model (rube.lm.p.1- the winbug version of lm.p).

	lm.p.1	lmer.p.2	lmer.p.3
AIC	11151.69	10462.11	10164.68
BIC	11180.79	10497.03	10193.79
DIC	11180.79	10497.03	9937.893

I omit Winbug version codes for lm.p.1, lmer.p.2 because it is same as those of classical rating.

```
> (DIC <- c(rube.lm.p.1.fit$DIC,rube.lmer.p.2.fit$DIC, rube.lmer.p.3.fit$DIC))
[1] 11151.815 10320.841 9937.893
> (AIC <- c(AIC(lm.p.1),AIC(lmer.p.2),AIC(lmer.p.3)))
[1] 11151.69 10462.11 10164.68
> (BIC <- c(BIC(lm.p.1),BIC(lmer.p.2),BIC(lmer.p.3)))
[1] 11180.79 10497.03 10193.79
```

AIC, BIC, and DIC prefer lmer.p.3

Same as the approach of classical rating fit, I check the following model but all three AIC, DIC, BIC do not improve, thus, I keep rube.lmer.p.3.

```
> display(lmer.p.4)
lmer(formula = Popular ~ harmony + voice + instrument + (1 |
fsubject:harmony) + (1 | fsubject:voice) + (1 | fsubject:instrument) - 1)
```


	coef.est	coef.se
harmony	0.02	0.04
voice	0.15	0.04
instrument	1.96	0.12

Error terms:

Groups	Name	Std.Dev.
fsubject:harmony	(Intercept)	0.62
fsubject:instrument	(Intercept)	3.26
fsubject:voice	(Intercept)	0.16
Residual		1.58

number of obs: 2493, groups: fsubject:harmony, 280;
fsubject:instrument, 210; fsubject:voice, 210

AIC = 10410.3, DIC = 10372.9
deviance = 10384.6

From, rube.lmer.p.3 fit,

```
#lmer.p.3<-
#lmer(Popular~(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument
#))
```

```
rube.lmer.p.3 <- "model {
  # LEVEL 01
  for (i in 1:N) {
    Popular[i] ~ dnorm(mu[i],sig2inv)
    mu[i] <- a0[subject[i],harmony[i]]
    +a1[subject[i],instrument[i]]+a2[subject[i],voice[i]]
  }

  # LEVEL 02
  for (j in 1:n.subject){
    for (k in 1:n.harmony){
      a0[j,k] ~ dnorm(b0,tau02inv)}

    for (l in 1:n.instrument){
      a1[j,l] ~ dnorm(b1,tau12inv)}

    for (m in 1:n.voice){
      a2[j,m] ~ dnorm(b2,tau22inv)}
  }
  b0 ~ dnorm(0,0.0001)
  b1 ~ dnorm(0,0.0001)
  b2 ~ dnorm(0,0.0001)

  tau02inv <- pow(tau0,-2)
```

```

tau0 ~ dunif(0,100)

tau12inv <- pow(tau1,-2)
tau1 ~ dunif(0,100)

tau22inv <- pow(tau2,-2)
tau2 ~ dunif(0,100)

sig2inv <- pow(sig,-2)
sig ~ dunif(0,100)
}"

rube.Imer.p.3.data.list <-list(Popular=Popular, harmony=harmony,
instrument=instrument,
voice=voice, subject=subject, N=length(Popular),
n.subject=length(unique(subject)),n.instrument=length(unique(instrument)),
n.voice=length(unique(voice)),n.harmony=length(unique(harmony)))

n.subject=length(unique(subject))
n.instrument=length(unique(instrument))
n.voice=length(unique(voice))
n.harmony=length(unique(harmony))

JK=n.subject*n.harmony
JL=n.subject*n.instrument
JM=n.subject*n.voice

rube.Imer.p.3.inits<-function(){
  list (b0=rnorm(1),b1=rnorm(1),b2=rnorm(1),

a0=matrix(rep(rnorm(JK)),n.subject,n.harmony),a1=matrix(rep(rnorm(JL)),n.subject,n.instrument),
a2=matrix(rep(rnorm(JM)),n.subject,n.voice),
tau0=runif(1,0,10),tau1=runif(1,0,10),tau2=runif(1,0,10),sig=runif(1,0,10))
}

rube(rube.Imer.p.3, rube.Imer.p.3.data.list,rube.Imer.p.3.inits)
rube.Imer.p.3.fit <-rube(rube.Imer.p.3,
rube.Imer.p.3.data.list,rube.Imer.p.3.inits,
parameters.to.save=c("b0","b1","b2","a0","a1","a2",
"sig","tau0","tau1","tau2"),
n.chains=3)

```

rube.Imer.p.3.fit

DIC= 9937.893

To look at three experimental influences, I look at the mean parameters and variance.

Voice has largest mean coefficient (2.7491748), thus it looks largest influential factor on popular ratings. The subject:voice intercept has lowest variance (0.03697154).

Instrument has 1.6766824 coefficients. The subject:instrument random effect has highest variance 3.32656836

The harmony has lowest mean coefficient (0.9499034). The subject:harmony has 0.37996987 variance.

```
> c(rube.lmer.p.3.fit$mean$b0, rube.lmer.p.3.fit$mean$b1,
rube.lmer.p.3.fit$mean$b2)
[1] 0.9499034 1.6766824 2.7491748

> c(rube.lmer.p.3.fit$mean$sig, rube.lmer.p.3.fit$mean$tau0,
rube.lmer.p.3.fit$mean$tau1, rube.lmer.p.3.fit$mean$tau2)
[1] 1.5818686 0.6164170 1.8238883 0.1922798

> c(rube.lmer.p.3.fit$mean$sig^2, rube.lmer.p.3.fit$mean$tau0^2,
rube.lmer.p.3.fit$mean$tau1^2, rube.lmer.p.3.fit$mean$tau2^2)
[1] 2.50230830 0.37996987 3.32656836 0.03697154
```

I compare lmer.p.3 result. Not a big difference with rube.lmer.p.3

```
lmer.p.3<-
lmer(Popular~(1 | fsubject:harmony)+(1 | fsubject:voice)+(1 | fsubject:instrument)
))

Random effects:
Groups          Name      Variance Std.Dev.
fsubject:harmony (Intercept) 0.38146 0.6176
fsubject:instrument (Intercept) 3.29133 1.8142
fsubject:voice    (Intercept) 0.02877 0.1696
Residual                                2.50105 1.5815
Number of obs: 2493, groups: fsubject:harmony, 280; fsubject:instrument, 210;
fsubject:voice, 210

Fixed effects:
              Estimate Std. Error t value
(Intercept)  5.3822    0.1349   39.91
```

b. Question 2c, for popular ratings.

From the best model from 4 a), I compare the following models

```
#display(lmer.p.6<-
#lmer(Popular~KnowRob+CollegeMusic+(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument))) #AIC = 9236.1, DIC = 9211.2

display(lmer.p.7<-
lmer(Popular~KnowRob+CollegeMusic+(1|fsubject:harmony)+(1|fsubject:voice)+(1+KnowRob|fsubject:instrument))) #AIC = 9240, DIC = 9211
display(lmer.p.8<-
lmer(Popular~KnowRob+CollegeMusic+(1|fsubject:harmony)+(1+KnowRob|fsubject:voice)+(1|fsubject:instrument))) #AIC = 9239.7, DIC = 9210.9
display(lmer.p.9<-
lmer(Popular~KnowRob+CollegeMusic+(1+KnowRob|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument))) # AIC = 9234.7, DIC = 9206
```

When we include KnowRob and CollegeMusic (lmer.p.6), Individual covariates improve AIC and DIC a lot. (Refer to lmer.p.3 as baseline model)

In the same way to 2 b), I check the interactions in the subject and stimuli random effects. lmer.p.9 shows some improvement but not a big. Thus, I choose lmer.p.6.

```
> display(lmer.p.6<-
lmer(Popular~KnowRob+CollegeMusic+(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)))
lmer(formula = Popular ~ KnowRob + CollegeMusic + (1 | fsubject:harmony) + (1 | fsubject:voice) + (1 | fsubject:instrument))
               coef.est coef.se
(Intercept)  5.08    0.32
KnowRob       0.23    0.08
CollegeMusic  0.15    0.35

Error terms:
Groups          Name      Std.Dev.
fsubject:harmony (Intercept) 0.63
fsubject:instrument (Intercept) 1.85
fsubject:voice    (Intercept) 0.17
Residual                      1.58
AIC = 9236.1, DIC = 9211.2
deviance = 9216.6
```

The coefficients for KnowRob is 0.23; the level difference in KnowRob for popular rating, after controlling for other variables and random effect.

The coefficient for CollegeMusic (0=no, 1=yes) is 0.15; the CollegeMusic (yes=1) influence for popular rating, after controlling for other variables, is 0.15.

The subject:harmony(intercept) errors have estimated standard deviation 0.63.

The subject:instrument (Intercept) errors have estimated standard deviation 1.85

The subject:harmony(intercept) errors have estimated standard deviation 0.17.

c. Question 3, for popular ratings.

```
display(lmer.p.6<-  
lmer(Popular~KnowRob+CollegeMusic+(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)))
```

AIC = 9236.1, DIC = 9211.2

```
display(lmer.p.10 <- lmer(Popular~KnowRob+CollegeMusic+ musician  
+(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)))
```

AIC = 9238.2, DIC = 9211

```
display(lmer.p.11 <- lmer(Popular~KnowRob+CollegeMusic*musician  
+(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)))
```

AIC = 9236, DIC = 9209.3

```
display(lmer.p.12 <- lmer(Popular~KnowRob*musician +CollegeMusic  
+(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)))
```

AIC = 9241, DIC = 9208.7

```
display(lmer.p.13 <- lmer(Popular~KnowRob +CollegeMusic+instrument*musician  
+(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)))
```

AIC = 9157.4, DIC = 9117

```
display(lmer.p.14 <- lmer(Popular~KnowRob +CollegeMusic+voice *musician  
+(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)))
```

AIC = 9244.2, DIC = 9198.6

```
display(lmer.p.15 <- lmer(Popular~KnowRob +CollegeMusic+harmony *musician  
+(1|fsubject:harmony)+(1|fsubject:voice)+(1|fsubject:instrument)))
```

AIC = 9242, DIC = 9196.9

```
> coef(lmer.p.13)
```

```
$ fsubject:harmony`
```

```
(Intercept) KnowRob CollegeMusic instrument musician instrument:musician
```

```
15:1 8.291065 0.2398161 0.1442141 -1.476287 -0.7572998 0.3485089
```

```
15:2 8.427678 0.2398161 0.1442141 -1.476287 -0.7572998 0.3485089
```

```
$ fsubject:instrument`
```

```
(Intercept) KnowRob CollegeMusic instrument musician instrument:musician
```

```
15:1 8.820362 0.2398161 0.1442141 -1.476287 -0.7572998 0.3485089
```

```
15:2 8.709980 0.2398161 0.1442141 -1.476287 -0.7572998 0.3485089
```

```
$ fsubject:voice`
```

```
(Intercept) KnowRob CollegeMusic instrument musician instrument:musician
```

```
15:1 8.003128 0.2398161 0.1442141 -1.476287 -0.7572998 0.3485089
```

```
15:2 8.070067 0.2398161 0.1442141 -1.476287 -0.7572998 0.3485089
```

```
> summary(lmer.p.3)
```

Random effects:			
Groups	Name	Variance	Std.Dev.
fsubject:harmony	(Intercept)	0.38146	0.6176
fsubject:instrument	(Intercept)	3.29133	1.8142
fsubject:voice	(Intercept)	0.02877	0.1696
Residual		2.50105	1.5815
Fixed effects:			
	Estimate	Std. Error	t value
(Intercept)	8.04527	0.40055	20.085
KnowRob	0.23982	0.07289	3.290
CollegeMusic	0.14421	0.28102	0.513
instrument	-1.47629	0.14944	-9.879
musician	-0.75730	0.68154	-1.111
instrument:musician	0.34851	0.30624	1.138
AIC = 9157.4, DIC = 9117			
deviance = 9127.2			

I rewrite the winbug version of lmer.p.13

```
> c(rube.lmer.p.13.fi.t$mean$sig, rube.lmer.p.13.fi.t$mean$tau0,
rube.lmer.p.13.fi.t$mean$tau1, rube.lmer.p.13.fi.t$mean$tau2)
[1] 1.5819441 0.6406454 1.8476294 0.1725627

> c(rube.lmer.p.13.fi.t$mean$sig^2,
rube.lmer.p.13.fi.t$mean$tau0^2, rube.lmer.p.13.fi.t$mean$tau1^2,
rube.lmer.p.13.fi.t$mean$tau2^2)
[1] 2.50254723 0.41042647 3.41373425 0.02977789

> c(rube.lmer.p.13.fi.t$mean$a3, rube.lmer.p.13.fi.t$mean$a4,
rube.lmer.p.13.fi.t$mean$a5)
[1] 0.2329654 0.1522885 -0.2537035

> c(rube.lmer.p.13.fi.t$mean$b0, rube.lmer.p.13.fi.t$mean$b1,
rube.lmer.p.13.fi.t$mean$b2)
[1] 0.3842028 4.0152647 0.7832560

> rube.lmer.p.13.fi.t$DIC
[1] 9015.953
```

Subject:instrument intercept error has 3.414 largest variance and Residual has 2.503 variance and subject:harmony has 0.410 variance and subject:voice has 0.028 variance.

The coefficient of KnowRob and CollegeMusic are 0.2329654 and CollegeMusic 0.1522 and the interaction between KnowRob and CollegeMusic is -0.2537035.

The mean intercept coefficient of three experimental factor and subject combination are 0.384202 (subject:harmony) and 4.0152647 (subject:instrument) and 0.7832560 (subject:voice)

```
rube.lmer.p.13 <- "model {
```

```
# LEVEL 01
for (i in 1:N) {
  Popular[i] ~ dnorm(mu[i],sig2inv)
  mu[i] <- a0[subject[i],harmony[i]]
+a1[subject[i],instrument[i]]+a2[subject[i],voice[i]]+ a3*KnowRob[i]+
  a4*CollegeMusic[i]+a5*instrument[i]*musician[i]
}
```

```
# LEVEL 02
for (j in 1:n.subject){
  for (k in 1:n.harmony){
    a0[j,k] ~ dnorm(b0,tau02inv)}
```

```
  for (l in 1:n.instrument){
    a1[j,l] ~ dnorm(b1,tau12inv)}
```

```
  for (m in 1:n.voice){
    a2[j,m] ~ dnorm(b2,tau22inv)}
}
```

```
b0 ~ dnorm(0,0.0001)
b1 ~ dnorm(0,0.0001)
b2 ~ dnorm(0,0.0001)
```

```
a3 ~ dnorm(0,0.0001)
a4 ~ dnorm(0,0.0001)
a5 ~ dnorm(0,0.0001)
```

```
tau02inv <- pow(tau0,-2)
tau0 ~ dunif(0,100)
```

```
tau12inv <- pow(tau1,-2)
tau1 ~ dunif(0,100)
```

```
tau22inv <- pow(tau2,-2)
tau2 ~ dunif(0,100)
```

```
sig2inv <- pow(sig,-2)
sig ~ dunif(0,100)
}"
```

```
rube.lmer.p.13.data.list <-list(Popular=Popular, harmony=harmony,
instrument=instrument,
voice=voice, subject=subject, N=length(Popular),
musician=musician,KnowRob=KnowRob,
CollegeMusic=CollegeMusic,n.subject=length(unique(subject)),n.instrument=length(unique(instrument)),
n.voice=length(unique(voice)),n.harmony=length(unique(harmony)))
```

```

n.subject=length(unique(subject))
n.instrument=length(unique(instrument))
n.voice=length(unique(voice))
n.harmony=length(unique(harmony))

JK=n.subject*n.harmony
JL=n.subject*n.instrument
JM=n.subject*n.voice

rube.lmer.p.13.inits<-function(){
  list (b0=rnorm(1),b1=rnorm(1),b2=rnorm(1),
a5=rnorm(1),a4=rnorm(1),a3=rnorm(1),

a0=matrix(rep(rnorm(JK)),n.subject,n.harmony),a1=matrix(rep(rnorm(JL)),n.subject,n.instrument),
      a2=matrix(rep(rnorm(JM)),n.subject,n.voice),
      tau0=runif(1,0,10),tau1=runif(1,0,10),tau2=runif(1,0,10),sig=runif(1,0,10
))
}

rube(rube.lmer.p.13, rube.lmer.p.13.data.list,rube.lmer.p.13.inits)
rube.lmer.p.13.fit <-rube(rube.lmer.p.13,
rube.lmer.p.13.data.list,rube.lmer.p.13.inits,

parameters.to.save=c("b0","b1","b2","a0","a1","a2","a3","a5","a4","sig","tau0",
"tau1","tau2"),n.chains=3) #"a5"
rube.lmer.p.13.fit

```

5. Brief write up. Write a one page professional-quality summary of your findings for Classical and Popular ratings, suitable for Dr. Jimenz. Be sure to address:

- a. The influence of the three main experimental factors (Instrument, Harmony & Voice):
All three main experimental factors are statistically important on both classical and popular ratings. Same as the researchers' prior hypothesis, instrument is the highest influential factor on both classical ratings. Harmony factor has relatively lower influential on rating than the other factors. For popular rating, voice is the largest influential factor.
Instrument has largest variance and Voice has low variance on ratings. Within the Voice stimuli, same as hypotheses, 'Conray Motion' could highly affects classical ratings than the other stimuli in voice. On the other hand, for popular ratings, 'Conray Montion' is relatively less effect than the two other stimuli in voice. Among Harmony stimuli, same as the research's hypotheses, 'I-V-vi' highly affects classical ratings than the other stimuli in harmonic. Within instrument stimuli, a string quartet highly affects classical ratings than the others; this implies that people are included to call music played by a string quarter classical.

- b. A brief discussion of variance components – is this a standard measure model, or did we need to include other variance components?

In part 1b/1c and 4a, we examine the spread between people using the simulated data, and also look at the model with random effects of person and experimental factor combination. When we consider the personal bias of each factor and combination in the model, we get more fitted model than a standard measure model.

Moreover, using variance component models, we could get more informative result.

For example, 1b) we see that the voice influence is negative, after controlling a random intercept for each participant. In a standard measure model, the effect of voice was positive. The estimated effects from the standard model might be influenced by extreme observations, causing large residual.

- c. A discussion of other individual covariates in the model.

For both popular/classical ratings, KnowRob (the exposure of Rob Paravonian's Pachelbel Rant) and CollegeMusic (taking music class in college) variables contribute a lot to model fit. These variables have positive rating effect on ratings.

In part 3 and 4c, we see interaction evidence between musician and instrument factor. It implies that the person who self-identify as musician may be influenced by instrument factor on ratings.