

Problem #1

Part (a)

Comparing the models between the one with the variable and one without using ANOVA gives all the p-values that are smaller than 0.05. Also, all the AIC and BIC values were smaller for the model with the addition of each variable. Therefore, all three variables, ‘Instrument’, ‘Harmony’, and ‘Voice’, makes the model better.

Looking the AIC and BIC decrease with addition of each variable, ‘Instrument’ variable has the overall biggest decrease with its addition to the model and ‘Voice’ variable has the overall smallest decrease. Therefore, influence of the three main factors are important in the order of ‘Instrument’, ‘Harmony’ and ‘Voice’.

Comparing one with instrument and one without	
<pre>> anova(fit1.2,fit2.3) Analysis of Variance Table Model 1: Classical ~ Harmony Model 2: Classical ~ Instrument + Harmony Res.Df RSS Df Sum of Sq F Pr(>F) 1 2489 17320 2 2487 13193 2 4127.1 389 < 2.2e-16 *** --- Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1</pre>	<pre>> cbind(AIC(fit1.2,fit2.3),BIC(fit1.2,fit2.3)) df AIC df BIC fit1.2 5 11917.23 5 11946.34 fit2.3 7 11242.69 7 11283.43</pre>
<pre>> anova(fit1.3,fit2.2) Analysis of Variance Table W Model 1: Classical ~ Voice Model 2: Classical ~ Instrument + Voice Res.Df RSS Df Sum of Sq F Pr(>F) 1 2490 17510 2 2488 13381 2 4128.3 383.8 < 2.2e-16 *** --- Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1</pre>	<pre>> cbind(AIC(fit1.3,fit2.2),BIC(fit1.3,fit2.2)) df AIC df BIC fit1.3 4 11942.32 4 11965.61 fit2.2 6 11275.96 6 11310.89</pre>
<pre>> anova(fit2.1,model1) Analysis of Variance Table Model 1: Classical ~ Harmony + Voice Model 2: Classical ~ Instrument + Harmony + Voice Res.Df RSS Df Sum of Sq F Pr(>F) 1 2487 17235 2 2485 13108 2 4127.6 391.26 < 2.2e-16 *** --- Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1</pre>	<pre>> cbind(AIC(fit2.1,model1),BIC(fit2.1,model1)) df AIC df BIC fit2.1 7 11908.94 7 11949.69 model1 9 11230.45 9 11282.84</pre>

Comparing one with harmony and one without	
<pre>> anova(fit1.1,fit2.3) Analysis of Variance Table Model 1: Classical ~ Instrument Model 2: Classical ~ Instrument + Harmony Res.Df RSS Df Sum of Sq F Pr(>F) 1 2490 13467 2 2487 13193 3 273.61 17.193 4.748e-11 *** --- Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1</pre>	<pre>> cbind(AIC(fit1.1,fit2.3),BIC(fit1.1,fit2.3)) df AIC df BIC fit1.1 4 11287.86 4 11311.14 fit2.3 7 11242.69 7 11283.43</pre>

<pre>Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 > anova(fit1.3,fit2.1) Analysis of Variance Table Model 1: Classical ~ Voice Model 2: Classical ~ Harmony + Voice Res.Df RSS Df Sum of Sq F Pr(>F) 1 2490 17510 2 2487 17235 3 274.44 13.2 1.502e-08 *** --- Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1</pre>	<pre>> cbind(AIC(fit1.3,fit2.1),BIC(fit 1.3,fit2.1)) df AIC df BIC fit1.3 4 11942.32 4 11965.61 fit2.1 7 11908.94 7 11949.69</pre>
<pre>> anova(fit2.2,model1) Analysis of Variance Table Model 1: Classical ~ Instrument + Voice Model 2: Classical ~ Instrument + Harmony + Voice Res.Df RSS Df Sum of Sq F Pr(>F) 1 2488 13381 2 2485 13108 3 273.65 17.293 4.107e-11 *** --- Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1</pre>	<pre>> cbind(AIC(fit2.2,model1),BIC(fit 2.2,model1)) df AIC df BIC fit2.2 6 11275.96 6 11310.89 model1 9 11230.45 9 11282.84</pre>

Comparing one with voice and one without	
<pre>> anova(fit1.1,fit2.2) Analysis of Variance Table Model 1: Classical ~ Instrument Model 2: Classical ~ Instrument + Voice Res.Df RSS Df Sum of Sq F Pr(>F) 1 2490 13467 2 2488 13381 2 85.603 7.9583 0.0003587 *** --- Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1</pre>	<pre>> cbind(AIC(fit1.1,fit2.2),BIC(fit 1.1,fit2.2)) df AIC df BIC fit1.1 4 11287.86 4 11311.14 fit2.2 6 11275.96 6 11310.89</pre>
<pre>> anova(fit1.2,fit2.1) Analysis of Variance Table Model 1: Classical ~ Harmony Model 2: Classical ~ Harmony + Voice Res.Df RSS Df Sum of Sq F Pr(>F) 1 2489 17320 2 2487 17235 2 85.216 6.1483 0.00217 ** --- Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1</pre>	<pre>> cbind(AIC(fit1.2,fit2.1),BIC(fit 1.2,fit2.1)) df AIC df BIC fit1.2 5 11917.23 5 11946.34 fit2.1 7 11908.94 7 11949.69</pre>
<pre>> anova(fit2.3,model1) Analysis of Variance Table Model 1: Classical ~ Instrument + Harmony Model 2: Classical ~ Instrument + Harmony + Voice Res.Df RSS Df Sum of Sq F Pr(>F) 1 2487 13193 2 2485 13108 2 85.64 8.1181 0.0003061 *** --- Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1</pre>	<pre>> cbind(AIC(fit2.3,model1),BIC(fit 2.3,model1)) df AIC df BIC fit2.3 7 11242.69 7 11283.43 model1 9 11230.45 9 11282.84</pre>

From the summary of the model, we can see how particular kinds of each variable affect ratings. As seen above, Instrument was highly significant variables with big impact on the Classical rating. In particular, indicator variable for string instrument increases the classical rating by 3.13 in average, compared to 1.37 for that for piano instrument. Among harmony variable, one noticeable kind is Harmony I-V-VI. While other kinds of harmony gives insignificant p-values, Harmony I-V-VI had very significant p-value, and it increases the Classical rating by 0.77 in average. All kinds of Voice variables were significant enough, but their impact was not as big as other variables.

```

Call:
lm(formula = Classical ~ Instrument + Harmony + Voice)

Residuals:
    Min       1Q   Median       3Q      Max
-6.8718 -1.7137 -0.0297  1.7576 11.4766

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.34016    0.12987  33.420 < 2e-16 ***
Instrumentpiano  1.37359    0.11298  12.158 < 2e-16 ***
Instrumentstring  3.13312    0.11230  27.899 < 2e-16 ***
HarmonyI-V-IV  -0.03108    0.13008  -0.239 0.811168
HarmonyI-V-VI   0.76909    0.13008   5.913 3.83e-09 ***
HarmonyIV-I-V   0.05007    0.12997   0.385 0.700092
Voicepar3rd    -0.41247    0.11271  -3.660 0.000258 ***
Voicepar5th    -0.37058    0.11264  -3.290 0.001016 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.297 on 2485 degrees of freedom
(27 observations deleted due to missingness)
Multiple R-squared:  0.255,    Adjusted R-squared:  0.2529
F-statistic: 121.5 on 7 and 2485 DF,  p-value: < 2.2e-16
    
```

Part (b)

i.

Level 1	$y_i \sim N(\alpha_{j[i]} + \beta \cdot X_i, \sigma^2) \leftarrow \text{when considering that } X_i \text{ includes indicators for all 3 main effects}$ <p style="text-align: center;">OR</p> $y_i \sim N(\alpha_{j[i]} + \beta_1 \cdot X_{i1} + \beta_2 \cdot X_{i2} + \beta_3 \cdot X_{i3}, \sigma^2)$
Level 2	$\alpha_j \sim N(\mu_\alpha, \tau^2)$

ii.

To test whether the random intercept is needed in the model, we can use exactRLRT function in R, since there is only one random effect to test. The p-value lot smaller than 0.05 strongly rejects the null hypothesis $H_0: \tau^2 = 0$. Therefore, we keep the random effect (1|Subject).

```

> model2 <- lmer(Classical ~ Instrument + Harmony + Voice + (1|Subject))
> exactRLRT(model2)
simulated finite sample distribution of RLRT.
(p-value based on 10000 simulated values)
data:
RLRT = 763.3759, p-value < 2.2e-16
    
```

Also, we can compare the AIC and BIC values after adding the random effect (1|Subject). Both values lowers by about 730. Thus, the random intercept is needed in the model.

<pre>> AIC(model1,model2) df AIC model1 9 11230.45 model2 10 10491.51</pre>	<pre>> BIC(model1,model2) df BIC model1 9 11282.84 model2 10 10549.73</pre>
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iii. We see that with the random intercept the influence of the three main experimental factors are similar but more significant and stronger. As the previous model – one without random intercept, Instrument was highly significant variables with big impact on the Classical rating. But now the indicator variable for string instrument increases the classical rating by 4.34 in average compared to 3.13 in average in the previous model. Among harmony variable, again Harmony I-V-VI is the most significant kinds of harmony with biggest t-value. We can also compared the F-values in ANOVA tables. F-values for all three main factors increased by about 50% of its original F-value.

<pre>Linear mixed model fit by REML ['lmerMod'] Formula: Classical ~ Instrument + Harmony + Voice + (1 Subject) REML criterion at convergence: 10471.51 Random effects: Groups Name Variance Std.Dev. Subject (Intercept) 1.702 1.305 Residual 3.581 1.892 Number of obs: 2493, groups: Subject, 70 Fixed effects: Estimate Std. Error t value (Intercept) 4.34374 0.18914 22.97 Instrumentpiano 1.37705 0.09318 14.78 Instrumentstring 3.13161 0.09257 33.83 HarmonyI-V-IV -0.03251 0.10718 -0.30 HarmonyI-V-VI 0.77096 0.10718 7.19 HarmonyIV-I-V 0.04989 0.10709 0.47 Voicepar3rd -0.41507 0.09287 -4.47 Voicepar5th -0.37439 0.09281 -4.03 Correlation of Fixed Effects: (Intr) Instrmntp Instrmnts HI-V-I HI-V-V HIV-I- Vcpr3r Instrumntpn -0.244 Instrmntstr -0.245 0.498 HrmnyI-V-IV -0.282 0.001 -0.001 HrmnyI-V-VI -0.282 0.001 -0.001 0.499 HrmnyIV-I-V -0.283 -0.001 -0.001 0.499 0.499 Voicepar3rd -0.245 -0.001 -0.001 -0.002 0.001 0.002 Voicepar5th -0.244 -0.001 0.000 -0.002 -0.003 -0.001 0.500</pre>							
ANOVA without random intercept				ANOVA with random intercept			
Analysis of Variance Table				Analysis of Variance Table			
Response: Classical				Response: Classical			
	Df	Sum Sq	Mean Sq	F value	Pr(>F)		
Instrument	2	4127.9	2063.96	391.2983	< 2.2e-16 ***	Instrument	2 4119.1 2059.53 575.147
Harmony	3	273.6	91.20	17.2911	4.121e-11 ***	Harmony	3 275.4 91.79 25.633
Voice	2	85.6	42.82	8.1181	0.0003061 ***	Voice	2 87.0 43.49 12.146
Residuals	2485	13107.5	5.27				

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.'							

Part (c)

i. To determine whether a model with all three main experimental factors is better or worse than two other previous models, we can compare the AIC and BIC values. For both values, they decreased in the order of model with only main effects, with main effects and (1|subject), and then the one with three random effects of person/instrument, person/harmony and person/voice combination. Therefore, the last model is preferred over the others.

> AIC(model11, model12, model13)				> BIC(model11, model12, model13)			
	df	AIC			df	BIC	
model11	9	11230.45		model11	9	11282.84	
model12	10	10491.51		model12	10	10549.73	
model13	12	10075.51		model13	12	10145.37	

ii. The influence of the three main experimental factors is still similar to the previous two models. Compared to the two other models, the F-values and t values for each main effect variables slightly decreased yet, still significant enough to consider them as significant factors. Even with slightly lower F-values and t-values the model is better since the random effects better accounts for the Classical ratings.

Looking at the size of the three estimated variance components, the estimated variance of (1|Subject:Instrument) was the biggest and that of (1|Subject:Voice) was the smallest. Therefore, the random effect of subject/instrument combination best accounts for the Classical ratings. Comparing these estimated variances with the estimated residual variance, we see that these random effects accounts for Classical ratings as much as the fixed effects do. In the previous model, one with only one random effect of (1|Subject), the estimated variance for random intercept was only half the estimated residual variance (1.702 compared to 3.581). Thus, having these three random effects better accounts for the Classical ratings.

```
Linear mixed model fit by REML ['lmerMod']
Formula: Classical ~ Instrument + Harmony + Voice + (1 | Subject:Instrument) + (1 | Subject:Harmony) + (1 | Subject:Voice)

REML criterion at convergence: 10051.51

Random effects:
Groups          Name                Variance Std.Dev.
Subject:Harmony (Intercept)    0.44307  0.6656
Subject:Voice   (Intercept)    0.02809  0.1676
Subject:Instrument (Intercept)  2.19850  1.4827
Residual                            2.43753  1.5613
Number of obs: 2493, groups: Subject:Harmony, 280; subject:voice, 210; Subject:Instrument, 210

Fixed effects:
              Estimate Std. Error t value
(Intercept)   4.34106   0.21435  20.252
Instrumentpiano  1.36384   0.26232   5.199
Instrumentstring 3.12836   0.26203  11.939
HarmonyI-V-IV  -0.03023   0.14317  -0.211
HarmonyI-V-VI  0.77063   0.14316   5.383
HarmonyIV-I-V  0.05618   0.14310   0.393
Voicepar3rd    -0.40699   0.08174  -4.979
Voicepar5th    -0.37084   0.08168  -4.540
```

iii.

Level 1	$y_i \sim N(\alpha_{1j[i],c[i]} + \alpha_{2j[i],h[i]} + \alpha_{3j[i],v[i]} + \beta \cdot X_i, \sigma_y^2),$ <p style="text-align: center;">when $\beta \cdot X_i = \beta_1 \cdot X_{i1} + \beta_2 \cdot X_{i2} + \beta_3 \cdot X_{i3}$</p>
Level 2	$\alpha_{1j,c} \sim N(\mu_{j,c} + \gamma_{1j} + \rho_c, \sigma_{jc}^2)$ $\alpha_{2j,h} \sim N(\mu_{j,h} + \gamma_{2j} + \rho_h, \sigma_{jh}^2)$ $\alpha_{3j,v} \sim N(\mu_{j,v} + \gamma_{3j} + \rho_v, \sigma_{jv}^2)$

Problem #2

Part (a)

First of all in order to determine which covariates to be added, I compared the model selected by problem 1, which is the last model with the one with added covariates, using AIC and BIC. Yet, since AIC and BIC significantly depends on the number of sample size, I needed to make sure the sample size used by two models were the same. This is relevant because lmer() function in R just drops whole observations if there are any missing values in the variables you specify for the model. So for example suppose variable A has no missing values but variable B has lots of missing values, and you want to compare $y \sim A + (1|\text{subject})$ with the model $y \sim A + B + (1|\text{subject})$, lmer() will use a smaller data set for the second model than the first, invalidating comparisons with AIC and BIC. To avoid this problem, I restricted my models to use the smaller data set that has no missing values in any of the fixed effects, so that we would not have change in the data set as we consider different models.

<code>> missing.value</code>	
Variable Name	Number of NAs
X	0
Subject	0
Harmony	0
Instrument	0
Voice	0
Selfdeclare	0
OMSI	0
X16.minus.17	0
ConsInstr	0
ConsNotes	360
Instr.minus.Notes	0
PachListen	72
ClsListen	36
KnowRob	180
KnowAxis	288
X1990s2000s	144
X1990s2000s.minus.1960s1970s	180
CollegeMusic	108
NoClass	288
APTheory	216
Composing	72
PianoPlay	0
GuitarPlay	0
X1stInstr	1512
X2ndInstr	2196
first12	0
Classical	27
Popular	27

One possible problem with this data formatting is that the data set might get too small to be meaningful. If we take out all the observation with any missing values in their measurements, it will get rid of about 2340 observations, only leaving 180 observations behind. To avoid such a small dataset, I checked the variables with biggest missing values first, so that we can ignore the missing values from those covariates. In terms of variable formatting, for the variables with 0 to 5 option with 0 as not at all, I dichotomized the variable into low and high: low being 0-2 and high being 3-5 to enable variable comparison.

First of all, I compared the models with the two variables that have the most missing values with the original model. Because both the AIC and BIC values are better for the original model, I could ignore the missing values from these two variables.

```
Original Model  
model3 <- lmer(Classical ~ Instrument + Harmony + Voice + (1|Subject:Instrument)+(1|Subject:Harmony)+(1|Subject:Voice))
```

Model	AIC	BIC
Original Model (model3)	761.6687	799.9841
Original Model (model3)+ X1stInstr	764.7309	806.2393

Model	AIC	BIC
Original Model (model3)	761.6687	799.9841
Original Model (model3)+ X2ndInstr	764.7056	806.2141

Then, I created my smallest data set to be used all over the course of model comparisons, which excludes all the observations with missing values from any one of the covariates but X1stInstr and X2ndInstr. This gave me a dataset with 1541 observations, which is not too small for the sample size.

Next, I compared the AIC and BIC values of the original model with the model added with new covariate for each covariates. Using the R function I made, I selected the covariates that has either lower AIC or BIC and further look into the values. Among these, I selected the ones with either AIC or BIC difference smaller than 3 but the other value should not have AIC or BIC value bigger than 3. As a result, I came up with 3 covariates: ClsListen, APTheory and PianoPlay

Model	AIC	BIC	AIC(new) – AIC (orginal)	BIC(new) – BIC (orginal)
Original Model (model3)	6262.207	6326.290	-	-
Original Model (model3)+ PachListen	6261.053	6330.476	-1.154224	4.185963
Original Model (model3)+ ClsListen	6253.177	6322.60	-9.029935	-3.689749
Original Model (model3)+ KnowRob	6261.730	6331.152	-0.4778973	4.8622895
Original Model (model3)+ KnowAxis	6261.358	6330.781	-0.8490432	4.4911437
Original Model (model3)+ APTheory	6257.857	6327.279	-4.3504835	0.9897033
Original Model (model3)+ PianoPlay	6253.808	6323.23	-8.399499	-3.059312

I compared the models with these 3 combinations of covariates, pick up the models with two lowest AIC and BIC and compared them in detail using anova.

	df	AIC	BIC	logLik	deviance
cand1	15	6233.7	6313.8	-3101.8	6203.7
cand2	14	6238.3	6313.0	-3105.1	6210.3
cand3	14	6233.4	6308.2	-3102.7	6205.4
cand4	14	6236.4	6311.2	-3104.2	6208.4
cand5	13	6238.2	6307.6	-3106.1	6212.2
cand6	13	6243.4	6312.8	-3108.7	6217.4
cand7	13	6239.4	6308.8	-3106.7	6213.4

Cand1	Original Model (model3)+ ClsListen + APTheory+ PianoPlay
Cand3	Original Model (model3) +ClsListen + PianoPlay
Cand5	Original Model (model3) +ClsListen)

From the ANOVA results, we see that the model is preferred in the order of cand3>cand1>cand5.

Therefore, as a final model with choose the model with two new covariates 'ClsListen' and 'PianoPlay'.

```
> anova(cand1,cand3)
      Df    AIC    BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
cand3 14 6233.4 6308.2 -3102.7  6205.4
cand1 15 6233.7 6313.8 -3101.8  6203.7  1.7914     1    0.1808

> anova(cand1,cand5)
      Df    AIC    BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
cand5 13 6238.2 6307.6 -3106.1  6212.2
cand1 15 6233.7 6313.8 -3101.8  6203.7  8.5258     2    0.01408 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

> anova(cand3,cand5)
      Df    AIC    BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
cand5 13 6238.2 6307.6 -3106.1  6212.2
cand3 14 6233.4 6308.2 -3102.7  6205.4  6.7344     1    0.009457 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Part (b)

Now, we will check whether the existing random effects should be subtracted or added, and whether the two new possible random effects, which are (1|Subject:ClsListen) and (1|Subject:PianoPlay) should be added.

(i) When I checked the AIC and BIC values for possible combinations of existing random effects, the best model given was one with (1|Subject:Instrument) and (1|Subject:Harmony).

```
m1: Classical ~ main effects + (1|Subject:Instrument)
m2: Classical ~ main effects + (1|Subject:Harmony)
m3: Classical ~ main effects + (1|Subject:Voice)
m4: Classical ~ main effects + (1|Subject:Instrument) + (1|Subject:Harmony)
m5: Classical ~ main effects + (1|Subject:Instrument) + (1|Subject:Voice)
```



```
m6: Classical ~ main effects + (1|Subject:Harmony) + (1|Subject:Voice)
```

	Df	AIC	BIC	logLik	deviance
m1	12	10048.2	10118	-5012.1	10024.2
m2	12	10504.5	10574	-5240.3	10480.5
m3	12	10586.1	10656	-5281.1	10562.1
m4	13	9949.3	10025	-4961.6	9923.3
m5	13	10050.2	10126	-5012.1	10024.2
m6	13	10506.5	10582	-5240.3	10480.5

(ii) Then, we check for additional new random effects to be added. Adding one of the new random effects gives you the same lowest AIC and BIC values. Among the two, I picked (1 | Subject:ClsListen) to continue with follow up questions.

```
m4: Classical ~ main effects + (1|Subject:Instrument) + (1|Subject:Harmony)
```

```
Random1: m4 + (1 | Subject:ClsListen) + (1 | Subject:PianoPlay)
```

```
Random2: m4 + (1 | Subject:ClsListen)
```

```
Random3: m4 + (1 | Subject:PianoPlay)
```

	Df	AIC	BIC	logLik	deviance
random2	14	9915.3	9996.7	-4943.7	9887.3
random3	14	9915.3	9996.7	-4943.7	9887.3
random1	15	9917.3	10004.5	-4943.7	9887.3

Therefore, the final model we get is :

```
Classical ~ Instrument + Harmony + Voice + ClsListen + PianoPlay + (1 | Subject:Instrument) + (1 | Subject:Harmony) + (1 | Subject:ClsListen)
```

Part (c)

Among 5 fixed effects variable, one with the biggest impact on classical rating is still the instrument, especially the string instrument. If the measurement has the Instrumentstring indicator than the Classical ratings increases by 3.15 in average. Among Harmony variable, I-V-Vi indicator is still the strongest indicator with largest t-value. This indicator increases the classical ratings. Voice variable seems to have a smallest in general, with their estimated coefficients of -0.40 and -0.35. Presence of the ClsListen indicator variable increases the Classical ratings by 0.42 in average. Unlike my expectation, PianoPlay indicator lowers the Classical ratings.

```
Linear mixed model fit by REML ['EminMod']
```

```
Formula: Classical ~ Instrument + Harmony + Voice + ClsListen + PianoPlay + (1 | Subject:Instrument) + (1 | Subject:Harmony) + (1 | Subject:ClsListen)
```

```
REML criterion at convergence: 9906.283
```

```
Random effects:
```

Groups	Name	Variance	Std.Dev.
Subject:Harmony	(Intercept)	0.4021	0.6341
Subject:Instrument	(Intercept)	1.2150	1.1023
Subject:ClsListen	(Intercept)	1.2569	1.1211
Residual		2.4480	1.5646

```
Number of obs: 2469, groups: Subject:Harmony, 276; Subject:Instrument, 207; Subject:ClsListen, 69
```

```
Fixed effects:
```

	Estimate	Std. Error	t value
(Intercept)	4.12408	0.28147	14.652
Instrumentpiano	1.37356	0.20309	6.763
Instrumentstring	3.15418	0.20281	15.552
HarmonyI-V-IV	-0.03194	0.13997	-0.228
HarmonyI-V-VI	0.78418	0.13999	5.602
HarmonyIV-I-V	0.05281	0.13993	0.377
Voicepar3rd	-0.39936	0.07716	-5.176
Voicepar5th	-0.35588	0.07714	-4.613
ClsListen1	0.42040	0.34340	1.224
PianoPlay1	-0.13088	0.43861	-0.298

Problem #3

To examine any interactions between the musician variable and other predictors in the model, I looked for the AIC and BIC decrease in adding new interaction variable. Since there are five predictors in the model, I checked five models with each interaction variables. Only one p-value from the ANOVA model comparison gave a significant result, which was musician*Harmony interaction term. Therefore, musicians are more influenced by Harmony than non-musicians.

```
> anova(t0,t1)
Data:
Models:
t0: Classical ~ Instrument + Harmony + Voice + ClsListen + PianoPlay +
t0: (1 | Subject:Instrument) + (1 | Subject:Harmony) + (1 | Subject:ClsListen)
t1: Classical ~ Instrument + Harmony + Voice + ClsListen + PianoPlay +
t1: (1 | Subject:Instrument) + (1 | Subject:Harmony) + (1 | Subject:ClsListen) +
t1: musician + Instrument:musician
  Df   AIC    BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
t0 14 9915.3 9996.7 -4943.7  9887.3
t1 17 9917.3 10016.1 -4941.6  9883.3 4.0559    3    0.2555

> anova(t0,t2) ##
Data:
Models:
t0: Classical ~ Instrument + Harmony + Voice + ClsListen + PianoPlay +
t0: (1 | Subject:Instrument) + (1 | Subject:Harmony) + (1 | Subject:ClsListen)
t2: Classical ~ Instrument + Harmony + Voice + ClsListen + PianoPlay +
t2: (1 | Subject:Instrument) + (1 | Subject:Harmony) + (1 | Subject:ClsListen) +
t2: musician + Harmony:musician
  Df   AIC    BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
t0 14 9915.3 9996.7 -4943.7  9887.3
t2 18 9896.2 10000.8 -4930.1  9860.2 27.184    4 1.824e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

> anova(t0,t3)
Data:
Models:
t0: Classical ~ Instrument + Harmony + Voice + ClsListen + PianoPlay +
t0: (1 | Subject:Instrument) + (1 | Subject:Harmony) + (1 | Subject:ClsListen)
t3: Classical ~ Instrument + Harmony + Voice + ClsListen + PianoPlay +
t3: (1 | Subject:Instrument) + (1 | Subject:Harmony) + (1 | Subject:ClsListen) +
t3: musician + Voice:musician
  Df   AIC    BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
t0 14 9915.3 9996.7 -4943.7  9887.3
t3 17 9920.3 10019.1 -4943.1  9886.3 1.0642    3    0.7857

> anova(t0,t4)
Data:
Models:
t0: Classical ~ Instrument + Harmony + Voice + ClsListen + PianoPlay +
t0: (1 | Subject:Instrument) + (1 | Subject:Harmony) + (1 | Subject:ClsListen)
t4: Classical ~ Instrument + Harmony + Voice + ClsListen + PianoPlay +
t4: (1 | Subject:Instrument) + (1 | Subject:Harmony) + (1 | Subject:ClsListen) +
t4: musician + ClsListen:musician
```

```

Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
t0 14 9915.3 9996.7 -4943.7 9887.3      0.5324      2 0.7663
t4 16 9918.8 10011.8 -4943.4 9886.8

> anova(t0,t5)
Data:
Models:
t0: Classical ~ Instrument + Harmony + Voice + CIsListen + PianoPlay +
t0:      (1 | Subject:Instrument) + (1 | Subject:Harmony) + (1 | Subject:CIsListen)
t5: Classical ~ Instrument + Harmony + Voice + CIsListen + PianoPlay +
t5:      (1 | Subject:Instrument) + (1 | Subject:Harmony) + (1 | Subject:CIsListen) +
t5:      musician + PianoPlay:musician
Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
t0 14 9915.3 9996.7 -4943.7 9887.3      0.9576      2 0.6195
t5 16 9918.4 10011.4 -4943.2 9886.4

```

Problem #4

Part (a)

To check the influence of Instrument, Harmony and Voice on Popular ratings, I compared the models of the one with the variable and one without, using ANOVA. If the p-values given are smaller than 0.05, then we conclude that adding a new variable does improve the model. All the p-values in the ANOVA table for ‘Instrument’ were highly significant, and those for ‘Harmony’ and ‘Voice’ were not.

Also, while all the AIC and BIC values got much smaller when the ‘Instrument’ variable was added, they got bigger or insignificantly smaller (difference less than 3) when the ‘Harmony’ and ‘Voice’ were added. Therefore, only ‘Instrument’ variable significantly influence the Popular ratings, and ‘Harmony’ and ‘Voice’ variables are not.

Comparing one with instrument and one without	
<pre> > anova(fit6,fit4) Analysis of Variance Table Model 1: Popular ~ Harmony Model 2: Popular ~ Instrument + Harmony Res.Df RSS Df Sum of Sq F Pr(>F) 1 2489 15596 2 2487 12672 2 2923.8 286.92 < 2.2e-16 *** --- Signif. codes: 0 '***' </pre>	<pre> > cbind(AIC(fit6,fit4),BIC(fit6,fit4)) df AIC df BIC fit6 5 11655.73 5 11684.84 fit4 7 11142.15 7 11182.90 </pre>
<pre> > anova(fit7,fit3) Analysis of Variance Table Model 1: Popular ~ Voice Model 2: Popular ~ Instrument + Voice Res.Df RSS Df Sum of Sq F Pr(>F) 1 2490 15612 2 2488 12688 2 2924.3 286.72 < 2.2e-16 *** --- Signif. codes: 0 '***' </pre>	<pre> > cbind(AIC(fit7,fit3),BIC(fit7,fit3)) df AIC df BIC fit7 4 11656.33 4 11679.62 fit3 6 11143.26 6 11178.19 </pre>
<pre> > anova(fit2,mdl1) Analysis of Variance Table Model 1: Popular ~ Harmony + Voice Model 2: Popular ~ Instrument + Harmony + Voice Res.Df RSS Df Sum of Sq F Pr(>F) </pre>	<pre> > cbind(AIC(fit2,mdl1),BIC(fit2,mdl1)) df AIC df BIC fit2 7 11657.31 7 11698.06 mdl1 9 11143.15 9 11195.54 </pre>

1	2487	15580						
2	2485	12656	2	2923.9	287.05	<	2.2e-16	***
--- Signif. codes: 0 '***'								

Comparing one with harmony and one without	
<pre>> anova(fit5,fit4) Analysis of Variance Table Model 1: Popular ~ Instrument Model 2: Popular ~ Instrument + Harmony Res.Df RSS Df Sum of Sq F Pr(>F) 1 2490 12703 2 2487 12672 3 31.119 2.0359 0.1068</pre>	<pre>> cbind(AIC(fit5,fit4),BIC(fit5,fit4)) df AIC df BIC fit5 4 11142.27 4 11165.55 fit4 7 11142.15 7 11182.90</pre>
<pre>> anova(fit7,fit2) Analysis of Variance Table Model 1: Popular ~ Voice Model 2: Popular ~ Harmony + Voice Res.Df RSS Df Sum of Sq F Pr(>F) 1 2490 15612 2 2487 15580 3 31.433 1.6725 0.1708</pre>	<pre>> cbind(AIC(fit7,fit2),BIC(fit7,fit2)) df AIC df BIC fit7 4 11656.33 4 11679.62 fit2 7 11657.31 7 11698.06</pre>
<pre>> anova(fit3,mdl1) Analysis of Variance Table Model 1: Popular ~ Instrument + Voice Model 2: Popular ~ Instrument + Harmony + Voice Res.Df RSS Df Sum of Sq F Pr(>F) 1 2488 12688 2 2485 12656 3 31.092 2.0349 0.1069</pre>	<pre>> cbind(AIC(fit3,mdl1),BIC(fit3,mdl1)) df AIC df BIC fit3 6 11143.26 6 11178.19 mdl1 9 11143.15 9 11195.54</pre>

Comparing one with voice and one without	
<pre>> anova(fit5,fit3) Analysis of Variance Table Model 1: Popular ~ Instrument Model 2: Popular ~ Instrument + Voice Res.Df RSS Df Sum of Sq F Pr(>F) 1 2490 12703 2 2488 12688 2 15.291 1.4993 0.2235</pre>	<pre>> cbind(AIC(fit5,fit3),BIC(fit5,fit3)) df AIC df BIC fit5 4 11142.27 4 11165.55 fit3 6 11143.26 6 11178.19</pre>
<pre>> anova(fit6,fit2) Analysis of Variance Table Model 1: Popular ~ Harmony Model 2: Popular ~ Harmony + Voice Res.Df RSS Df Sum of Sq F Pr(>F) 1 2489 15596 2 2487 15580 2 15.152 1.2093 0.2986</pre>	<pre>> cbind(AIC(fit6,fit2),BIC(fit6,fit2)) df AIC df BIC fit6 5 11655.73 5 11684.84 fit2 7 11657.31 7 11698.06</pre>
<pre>> anova(fit4,mdl1) Analysis of Variance Table Model 1: Popular ~ Instrument + Harmony Model 2: Popular ~ Instrument + Harmony + Voice Res.Df RSS Df Sum of Sq F Pr(>F) 1 2487 12672 2 2485 12656 2 15.263 1.4984 0.2237</pre>	<pre>> cbind(AIC(fit4,mdl1),BIC(fit4,mdl1)) df AIC df BIC fit4 7 11142.15 7 11182.90 mdl1 9 11143.15 9 11195.54</pre>

From the summary of the model, we can see how particular kinds of each variable affect ratings. As seen above, Instrument was highly significant variables with big impact on the Popular rating. Among harmony variable, one noticeable kind is Harmony I-V-VI. While other kinds of harmony gives insignificant p-values, Harmony I-V-VI had a significant p-value, and it decreases the Popular rating by -0.27 in average. All kinds of Voice variables were not significant enough.

```
Call:
lm(formula = Popular ~ Instrument + Harmony + Voice)

Residuals:
    Min       1Q   Median       3Q      Max
-6.7218 -1.7026  0.2008  1.4691 13.2248

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   6.58263    0.12761   51.583 <2e-16 ***
Instrumentpiano -0.95200    0.11102   -8.575 <2e-16 ***
Instrumentstring -2.61173    0.11035  -23.667 <2e-16 ***
HarmonyI-V-IV  -0.02405    0.12782   -0.188  0.8508
HarmonyI-V-VI  -0.26829    0.12782   -2.099  0.0359 *
HarmonyIV-I-V  -0.18564    0.12772   -1.454  0.1462
Voicepar3rd     0.16859    0.11075    1.522  0.1281
Voicepar5th     0.16326    0.11068    1.475  0.1403
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.257 on 2485 degrees of freedom
(27 observations deleted due to missingness)
Multiple R-squared:  0.1901,    Adjusted R-squared:  0.1878
F-statistic: 83.32 on 7 and 2485 DF,  p-value: < 2.2e-16
```

Part (b)

To answer the question, I tried to find the best model for Popular rating as I did for Classical rating. First, as in problem 1, I compared three models (i) with only three main effects, (ii) with main effects and (1|Subject), and (iii) with main effects and (1|Subject:Instrument), (1|Subject:Harmony), (1|Subject:Voice). Comparing the AIC and BIC values, I chose (iii) as the best model.

	df	AIC	BIC
(i)	md11 9	11143.15	11195.54
(ii)	md12 10	10453.12	10511.34
(iii)	md13 12	10097.24	10167.09

Then, I continued to find the covariates to be added. Again, I compared the models with the two variables that have the most missing values with the original model. Because both the AIC and BIC values are better for the original model, I could ignore the missing values from these two variables.

Original Model

```
mdl3 <- lmer(Popular ~ Instrument + Harmony + Voice + (1|Subject:Instrument)+(1|Subject:Harmony)+(1|Subject:Voice))
```

Model	AIC	BIC
Original Model (mdl3)	776.4435	814.7590
Original Model (mdl3)+ X1stInstr	777.6879	819.1964

Model	AIC	BIC
Original Model (mdl3)	776.4435	814.7590
Original Model (mdl3)+ X2ndInstr	776.9491	818.4576

Using the same smaller data set as I did for Classical ratings, which excludes all the observations with missing values from any one of the covariates but X1stInstr and X2ndInstr, I compared the AIC and BIC values of the original model with the model added with new covariate for each covariates. There are only two models that has either lower AIC or BIC than the original model. I reject the GuitarPlay variable since BIC is significantly bigger ($4.36 > 3$) and the decrease is AIC is not significant enough

Model	AIC	BIC	AIC(new) – AIC (original)	BIC(new) – BIC (original)
Original Model (model3)	6262.207	6326.290	-	-
Original Model (model3)+ PachListen	6261.053	6330.476	-2.960526	2.379661
Original Model (model3)+ GuitarPlay	6263.539	6332.962	-0.9763054	4.3638815

To make sure PachListen to be included in the model, I checked the p-value in the ANOVA table. The p-value was smaller than 0.05. Therefore, I decide to include PachListen variable into the model.

	Df	AIC	BIC	logLik	deviance	Chisq	Chi	Df	Pr(>Chisq)
mdl3	12	6343.9	6407.9	-3159.9	6319.9				
vr7	13	6341.8	6411.2	-3157.9	6315.8	4.1052		1	0.04275 *

Then, checking for the random effects using AIC and BIC tells us to take out (1 | Subject:voice) and add (1 | Subject:PachListen). With this adjusted random effects the final model we get for Popular ratings is :

Popular ~ Instrument + Harmony + Voice + (1|Subject:Instrument) + (1|Subject:Harmony) + + PachListen + (1 | Subject:PachListen)

From the summary of this model we see that impact of Instrument is very significant vompared to 3 other variables: the F-value of the Instrument is tremendously bigger than the other F-values.

	Df	Sum Sq	Mean Sq	F value
Instrument	2	456.89	228.445	90.8257
Harmony	3	15.38	5.126	2.0379
Voice	2	15.94	7.969	3.1685
PachListen	1	0.39	0.394	0.1565

As in the Classical ratings, string instrument indicator has the biggest impact on the Popular rating. It decreases the popular rating by 2.59 in average. All the harmony indicators will give negative impact on Popular rating compared to its standard zero indicator I-VI-V harmony. voicepar3rd and voicepar5indicators will increase

the Popular ratings by 0.18 and 0.14 in average. The new covariate PachListen will lower the rating by -0.21 in average.

```
Linear mixed model fit by REML ['lmerMod']
Formula: Popular ~ Instrument + Harmony + Voice + PachListen + (1 | Subject:Instrument) +
(1 | Subject:Harmony) + (1 | Subject:PachListen)

REML criterion at convergence: 9791.452

Random effects:
Groups          Name                Variance Std.Dev.
Subject:Harmony (Intercept)  0.3741   0.6117
Subject:Instrument (Intercept) 1.0829   1.0406
Subject:PachListen (Intercept) 1.1306   1.0633
Residual                2.5152   1.5859
Number of obs: 2433, groups: Subject:Harmony, 272; Subject:Instrument, 204;
Subject:PachListen, 68

Fixed effects:
              Estimate Std. Error t value
(Intercept)    6.80116    0.54572  12.463
Instrumentpiano -0.91103    0.19528  -4.665
Instrumentstring -2.59014    0.19499 -13.284
HarmonyI-V-IV   -0.02128    0.13884  -0.153
HarmonyI-V-VI  -0.29114    0.13887  -2.096
HarmonyIV-I-V  -0.19530    0.13881  -1.407
Voicepar3rd     0.18482    0.07879   2.346
Voicepar5th     0.15478    0.07877   1.965
PachListen1    -0.21759    0.55003  -0.396
```

choices of random effects here are somewhat unusual, and omit some aspects of the experimental design that should be kept in the model

Part (c)

To examine the interactions between the musician variable and predictors in my model, again I looked for the AIC and BIC decrease in adding new interaction variable. Since there are four predictors in the model, I checked four models with each interaction variables. Two p-value from the ANOVA model comparison gave a significant result, which was musician*Harmony and musician*PachListen interaction term. Therefore, musicians are more influenced by Harmony and PachListen than non-musicians.

```
> anova(mu0,mu1)
Data:
Models:
mu0: Popular ~ Instrument + Harmony + Voice + PachListen + (1 | Subject:Instrument) +
mu0:      (1 | Subject:Harmony) + (1 | Subject:PachListen)
mu1: Popular ~ Instrument + Harmony + Voice + PachListen + (1 | Subject:Instrument) +
mu1:      (1 | Subject:Harmony) + (1 | Subject:PachListen) + musician +
mu1:      Instrument:musician
      Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
mu0 13 9799.2 9874.6 -4886.6  9773.2
mu1 16 9799.5 9892.2 -4883.7  9767.5  5.7186    3    0.1261
> anova(mu0,mu2)
Data:
Models:
mu0: Popular ~ Instrument + Harmony + Voice + PachListen + (1 | Subject:Instrument) +
mu0:      (1 | Subject:Harmony) + (1 | Subject:PachListen)
mu2: Popular ~ Instrument + Harmony + Voice + PachListen + (1 | Subject:Instrument) +
mu2:      (1 | Subject:Harmony) + (1 | Subject:PachListen) + musician +
mu2:      Harmony:musician
      Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
mu0 13 9799.2 9874.6 -4886.6  9773.2
mu2 17 9791.7 9890.3 -4878.9  9757.7 15.459    4 0.003839 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

exploration of musician interactions is rather incomplete

```

> anova(mu0,mu3)
Data:
Models:
mu0: Popular ~ Instrument + Harmony + Voice + PachListen + (1 | Subject:Instrument) +
mu0:      (1 | Subject:Harmony) + (1 | Subject:PachListen)
mu3: Popular ~ Instrument + Harmony + Voice + PachListen + (1 | Subject:Instrument) +
mu3:      (1 | Subject:Harmony) + (1 | Subject:PachListen) + musician +
mu3:      Voice:musician
      Df      AIC      BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
mu0 13 9799.2 9874.6 -4886.6  9773.2
mu3 16 9800.7 9893.5 -4884.4  9768.7 4.4741    3 0.2146
> anova(mu0,mu4)
Data:
Models:
mu0: Popular ~ Instrument + Harmony + Voice + PachListen + (1 | Subject:Instrument) +
mu0:      (1 | Subject:Harmony) + (1 | Subject:PachListen)
mu4: Popular ~ Instrument + Harmony + Voice + PachListen + (1 | Subject:Instrument) +
mu4:      (1 | Subject:Harmony) + (1 | Subject:PachListen) + musician +
mu4:      PachListen:musician
      Df      AIC      BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
mu0 13 9799.2 9874.6 -4886.6  9773.2
mu4 15 9797.2 9884.1 -4883.6  9767.2 6.0392    2 0.04882 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```


Problem 5

To examine the influence of three main experimental factors, I compared the AIC and BIC values before and after including the variable to the model, using the ANOVA table comparison I also checked for the p-value to see if this difference in AIC and BIC values are statistically significant. Then, looking at the estimated coefficients of each indicator variables, I could see the extent to which each variable affects the ratings and whether it increases or decreases the ratings. Bigger the coefficient value, bigger the impact of the variable on the ratings. Then, we checked whether we could adopt a “repeated measures” model by testing if the random intercept by subject makes the model better. Taking one step further, we see if the model is better if we could adopt personal biases vary with the types of instrument, harmony and/or voice leading. Then, we take into account the other possible covariates by checking whether adding certain covariate makes the model better.

which model was better?

For the Classical Ratings, as I showed in part1(a), all three main experimental factors influenced the ratings significantly. Among these ‘Instrument’ variable had the biggest and most statistically significant influence on the ratings. In particular, string instrument indicator increases the classical ratings by about 3.13 in average, which is a huge impact in a 1-10 rating scale. Among the ‘Harmony’ variable Harmony I-V-VI had was the most statistically significant, but the extent to which it impacts the rating was not that big, 0.77 in average. In ‘Voice’ variable, Parallel 3rds and 5ths had a negative impact on the classical rating compared to Contrary motion. The extent of the impact was not big, -0.41 and -0.37, compared to the Contrary motion, but the difference was statistically significant.

When I checked whether “repeated measures” model should be adopted, it showed positive results by lowering AIC and BIC levels by more than 1000. Therefore, I continued with the “repeated measures” model. When we considered that the personal biases to vary with the types of instrument, harmony and/or voice leading, we see that these random effects account for Classical ratings ratings as much as the fixed effects do. This was seen by comparing the estimated variances with the estimated residual variance. When we did not considered variation of personal bias according to the types of main effects, the estimated variance for random intercept was only half the estimated residual variance (1.702 compared to 3.581). On the other hand, when we took the variation into account, the size of the estimated variance for random intercepts was as big as the estimated residual variance. From the other survey factors, we could see that ‘ClisListen’, ‘APTheory’ and ‘PianoPlay’ does impact the classical rating but taking correlation into accounts, we were left with ‘ClisListen’ and ‘PianoPlay’. High ratings of ‘ClisListen’(higher than 3) increases the rating by 0.42 in average, and that of ‘PianoPlay’ lowers the rating by 0.13.

this parag is difficult to follow. probably should break into two parags and state firm conclusion s at the end of each parag.

For Popular Ratings, as I showed in part4(a), only ‘Instrument’ variable was statically significant. Other two variables did not help much determining the Popular ratings. As in classical ratings, string instrument indicator had the biggest impact, lowering the rating by 2.61 in average. Once again, Harmony I-V-VI had a significant impact on the rating, lowering the rating by 0.27 in average. Unlike in Classical rating, Parallel 3rds and 5ths had a positive impact compared to Contrary motion. The same were the true for the “repeated measures” model analysis as in Classical rating. It showed positive results by lowering AIC and BIC levels by more than 1000. One extra covariate I added for Popular rating is ‘PachListen’, people who are more familiar with (3 or higher in 0-5 rating scale) Pachelbel’s Canon was in average 0.21 lower in Popular rating.

the unusual random effect involving pachlisten should be discussed.

As researchers’ expected, indeed the instrument had the largest influence on rating and Harmony I-V-VI was frequently rated as classical. Also, the contrary motion was frequently rated as classical compared to other types. Yet, the extent of this influence was smaller than 1.