

# The Relationship between Per Capita Income and the County's Economic, Health and Social Well-being

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## Abstract

This paper is interested in analyzing the relationship between average income per person and variables related to economic, health, and social well-being. We examined the data of selected county demographic information from Kutner et al. (2005). We created regression models with possible transformations, interactions based on F-tests, AIC, BIC, and VIF values. It appears that the land area, the percent of unemployment, the percent of population aged 18-34, the number of doctors, the percent of with income below poverty level, and geographic region are better in predicting the per capita income. However, it is also surprising to see the negative correlations between per capita income and unemployment or the percent of the population aged 18-34.

## 1 Introduction

The income per person measures the economic well-being of the person. The local agencies, for example, the agencies at the county level, use the income data to plan and fund programs provide economic assistance for populations in need. In this report, we will investigate the relationship between average income per person and some variables that indicate the economic, health and social well-being at the county level.

We will

- Illustrate the relationship between any two variables in our analyses
- Identify the relationship between per-capita income and crime rate specifically
- Develop regression models to predict per-capita income from other economic, health, and social well-being related variables

## 2 Data

The data provides selected county demographic information (CDI) for 440 of the most populous counties in the United States. We took the data from Kutner et al. (2005). The data are given in the file cdi.dat in the files area for our course on Canvas. Each data set line has an identification number with a county name and state abbreviation and provides information on 14 variables for a single county. The report generally pertains to the years 1990 and 1992.

The variables in the data set are shown in Table 1.

Table 1: Variable Definitions for the cdi.dat data set.

| Variable       | Definition & Comments   |
|----------------|---|
| id             | The identification number of each record in the data  |
| county         | Categorical Variable: County name   |
| state          | Categorical Variable: Two-letter state abbreviation   |
| land.area      | Land area (in square miles)   |
| pop            | Estimated 1990 population   |
| pop.18_34      | Percent of 1990 CDI population aged 18–34   |
| pop.65_plus    | Percent of 1990 CDI population aged 65 or old   |
| doctors        | Number of professionally active nonfederal physicians during 1990   |
| hosp.beds      | Total number of beds, cribs, and bassinets during 1990  |
| crimes         | Total number of serious crimes in 1990, including murder, rape, robbery, aggravated assault, burglary, larceny-theft, and motor vehicle theft, as reported by law enforcement agencies  |
| pct.hs.grad    | Percent of adult population (persons 25 years old or older) who completed 12 or more years of school  |
| pct.bach.deg   | Percent of adult population (persons 25 years old or older) with bachelor's degree  |
| pct.below.pov  | Percent of 1990 CDI population with income below poverty level  |
| pct.unemp      | Percent of 1990 CDI population that is unemployed   |
| per.cap.income | Per-capita income (i.e. average income per person) of 1990 CDI population (in dollars)  |
| tot.income     | Total personal income of 1990 CDI population (in millions of dollars)   |
| region         | Categorical Variable: Geographic region classification used by the US Bureau of the Census, NE (northeast region of the US), NC (north-central region of the US), S (southern region of the US), and W (Western region of the US) |
| per.cap.crime  | Per-capita income (i.e. average income per person) of 1990 CDI population (in dollars)  |

The id variable here is the same as the row number in our data, and we did not include it in our data analyses.

The summary statistics for the quantitative variables in the data are given in Table 2 below.

Table 2: Summary Statistics for Continuous Variables in cdi.dat dataset

|                | Min.     | 1st Qu.   | Median    | Mean      | 3rd Qu.   | Max.      | SD        |
|----------------|----------|-----------|-----------|-----------|-----------|-----------|-----------|
| land.area      | 15.0     | 451.25    | 656.50    | 1041.41   | 946.75    | 20062.0   | 1549.92   |
| pop            | 100043.0 | 139027.25 | 217280.50 | 393010.92 | 436064.50 | 8863164.0 | 601987.02 |
| pop.18_34      | 16.4     | 26.20     | 28.10     | 28.57     | 30.02     | 49.7      | 4.19      |
| pop.65_plus    | 3.0      | 9.88      | 11.75     | 12.17     | 13.62     | 33.8      | 3.99      |
| doctors        | 39.0     | 182.75    | 401.00    | 988.00    | 1036.00   | 23677.0   | 1789.75   |
| hosp.beds      | 92.0     | 390.75    | 755.00    | 1458.63   | 1575.75   | 27700.0   | 2289.13   |
| crimes         | 563.0    | 6219.50   | 11820.50  | 27111.62  | 26279.50  | 688936.0  | 58237.51  |
| pct.hs.grad    | 46.6     | 73.88     | 77.70     | 77.56     | 82.40     | 92.9      | 7.02      |
| pct.bach.deg   | 8.1      | 15.28     | 19.70     | 21.08     | 25.33     | 52.3      | 7.65      |
| pct.below.pov  | 1.4      | 5.30      | 7.90      | 8.72      | 10.90     | 36.3      | 4.66      |
| pct.unemp      | 2.2      | 5.10      | 6.20      | 6.60      | 7.50      | 21.3      | 2.34      |
| per.cap.income | 8899.0   | 16118.25  | 17759.00  | 18561.48  | 20270.00  | 37541.0   | 4059.19   |
| tot.income     | 1141.0   | 2311.00   | 3857.00   | 7869.27   | 8654.25   | 184230.0  | 12884.32  |
| per.cap.crime  | 0.0      | 0.04      | 0.05      | 0.06      | 0.07      | 0.3       | 0.03      |

In Table 8, there are several variables with Mean substantially larger than Median (land.area, pop, doctors, hosp.beds, crimes, per.cap.income, and tot.income), indicating possible right-skewing in their distributions. There are no variables with a mean substantially smaller than the median.

Since all categorical variables in the data describe the geographic location, the summary statistics of one categorical variable could at least provide us some geographic information of the data. The summary statistics for the categorical variable region in the data are given in Table 3 below. We see that most counties are in the South (region 'S'), and the least are in the west (region 'W').

Table 3: Summary  
Statistics of Region in CDI

|      | NC  | NE  | S   | W  |
|------|-----|-----|-----|----|
| Freq | 108 | 103 | 152 | 77 |

### 3 Methods

Our analyses consist of four parts. First, we used the correlation matrix to investigate the relationship between any two variables in our data. As the per capita income is the main interest of the response variable in our data, we also checked to see how other predictors in our data are related to the per capita income.

Second, to develop a regression model to predict the per capita income from crime rate and region of the county, we considered regression models using untransformed per capita income and crime rate or per capita crime rate with or without interaction with geographic region. We chose our final model based on F-tests, AIC, and BIC values of each regression model, accompanied by a summary of each regression analysis and an examination of residual diagnostic plots.

Moreover, to develop a regression model to predict the per capita income from the other variables in our data, we started with a complete model using logarithmic, and Box-Cox power transforms of the right-skewed predictors and power transforms the left-skewed predictors without region. We found all subsets of the whole model. We chose our final model based on the minimum BIC value, accompanied by a summary of each regression analysis and an examination of residual diagnostic plots, VIF values, and marginal model plots. Then, we added the interactions with the region in our last model. We kept the interaction term based on the p-values. At last, we chose our final model based on F-tests, AIC, and BIC values, accompanied by a summary of each regression analysis and an examination of residual diagnostic plots.

At last, as states are nested in the region, we fitted our final all-subsets regression model with state, and we omitted the region and the interaction with region here. We also chose our final model based on F-tests, AIC, and BIC values, accompanied by a summary of each regression analysis and an examination of residual diagnostic plots. We used the same criteria to compare this final model for the state with the last model for the region.

### 4 Results

#### 4.1 Relationship Among Variables

We considered a correlation matrix presented in Figure 1 below to capture the correlation coefficient between any two original variables in our data. A value greater than zero indicates there is a positive relationship between the two variables. A value less than zero means there is a negative relationship between the two variables. An absolute correlation coefficient being one indicates a perfect correlation between the two variables.

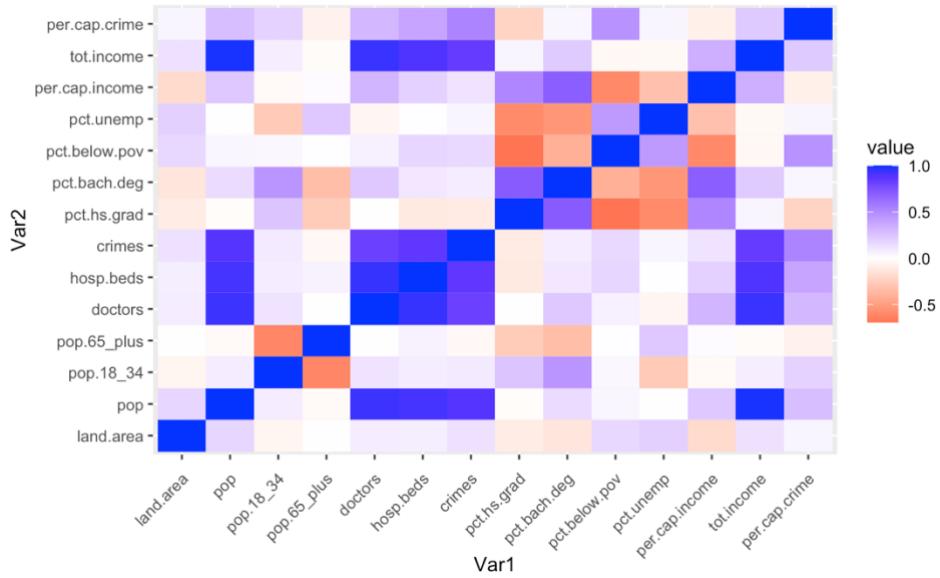


Figure 1: Correlation Matrix of Variables in cdi.dat data

In Figure 1, the tot.income and pop are highly correlated, which is not surprising as the tot.income is the total personal income over the population. These two variables are reasonably highly correlated with crimes, hosp.beds, and doctors since we need more doctors, number of beds, cribs, and bassinets for a larger population. The incidents of crimes are more frequent for a larger population. The three variables—crimes, hosp.beds, and doctors—seem strongly positively correlated with one another. The per.cap.income is not highly correlated with any variable. The pct.hs.grad or pct.bach.deg is positively correlated with the per.cap.income since people with higher education tend to more expertise in solving more difficult problems and then earn more. The pct.below.pov or pct.unemp is negatively correlated with the per.cap.income since we have less income in total and less income per capita if more people are in poverty or unemployed. These four variables pct.hs.grad, pct.bach.deg, pct.below.pov, pct.unemp are moderately correlated with each other. Further EDA analyses on the relationship of each variable against per.cap.income are in Appendix A (see page 20 below).

## 4.2 Predicting Per Capita Income from Crime Rate and Region

We considered one regression model using the original variables per.cap.income, crimes, and region and another using per.cap.income, region, and created per.cap.crime which is the number of crimes over the population. Then, we investigated whether the associations between per.cap.income and crimes or per.cap.crime differentiate across different regions.

### Regression Models without Interaction

The two regression models with crime or per.cap.crime were the following models, shown with estimated regression coefficients:

$$\text{per.cap.income} = 18110 + 0.009 * \text{crime} + 2286 * \text{regionNE} - 860.6 * \text{regionS} - 142.8 * \text{regionW} \quad (1)$$

$$\begin{aligned} \text{per.cap.income} = & 18006.04 + 5773.20 * \text{per.cap.crime} + 2354.70 * \text{regionNE} \\ & - 927.45 * \text{regionS} - 34.92 * \text{regionW} \end{aligned} \quad (2)$$

Models (1) and (2) had similar low R2 values (0.09288 and 0.07782, respectively). Still, we detected a non-random pattern and highly influential points in the residual diagnostic plots for model (1) (Appendix B, see page 21 below). Model (2) has better diagnostic plots, and we prefer model (2), which uses per.cap.crime. Table 4 gives the full table of estimated coefficients and standard errors for model (2).

|               | Estimate | Std. Error | t value | Pr(> t )     |
|---------------|----------|------------|---------|--------------|
| (Intercept)   | 18006.04 | 537.04     | 33.528  | < 2e-16 ***  |
| per.cap.crime | 5773.20  | 7520.41    | 0.768   | 0.4431       |
| regionNE      | 2354.70  | 541.97     | 4.345   | 1.74e-05 *** |
| regionS       | -927.45  | 512.31     | -1.810  | 0.0709 .     |
| regionW       | -34.92   | 586.03     | -0.060  | 0.9525       |

Table 4: Estimated coefficients and standard errors for model (2).

### Regression Models with Interaction

To investigate whether the correlation between per.cap.income and crime differentiate across the different regions, we build on the model (2) with interaction terms between per.cap.crime and region. The model with interaction was shown below with estimated regression coefficients:

$$\begin{aligned} \text{per.cap.income} = & 4379.1 * \text{per.cap.crime} + 2329.0 * \text{regionNE} - 1010.4 * \text{regionS} - 670.0 * \text{region} \\ & + 288.4 * \text{per.cap.income} * \text{regionNE} + 1558.9 * \text{per.cap.income} * \text{regionS} \\ & + 10655.5 * \text{per.cap.income} * \text{regionW} \end{aligned} \quad (3)$$

Based on the F-statistics in the ANOVA table below, there is little evidence to support the full model (3). We were happy to adopt the reduced model (2) that contains no interactions between per.cap.crime and region. According to the model, the salary level varies with the region in the US, but the way it is related to crime does not.

#### Analysis of Variance Table

| Model 1: per.cap.income ~ per.cap.crime + region |        |            |    |           |               |
|--|--------|------------|----|-----------|---------------|
| Model 2: per.cap.income ~ per.cap.crime * region |        |            |    |           |               |
|  | Res.Df | RSS        | Df | Sum of Sq | F Pr(>F)      |
| 1  | 435    | 6609753963 |    |           |               |
| 2  | 432    | 6607856753 | 3  | 1897210   | 0.0413 0.9888 |

Table 5: ANOVA table of Models (2) and (3)

Based on table 4 above, for one unit increase in the number of crime per capita, the per capita income increases 5773.2 dollars on average. The average per capita income in the North Central region is 18006.04 dollars. The average per capita income in the Northeastern region is 20360.74 dollars. The average per capita in the South is 17078.59 dollars. The average per capita income in the West region 17971.12 dollars.

### 4.2 Predicting Per capita income from other variables

#### Logarithmic and Power Transformations

To address right skewness, potential leverage, and influence issues, we took the logarithms of the variables we identified with some or severe right skewness in their univariate distributions (Appendix B, see pages 39 to 52). To address left skewness, potential leverage, and influence issues, we raised the

power of the variables we identified with some severe left skewness in their univariate distributions (Appendix B, see page 46).

Before we go through our model selections, we should first mention the inclusion of three categorical variables: county, state, and region in our models. Looking at the unique values for the county (Appendix A, see page 11 below), it has 373 unique values, which is nearly as many as the rows in the cdi.dat dataset (440). A graph describing the combination of county and state showed that we only had one observation per unique county, so the county is not a valid variable to include in our models (Appendix A, see pages 11 to 13).

### Variable selection using All Subsets

We started with fitting the following full model

$$\text{per.cap.income} = \log(\text{land.area}) + \log(\text{per.cap.crime}) + \log(\text{pct.unemp}) + \log(\text{pct.below.pov}) + \log(\text{pct.bach.deg}) + (\text{pct.hs.grad})^3 + \log(\text{hosp.beds}) + \log(\text{doctors}) + \log(\text{pop.65_plus}) + \log(\text{pop.18_34}) + e \quad (4)$$

Based on the inverse response plot and Box-Cox statistics, we found no need to transform the response variable per.cap.income. A detailed graphical summary of using all possible subsets can be found in Appendix C (see pages 55 to 65 below). We chose our final model with a minimum BIC value (Appendix C, see page 56). The final model is shown below with estimated regression coefficients:

$$\text{per.cap.income} = 32700.93 - 681.97 \log(\text{land.area}) + 1712.85 \log(\text{pct.unemp}) - 4165.28 \log(\text{pct.below.pov}) + 6137.60 \log(\text{pct.bach.deg}) - 84.97 (\text{pct.hs.grad})^3 + 1111.08 \log(\text{doctors}) - 6863.47 \log(\text{pop.18_34}) \quad (5)$$

Table 6 gives the full table of estimated coefficients and standard errors for model (5).

|                              | Coefficients: | Estimate | Std. Error | t value  | Pr(> t ) |
|------------------------------|---------------|----------|------------|----------|----------|
| (Intercept)                  | 32700.93      | 2806.52  | 11.652     | < 2e-16  | ***      |
| $\log(\text{land.area})$     | -681.97       | 100.57   | -6.781     | 3.93e-11 | ***      |
| $\log(\text{pct.unemp})$     | 1712.85       | 339.06   | 5.052      | 6.47e-07 | ***      |
| $\log(\text{pct.below.pov})$ | -4165.28      | 229.08   | -18.183    | < 2e-16  | ***      |
| $\log(\text{pct.bach.deg})$  | 6137.60       | 485.54   | 12.641     | < 2e-16  | ***      |
| $\text{pct.hs.grad}$         | -84.97        | 21.70    | -3.916     | 0.000105 | ***      |
| $\log(\text{doctors})$       | 1111.08       | 91.42    | 12.153     | < 2e-16  | ***      |
| $\log(\text{pop.18_34})$     | -6863.47      | 729.42   | -9.409     | < 2e-16  | ***      |

Table 6: Estimated coefficients and standard errors for model (5).

The blue data-based curves line up well with the red model-based curves in the marginal model plots of the model (5). The coefficients on pct.unemp, pct.hs.grad and pop.18\_34 do not meet what we expected. One possible reason would be that we did not consider geographic effects.

### Regression Model with Region

We started with interacting all the variables in the final subset model with region. We kept the whole categorical variable if any indicator for a categorical variable or an interaction with categorical variable is statistically significant coefficient. If none of them is statistically significant, we dropped the variable or the interaction of the variable with the categorical variable. The VIF values are not excessively large and only two of them exceed five (Appendix C, see page 62 below). The F-statistics, AIC, and BIC values in Appendix C suggested that we should choose the model with interaction between some variables and region (see page 64 below).

## Regression Model with State

Then, we considered model (5) with state and interaction with the state. The following ANOVA table, AIC, and BIC values in Appendix D suggest choosing the model with the state but without interaction (see pages 68 and 69 below).

| Analysis of Variance Table   |        |            |     |           |                      |
|--|--------|------------|-----|-----------|----------------------|
|  | Res.Df | RSS        | DF  | Sum of Sq | F                    |
| Model 1: per.cap.income ~ log(land.area) + log(pct.unemp) + log(pct.below.pov) + log(pct.bach.deg) + (pct.hs.grad)^3 + log(doctors) + log(pop.18_34)           | 432    | 1314204306 |     |           |                      |
| Model 2: per.cap.income ~ log.land.area + log.pop.18_34 + log.doctors + pct.hs.grad.3 + log.pct.bach.deg + log.pct.below.pov + log.pct.unemp + state           | 385    | 967714409  | 47  | 346489897 | 3.0968 3.526e-08 *** |
| Model 3: per.cap.income ~ (log.land.area + log.pop.18_34 + log.doctors + pct.hs.grad.3 + log.pct.bach.deg + log.pct.below.pov + log.pct.unemp + state) * state | 180    | 428500031  | 205 | 539214379 | 1.1049 0.2464        |

Table 7: ANOVA table for the model (5), with State, and Interaction with State

A full table of estimated coefficients and standard errors for our model with state is listed in Appendix D (see pages 69 and 70 below).

## Final Model

Then we compared the model with some interaction with the region and the model with the state. Looking at the diagnostic plots for the model with state, we saw more deviation in the normal Q-Q plots, and there are some highly influential points in the leverage graph. In addition, the VIF values of the model with state suggest that adding the categorical variable state would result in a severe multicollinearity issue here (Appendix D, see page 73 below). Considering the correlations between the predictor and the response variable across the region, we chose the model with some interaction with the region even though the ANOVA table and AIC values prefer the model with the state (Appendix D, see page 73 below).

The following table gives the full table of estimated coefficients and standard errors for our final model.

| Coefficients:              | Estimate   | Std. Error | t value | Pr(> t )     |
|----------------------------|------------|------------|---------|--------------|
| (Intercept)                | 3.360e+04  | 4.645e+03  | 7.235   | 2.25e-12 *** |
| log.land.area              | -7.042e+02 | 1.160e+02  | -6.070  | 2.88e-09 *** |
| log.pop.18_34              | -7.613e+03 | 1.676e+03  | -4.543  | 7.28e-06 *** |
| log.doctors                | 9.303e+02  | 9.476e+01  | 9.818   | < 2e-16 ***  |
| pct.hs.grad.3              | -1.494e-03 | 3.493e-03  | -0.428  | 0.669130     |
| log.pct.bach.deg           | 4.676e+03  | 1.022e+03  | 4.574   | 6.33e-06 *** |
| log.pct.below.pov          | -3.094e+03 | 5.243e+02  | -5.901  | 7.49e-09 *** |
| log.pct.unemp              | 1.264e+03  | 3.599e+02  | 3.513   | 0.000491 *** |
| regionNE                   | 6.507e+03  | 6.535e+03  | 0.996   | 0.319934     |
| regionS                    | -7.750e+03 | 5.118e+03  | -1.514  | 0.130705     |
| regionW                    | 8.801e+02  | 7.303e+03  | 0.120   | 0.904145     |
| log.pop.18_34:regionNE     | -4.386e+03 | 2.371e+03  | -1.849  | 0.065105 .   |
| log.pop.18_34:regionS      | 1.143e+03  | 1.926e+03  | 0.594   | 0.553079     |
| log.pop.18_34:regionW      | 2.167e+03  | 2.495e+03  | 0.869   | 0.385500     |
| pct.hs.grad.3:regionNE     | -6.806e-03 | 4.468e-03  | -1.523  | 0.128469     |
| pct.hs.grad.3:regionS      | -4.696e-03 | 4.207e-03  | -1.116  | 0.264941     |
| pct.hs.grad.3:regionW      | -1.791e-02 | 4.387e-03  | -4.082  | 5.34e-05 *** |
| log.pct.bach.deg:regionNE  | 4.292e+03  | 1.285e+03  | 3.340   | 0.000913 *** |
| log.pct.bach.deg:regionS   | 2.482e+03  | 1.147e+03  | 2.164   | 0.030997 *   |
| log.pct.bach.deg:regionW   | 3.229e+03  | 1.374e+03  | 2.350   | 0.019231 *   |
| log.pct.below.pov:regionNE | -7.567e+02 | 7.117e+02  | -1.063  | 0.288320     |
| log.pct.below.pov:regionS  | -8.122e+02 | 6.451e+02  | -1.259  | 0.208756     |
| log.pct.below.pov:regionW  | -3.949e+03 | 9.132e+02  | -4.324  | 1.92e-05 *** |

Table 8: Estimated coefficients and standard errors for the final model with region and interaction.

## 5 Discussion

Among models we considered, we found the model that fits per.cap.income and other variables best, including log-transformed land.area, pop.18\_34, doctors, pct.hs.grad.3, pct.bach.deg, pct.below.pov, pct.unemp, region and their interaction with the region. The model explains 83.8% of the variation in the per.cap.income around its mean.

For every 1% increase in a county's land area, the average income per person decreases by about 7.04 dollars on average. For every 1% increase in the percent of the population aged 18-34, the average per capita income decreases by 76.13 dollars on average. We might conjecture that people aged 18-34 are not at peak earning capacity yet, and so perhaps their lower incomes drag down the per-capita average. For every 1% increase in doctors in a county, the average per capita income increases by 9.30 dollars on average. The percent of the population that are high school graduates doesn't have much effect. It might depend on whether college graduates are counted as a subset of high school graduates rather than calculating them separately. For every 1% increase in the percent of the adult population (persons 25 years old or older) with a bachelor's degree, the average income per person increases by about 46.76 dollars on average. For every 1% increase in the population with income below the poverty level, the average per capita income decreases by 30.94 dollars on average. For every 1% increase in the percent of unemployed people, the average per capita income increases by about 12.64 dollars on average. In several of the interactions for the region, the West shows up as deviating significantly from the North Central part of the US.

Among all models, we notice a negative correlation between the percent of unemployment and per capita income. It could be the case that we missed some crucial variables related to both unemployment and income.

A fundamental limitation of this work is that we cannot refer our model to a more specific place such as counties or states since we do not have enough data for each county or state here. Moreover, given the VIF values of the model with the state (Appendix D, see page 63 below), it suggests that adding more state values could not provide us with any critical information as different levels are strongly correlated with each other. In this case, we believe that there is no need to worry about missing counties or states, but we need to take the geographic location (region) into account in predicting the per capita income.

## References

Kutner, M.H., Nachsheim, C.J., Neter, J. & Li, W. (2005) Applied Linear Statistical Models, Fifth Edition.  
NY: McGraw-Hill/Irwin.

## Technical Appendix

### Reading Data

```
cdi <- read.table("/Users/rosahhh/Desktop/Fall 2021/36617 Applied Linear Mode  
l/project 01/cdi.dat")  
  
attach(cdi)
```

### Appendix A

```
pandoc.table(head(cdi[,1:10]), caption="Partial Summary of CDI dataset", styl  
e="grid")  
  
##  
##  
## +-----+-----+-----+-----+-----+  
## | id | county | state | land.area | pop | pop.18_34 |  
## +=====+=====+=====+=====+=====+  
## | 1 | Los_Angeles | CA | 4060 | 8863164 | 32.1 |  
## +-----+-----+-----+-----+-----+  
## | 2 | Cook | IL | 946 | 5105067 | 29.2 |  
## +-----+-----+-----+-----+-----+  
## | 3 | Harris | TX | 1729 | 2818199 | 31.3 |  
## +-----+-----+-----+-----+-----+  
## | 4 | San_Diego | CA | 4205 | 2498016 | 33.5 |  
## +-----+-----+-----+-----+-----+  
## | 5 | Orange | CA | 790 | 2410556 | 32.6 |  
## +-----+-----+-----+-----+-----+  
## | 6 | Kings | NY | 71 | 2300664 | 28.3 |  
## +-----+-----+-----+-----+-----+  
##  
## Table: Partial Summary of CDI dataset (continued below)  
##  
##  
##  
## +-----+-----+-----+-----+  
## | pop.65_plus | doctors | hosp.beds | crimes |  
## +=====+=====+=====+=====+  
## | 9.7 | 23677 | 27700 | 688936 |  
## +-----+-----+-----+-----+  
## | 12.4 | 15153 | 21550 | 436936 |  
## +-----+-----+-----+-----+  
## | 7.1 | 7553 | 12449 | 253526 |
```

```

## +-----+-----+-----+-----+
## | 10.9 | 5905 | 6179 | 173821 |
## +-----+-----+-----+-----+
## | 9.2 | 6062 | 6369 | 144524 |
## +-----+-----+-----+-----+
## | 12.4 | 4861 | 8942 | 680966 |
## +-----+-----+-----+-----+

pandoc.table(head(cdi[,c(1,11:17)]), caption="Partial Summary of CDI dataset  
(Continued)",  
                  style="grid")

##  

##  

## +-----+-----+-----+-----+-----+  

## | id | pct.hs.grad | pct.bach.deg | pct.below.pov | pct.unemp |  

## +=====+=====+=====+=====+=====+  

## | 1 | 70 | 22.3 | 11.6 | 8 |  

## +-----+-----+-----+-----+-----+  

## | 2 | 73.4 | 22.8 | 11.1 | 7.2 |  

## +-----+-----+-----+-----+-----+  

## | 3 | 74.9 | 25.4 | 12.5 | 5.7 |  

## +-----+-----+-----+-----+-----+  

## | 4 | 81.9 | 25.3 | 8.1 | 6.1 |  

## +-----+-----+-----+-----+-----+  

## | 5 | 81.2 | 27.8 | 5.2 | 4.8 |  

## +-----+-----+-----+-----+-----+  

## | 6 | 63.7 | 16.6 | 19.5 | 9.5 |  

## +-----+-----+-----+-----+-----+  

##  

## Table: Partial Summary of CDI dataset (Continued) (continued below)
##  

##  

##  

## +-----+-----+-----+  

## | per.cap.income | tot.income | region |  

## +=====+=====+=====+  

## | 20786 | 184230 | W |  

## +-----+-----+-----+  

## | 21729 | 110928 | NC |  

## +-----+-----+-----+  

## | 19517 | 55003 | S |  

## +-----+-----+-----+  

## | 19588 | 48931 | W |  

## +-----+-----+-----+  

## | 24400 | 58818 | W |  

## +-----+-----+-----+  

## | 16803 | 38658 | NE |  

## +-----+-----+-----+

```

```

b <- apply(cdi, 2, function(x) {length(unique(x))})
pandoc.table(b, caption="The Count of Unique Values for Variables in CDI data
set", style="grid")

##
##
## +-----+-----+-----+-----+
## | id   | county | state | land.area | pop   | pop.18_34 | pop.65_plus |
## +=====+=====+=====+=====+=====+=====+=====+
## | 440  | 373   | 48    | 384    | 440   | 149    | 137    |
## +-----+-----+-----+-----+
##
## Table: The Count of Unique Values for Variables in CDI dataset (continued
below)
##
##
##
## +-----+-----+-----+-----+
## | doctors | hosp.beds | crimes | pct.hs.grad | pct.bach.deg |
## +=====+=====+=====+=====+=====+
## | 360   | 391   | 437   | 223   | 220   |
## +-----+-----+-----+
##
## Table: Table continues below
##
##
##
## +-----+-----+-----+-----+
## | pct.below.pov | pct.unemp | per.cap.income | tot.income | region |
## +=====+=====+=====+=====+=====+
## | 155   | 97    | 436   | 428   | 4    |
## +-----+-----+-----+
county.state <- with(cdi,paste(county,state))
tmp <- as.data.frame(matrix(sort(county.state),ncol=4))
names(tmp) <- paste("Counties",c("1-110","111-220","221-330","331-440"))

pandoc.table(tmp[1:30,], caption="Summary of County and State in CDI", style=
"grid")

##
##
## +-----+-----+-----+
## | Counties 1-110   | Counties 111-220 | Counties 221-330 |
## +=====+=====+=====+
## | Ada ID          | Ector TX       | Lycoming PA     |
## +-----+-----+-----+
## | Adams CO        | El_Dorado CA  | Macomb MI      |
## +-----+-----+-----+
## | Aiken SC        | El_Paso CO    | Macon IL       |

```

|  |
|--|
| ## +-----+-----+-----+                             |
| ##   Alachua FL   El_Paso TX   Madison AL          |
| ## +-----+-----+-----+                             |
| ##   Alamance NC   Elkhart IN   Madison IL         |
| ## +-----+-----+-----+                             |
| ##   Alameda CA   Erie NY   Madison IN             |
| ## +-----+-----+-----+                             |
| ##   Albany NY   Erie PA   Mahoning OH             |
| ## +-----+-----+-----+                             |
| ##   Alexandria_City VA   Escambia FL   Manatee FL |
| ## +-----+-----+-----+                             |
| ##   Allegheny PA   Essex MA   Marathon WI         |
| ## +-----+-----+-----+                             |
| ##   Allen IN   Essex NJ   Maricopa AZ             |
| ## +-----+-----+-----+                             |
| ##   Allen OH   Fairfax_County VA   Marin CA       |
| ## +-----+-----+-----+                             |
| ##   Anderson SC   Fairfield CT   Marion FL        |
| ## +-----+-----+-----+                             |
| ##   Androscoggin ME   Fairfield OH   Marion IN    |
| ## +-----+-----+-----+                             |
| ##   Anne_Arundel MD   Fayette KY   Marion OR      |
| ## +-----+-----+-----+                             |
| ##   Arapahoe CO   Fayette PA   Martin FL          |
| ## +-----+-----+-----+                             |
| ##   Arlington_County VA   Florence SC   Maui HI   |
| ## +-----+-----+-----+                             |
| ##   Atlantic NJ   Forsyth NC   McHenry IL         |
| ## +-----+-----+-----+                             |
| ##   Baltimore MD   Fort_Bend TX   McLean IL       |
| ## +-----+-----+-----+                             |
| ##   Baltimore_City MD   Franklin OH   McLennan TX |
| ## +-----+-----+-----+                             |
| ##   Barnstable MA   Franklin PA   Mecklenburg NC  |
| ## +-----+-----+-----+                             |
| ##   Bay FL   Frederick MD   Medina OH             |
| ## +-----+-----+-----+                             |
| ##   Bay MI   Fresno CA   Merced CA                |
| ## +-----+-----+-----+                             |
| ##   Beaver PA   Fulton GA   Mercer NJ             |
| ## +-----+-----+-----+                             |
| ##   Bell TX   Galveston TX   Mercer PA            |
| ## +-----+-----+-----+                             |
| ##   Benton WA   Gaston NC   Merrimack NH          |
| ## +-----+-----+-----+                             |
| ##   Bergen NJ   Genesee MI   Middlesex CT         |
| ## +-----+-----+-----+                             |
| ##   Berks PA   Gloucester NJ   Middlesex MA       |
| ## +-----+-----+-----+                             |
| ##   Berkshire MA   Greene MO   Middlesex NJ       |

```

## +-----+-----+-----+
## | Bernalillo NM | Greene OH | Midland TX |
## +-----+-----+-----+
## | Berrien MI | Greenville SC | Milwaukee WI |
## +-----+-----+-----+
##
## Table: Summary of County and State in CDI (continued below)
##
##
##
## +-----+
## | Counties 331-440 |
## +=====+
## | Rockingham NH |
## +-----+
## | Rockland NY |
## +-----+
## | Rowan NC |
## +-----+
## | Rutherford TN |
## +-----+
## | Sacramento CA |
## +-----+
## | Saginaw MI |
## +-----+
## | Salt_Lake UT |
## +-----+
## | San_Bernardino CA |
## +-----+
## | San_Diego CA |
## +-----+
## | San_Francisco CA |
## +-----+
## | San_Joaquin CA |
## +-----+
## | San_Luis_Obispo CA |
## +-----+
## | San_Mateo CA |
## +-----+
## | Sangamon IL |
## +-----+
## | Santa_Barbara CA |
## +-----+
## | Santa_Clara CA |
## +-----+
## | Santa_Cruz CA |
## +-----+
## | Sarasota FL |
## +-----+
## | Saratoga NY |

```

```

## +-----+
## |      Sarpy NE      |
## +-----+
## |      Schenectady NY |
## +-----+
## |      Schuylkill PA  |
## +-----+
## |      Sedgwick KS   |
## +-----+
## |      Seminole FL   |
## +-----+
## |      Shasta CA     |
## +-----+
## |      Shawnee KS    |
## +-----+
## |      Sheboygan WI  |
## +-----+
## |      Shelby TN     |
## +-----+
## |      Smith TX      |
## +-----+
## |      Snohomish WA  |
## +-----+



cdinumeric <- cdi[,-c(1:3,17)]
a <- apply(cdinumeric,2,function(x) c(summary(x),SD=sd(x))) %>%
  as.data.frame %>%
  t() %>%
  round(digits=2)

pandoc.table(a, caption="Summary Statistics for Continuous Variables in cdi
.dat dataset ", style="grid")

##



## +-----+-----+-----+-----+-----+-----+
## |      &nbs;      | Min.   | 1st Qu. | Median  | Mean    | 3rd Qu. |
## +=====+=====+=====+=====+=====+=====+
## |      **land.area** | 15     | 451.2   | 656.5   | 1041    | 946.8   |
## +-----+-----+-----+-----+-----+-----+
## |      **pop**       | 1e+05  | 139027  | 217280  | 393011  | 436064  |
## +-----+-----+-----+-----+-----+-----+
## |      **pop.18_34** | 16.4   | 26.2    | 28.1    | 28.57   | 30.02   |
## +-----+-----+-----+-----+-----+-----+
## |      **pop.65_plus** | 3       | 9.88    | 11.75   | 12.17   | 13.62   |
## +-----+-----+-----+-----+-----+-----+
## |      **doctors**   | 39     | 182.8   | 401     | 988    | 1036   |
## +-----+-----+-----+-----+-----+-----+
## |      **hosp.beds** | 92     | 390.8   | 755    | 1459   | 1576   |
## +-----+-----+-----+-----+-----+-----+

```

```

## |      **crimes** | 563 | 6220 | 11820 | 27112 | 26280 |
## +-----+-----+-----+-----+-----+-----+
## | **pct.hs.grad** | 46.6 | 73.88 | 77.7 | 77.56 | 82.4 |
## +-----+-----+-----+-----+-----+-----+
## | **pct.bach.deg** | 8.1 | 15.28 | 19.7 | 21.08 | 25.33 |
## +-----+-----+-----+-----+-----+-----+
## | **pct.below.pov** | 1.4 | 5.3 | 7.9 | 8.72 | 10.9 |
## +-----+-----+-----+-----+-----+-----+
## | **pct.unemp** | 2.2 | 5.1 | 6.2 | 6.6 | 7.5 |
## +-----+-----+-----+-----+-----+-----+
## | **per.cap.income** | 8899 | 16118 | 17759 | 18561 | 20270 |
## +-----+-----+-----+-----+-----+-----+
## | **tot.income** | 1141 | 2311 | 3857 | 7869 | 8654 |
## +-----+-----+-----+-----+-----+-----+
##
## Table: Summary Statistics for Continuous Variables in cdi.dat dataset (continued below)
##
##
##
## +-----+-----+-----+
## |        | Max. | SD |
## +=====+=====+=====+
## | **land.area** | 20062 | 1550 |
## +-----+-----+
## | **pop** | 8863164 | 601987 |
## +-----+-----+
## | **pop.18_34** | 49.7 | 4.19 |
## +-----+-----+
## | **pop.65_plus** | 33.8 | 3.99 |
## +-----+-----+
## | **doctors** | 23677 | 1790 |
## +-----+-----+
## | **hosp.beds** | 27700 | 2289 |
## +-----+-----+
## | **crimes** | 688936 | 58238 |
## +-----+-----+
## | **pct.hs.grad** | 92.9 | 7.02 |
## +-----+-----+
## | **pct.bach.deg** | 52.3 | 7.65 |
## +-----+-----+
## | **pct.below.pov** | 36.3 | 4.66 |
## +-----+-----+
## | **pct.unemp** | 21.3 | 2.34 |
## +-----+-----+
## | **per.cap.income** | 37541 | 4059 |
## +-----+-----+
## | **tot.income** | 184230 | 12884 |
## +-----+-----+

```

```

tmp <- rbind(with(cdi,table(region)))
row.names(tmp) <- "Freq"
pandoc.table(tmp,caption="Summary Statistics of Region in CDI", style="grid")

##
##
## +-----+-----+-----+-----+
## |   &nbsp; | NC | NE | S  | W  |
## +=====+=====+=====+=====+
## | **Freq** | 108 | 103 | 152 | 77 |
## +-----+-----+-----+-----+
##
## Table: Summary Statistics of Region in CDI

```

**1. Indicate where (in which variables) there is missing data (NA's), if any, how much there is (in each variable) and why it might be there.**

```

colSums(is.na(cdi))

##          id      county       state    land.area      pop
##          0          0          0          0          0          0
## pop.18_34 pop.65_plus doctors hosp.beds    crimes
##          0          0          0          0          0          0
## pct.hs.grad pct.bach.deg pct.below.pov pct.unemp per.cap.income
##          0          0          0          0          0          0
## tot.income      region
##          0          0

cdi$county <- as.factor(cdi$county)
cdi$state <- as.factor(cdi$state)
cdi$region <- as.factor(cdi$region)

```

**2. Make some appropriate descriptive EDA plots to illustrate any important features of the variables or possible important relationships among them.**

```
cdigood <- data.frame(cdinumeric,region=cdi$region)
```

**i. Univariate Distributions**

```

cdigood$land.area <- as.numeric(cdigood$land.area)
cdigood$pop <- as.numeric(cdigood$pop)
cdigood$doctors <- as.numeric(cdigood$doctors)
cdigood$hosp.beds <- as.numeric(cdigood$hosp.beds)
cdigood$crimes <- as.numeric(cdigood$crimes)
cdigood$per.cap.income <- as.numeric(cdigood$per.cap.income)
cdigood$tot.income <- as.numeric(cdigood$tot.income)

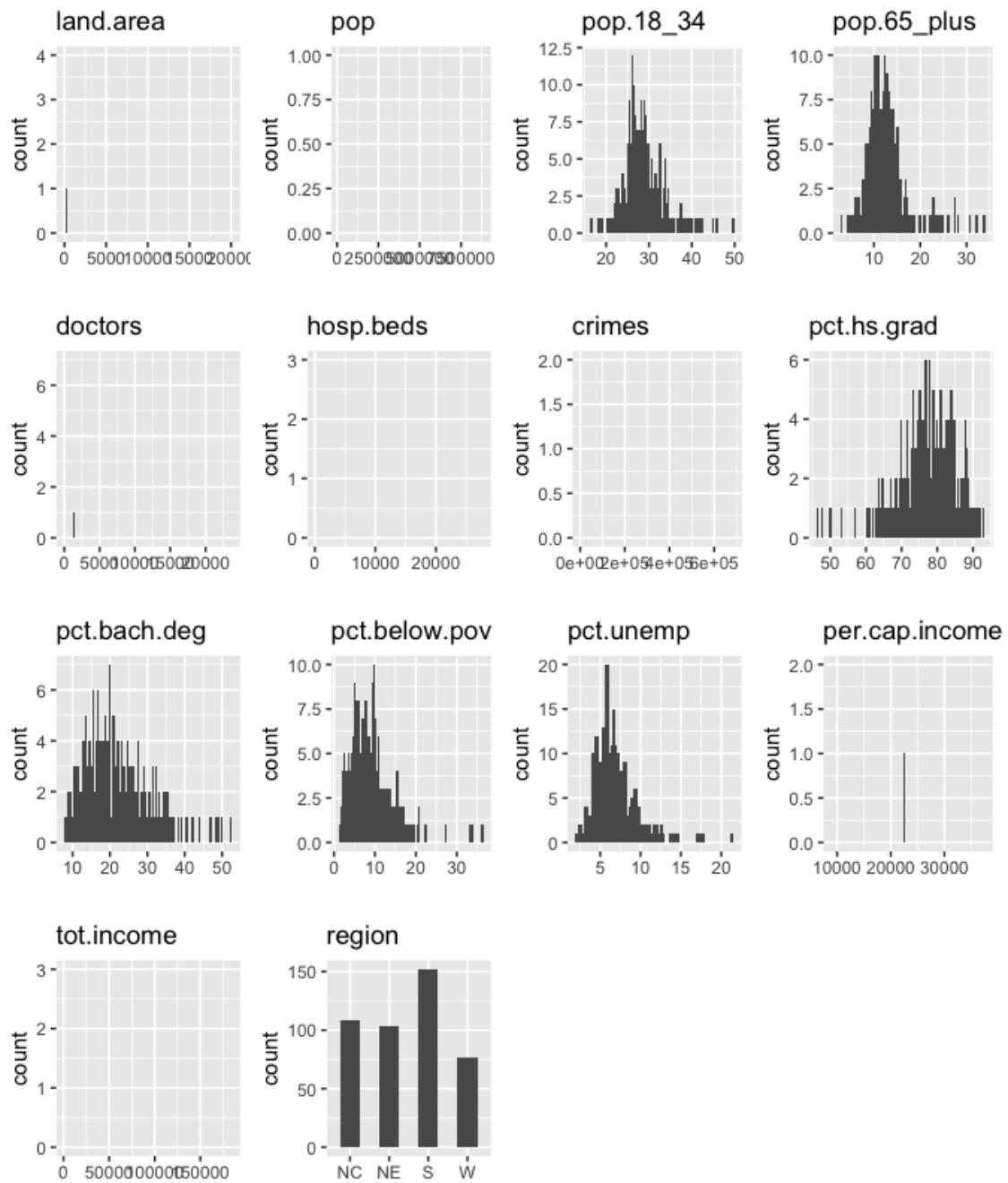
```

```

hist.builder <- function(df) { ## creates a list of graphs
  result <- NULL
  for (var in names(df)) {
    d <- data.frame(dd=df[,var])
    if(mode(df[,var])=="numeric") {
      print(var)
      p <- ggplot(d,aes(x=dd)) + stat_count(width = 0.5) +
        ggttitle(var) + xlab("")
    } else {
      p <- ggplot(d,aes(x=dd)) + stat_count(width = 0.5) +
        ggttitle(var) + xlab("")
    }
    result <- c(result,list(p))
  }
  return(result)
}

grid.arrange(grobs=hist.builder(cdigood))

```



### Distributions of Variables

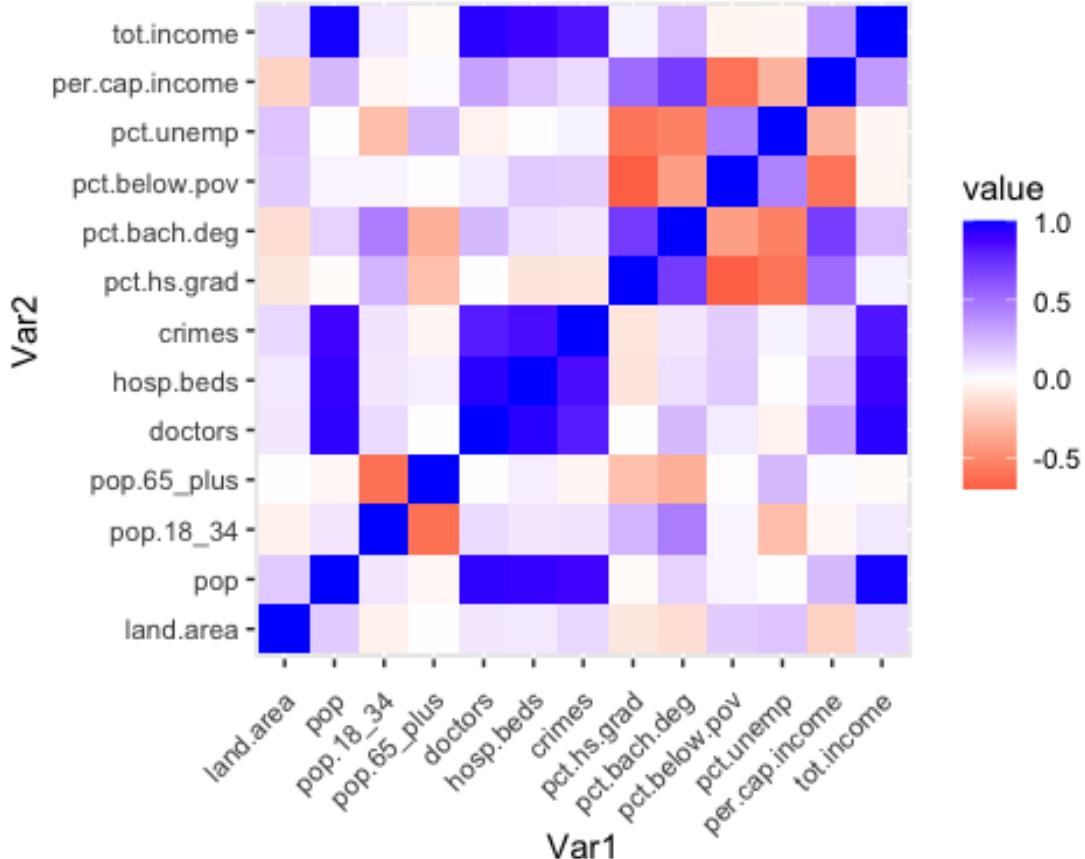
```
## [1] "land.area"
## [1] "pop"
## [1] "pop.18_34"
## [1] "pop.65_plus"
## [1] "doctors"
## [1] "hosp.beds"
## [1] "crimes"
## [1] "pct.hs.grad"
```

```

## [1] "pct.bach.deg"
## [1] "pct.below.pov"
## [1] "pct.unemp"
## [1] "per.cap.income"
## [1] "tot.income"
## [1] "region"

Bivariate Relationship
corgraph <-function(df) {
  cormat <- cor(df)
  melted_cormat <- melt(cormat) ## need library(reshape2) for this...
  ggplot(data = melted_cormat, aes(x=Var1, y=Var2, fill=value)) +
    geom_tile() +
    theme(axis.text.x = element_text(angle = 45,vjust=0.9,hjust=1)) +
    scale_fill_gradient2(low="red",mid="white",high="blue")
}
corgraph(cdinumeric)

```



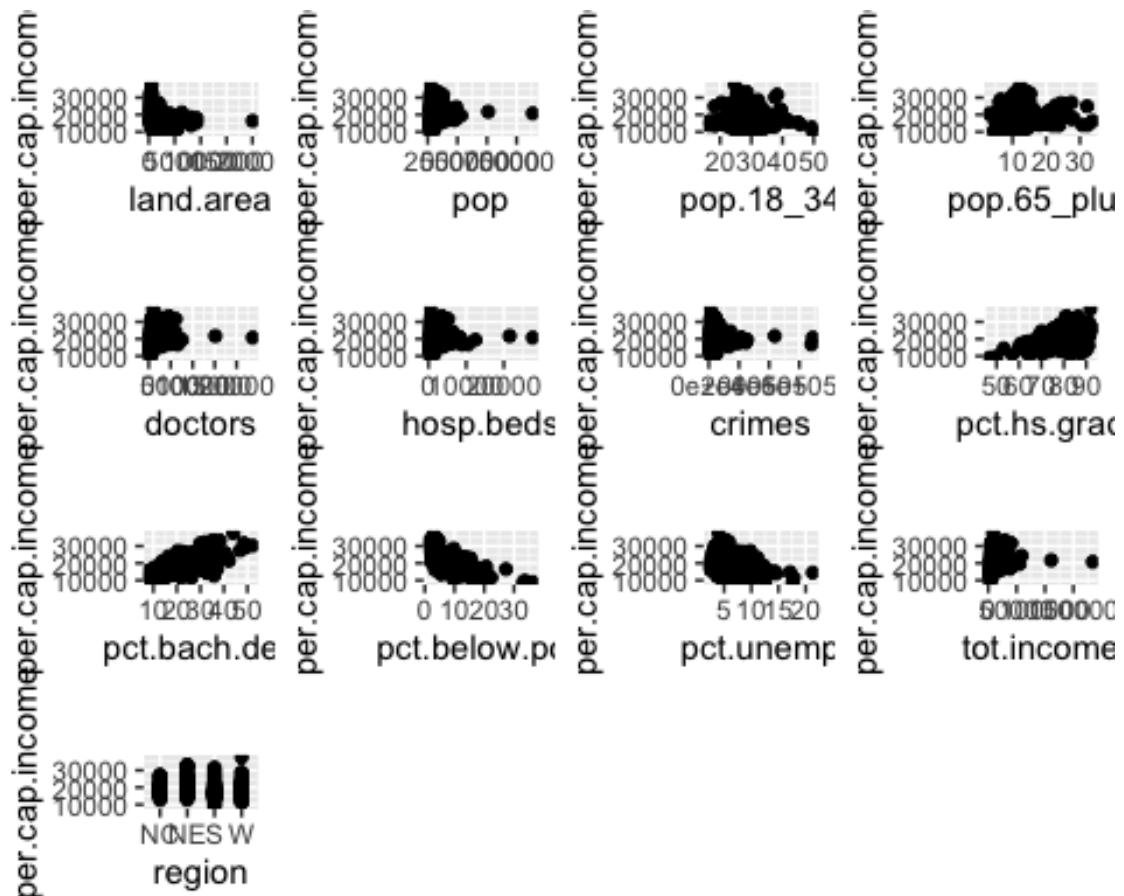
```

scatter.builder <-function(df,yvar="per.cap.income") {
  result <- NULL
  y.index <- grep(yvar,names(df))
  for(xvar in names(df)[-y.index]) {
    d <- data.frame(xx=df[,xvar],yy=df[,y.index])
  }
}
```

```

if(mode(df[,xvar])=="numeric") {
  p <- ggplot(d,aes(x=xx,y=yy)) + geom_point() +
    ggttitle("") + xlab(xvar) + ylab(yvar)
} else {
  p <- ggplot(d,aes(x=xx,y=yy)) + geom_boxplot(notch=F) +
    ggttitle("") + xlab(xvar) + ylab(yvar)
}
result <- c(result,list(p))
}
return(result)
}
grid.arrange(grobs=scatter.builder(cdigood))

```



*Distributions of Variables against Income per capita*

#### Appendix B

```

lm1 <- lm(per.cap.income ~ crimes + region)

summary(lm1)

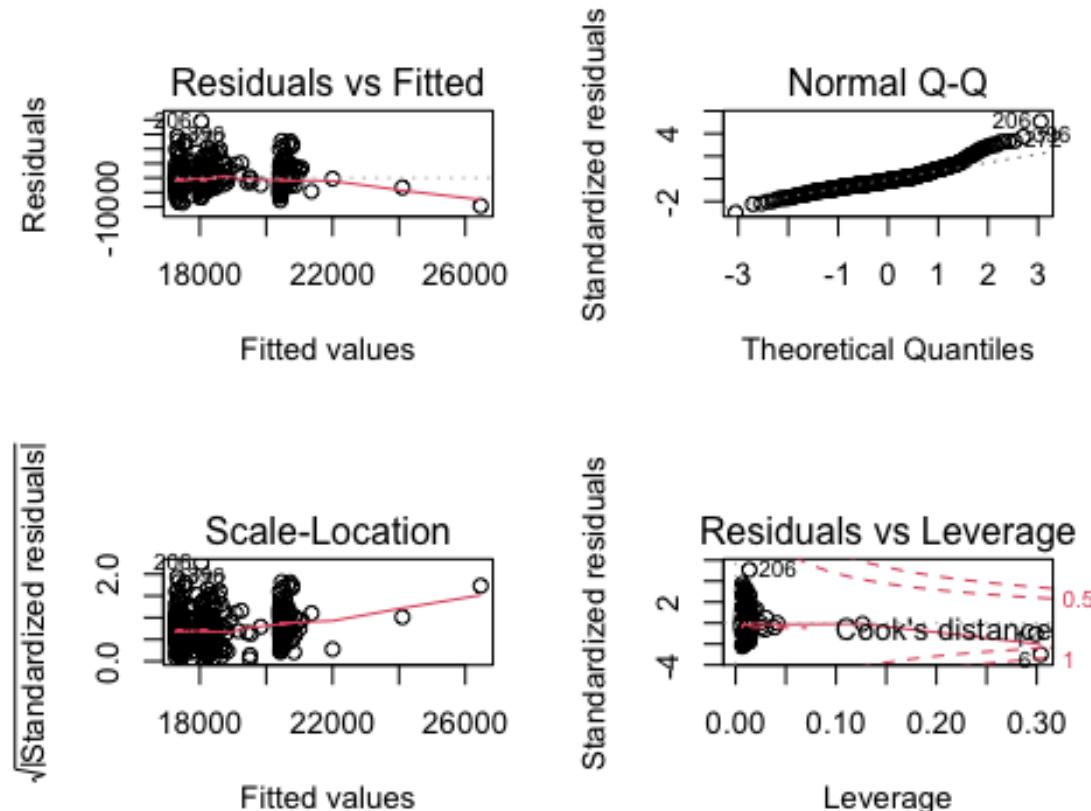
##
## Call:
## lm(formula = per.cap.income ~ crimes + region)
## 
```

```

## Residuals:
##      Min     1Q Median     3Q    Max
## -9661.0 -2260.7 -618.3 1650.0 19492.6
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 1.811e+04 3.784e+02 47.846 < 2e-16 ***
## crimes      8.915e-03 3.188e-03  2.797 0.00539 **  
## regionNE    2.286e+03 5.325e+02  4.293 2.17e-05 *** 
## regions     -8.606e+02 4.868e+02 -1.768 0.07782 .    
## regionW     -1.428e+02 5.796e+02 -0.246 0.80548  
## ---        
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3866 on 435 degrees of freedom
## Multiple R-squared:  0.1011, Adjusted R-squared:  0.09288 
## F-statistic: 12.24 on 4 and 435 DF,  p-value: 1.946e-09

par(mfrow=c(2,2))
plot(lm1)

```



```

cdi$per.cap.crime <- crimes/pop

names(cdi)

## [1] "id"           "county"        "state"         "land.area"
## [5] "pop"          "pop.18_34"      "pop.65_plus"   "doctors"
## [9] "hosp.beds"    "crimes"        "pct.hs.grad"   "pct.bach.deg"
## [13] "pct.below.pov" "pct.unemp"     "per.cap.income" "tot.income"
## [17] "region"       "per.cap.crime"

detach(cdi)
attach(cdi)

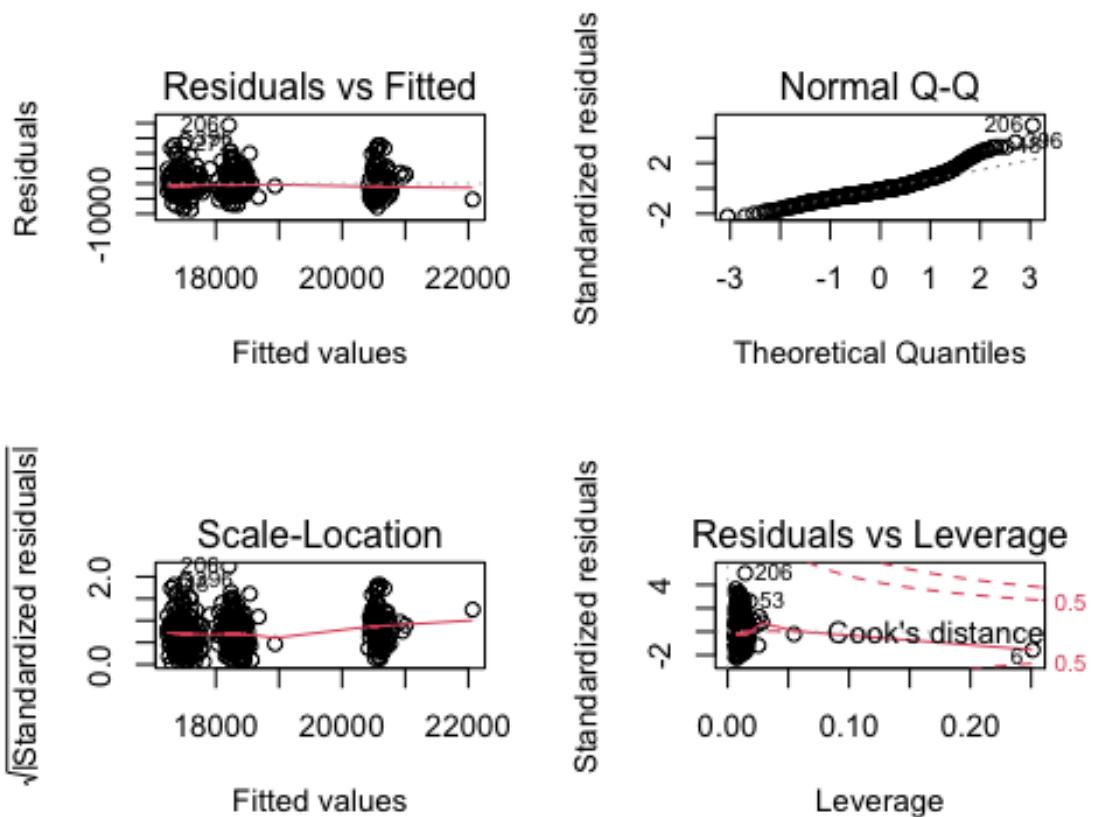
lm2 <- lm(per.cap.income ~ per.cap.crime + region, data=cdi)

summary(lm2)

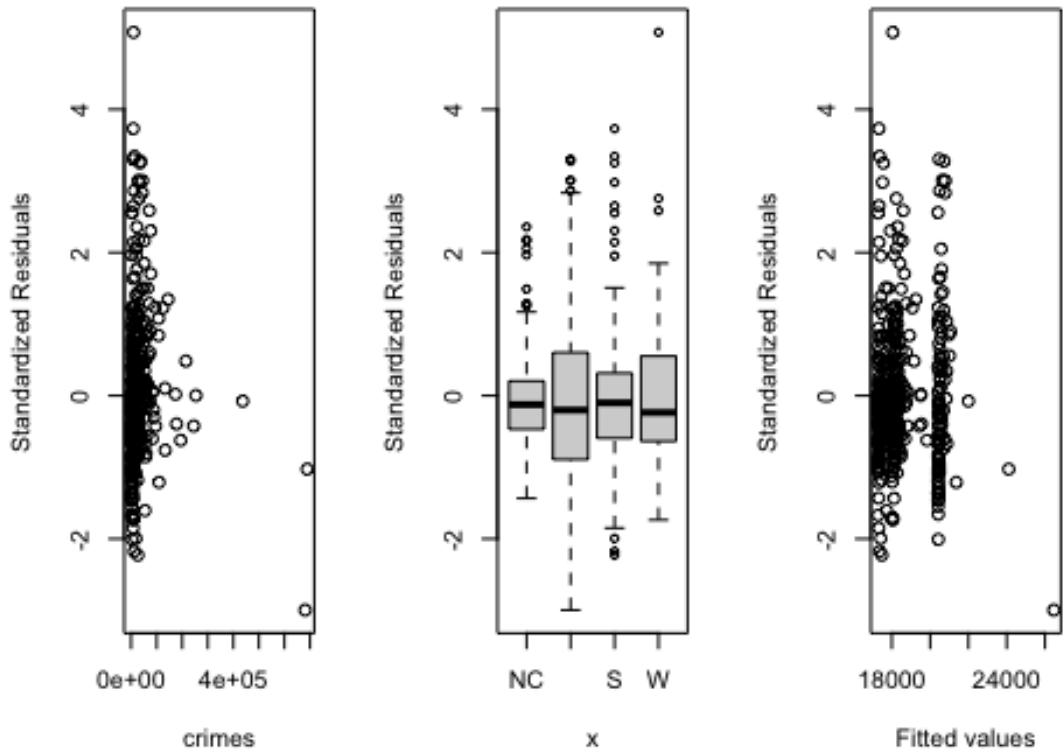
##
## Call:
## lm(formula = per.cap.income ~ per.cap.crime + region, data = cdi)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -8634 -2300   -631   1710  19332
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 18006.04    537.04  33.528 < 2e-16 ***
## per.cap.crime 5773.20    7520.41   0.768  0.4431    
## regionNE     2354.70    541.97   4.345 1.74e-05 ***
## regions      -927.45    512.31  -1.810  0.0709    
## regionW      -34.92     586.03  -0.060  0.9525    
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3898 on 435 degrees of freedom
## Multiple R-squared:  0.08622,    Adjusted R-squared:  0.07782 
## F-statistic: 10.26 on 4 and 435 DF,  p-value: 6.007e-08

par(mfrow=c(2,2))
plot(lm2)

```

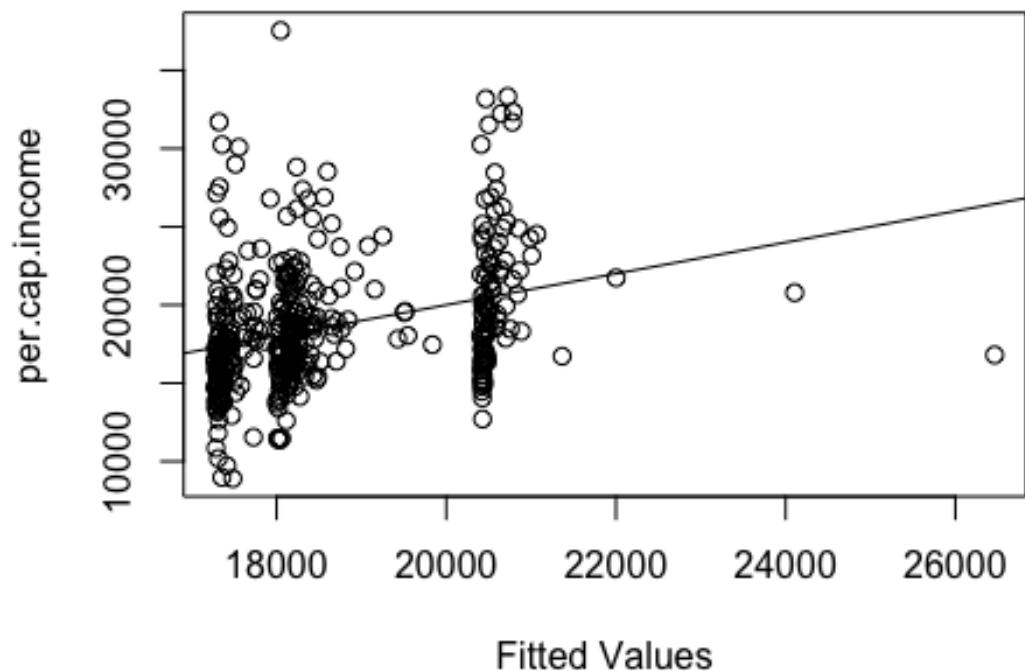


```
StanRes1 <- rstandard(lm1)
par(mfrow=c(1,3))
plot(crimes, StanRes1, ylab="Standardized Residuals")
plot(region, StanRes1, ylab="Standardized Residuals")
plot(lm1$fitted.values, StanRes1, ylab="Standardized Residuals", xlab="Fitted values")
```



Plots of the standardized residuals from model lm1

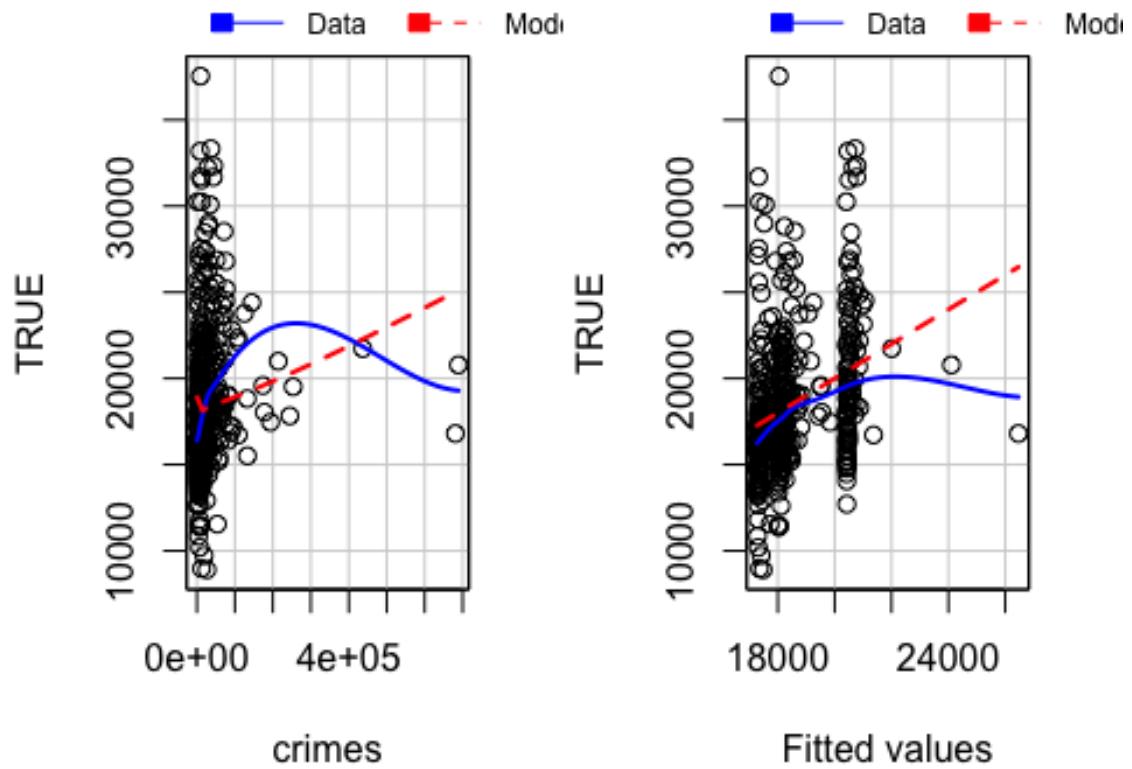
```
par(mfrow=c(1,1))
plot(lm1$fitted.values, per.cap.income, xlab="Fitted Values")
abline(lsfit(lm1$fitted.values, per.cap.income))
```



*A plot of per.cap.income against fitted values*

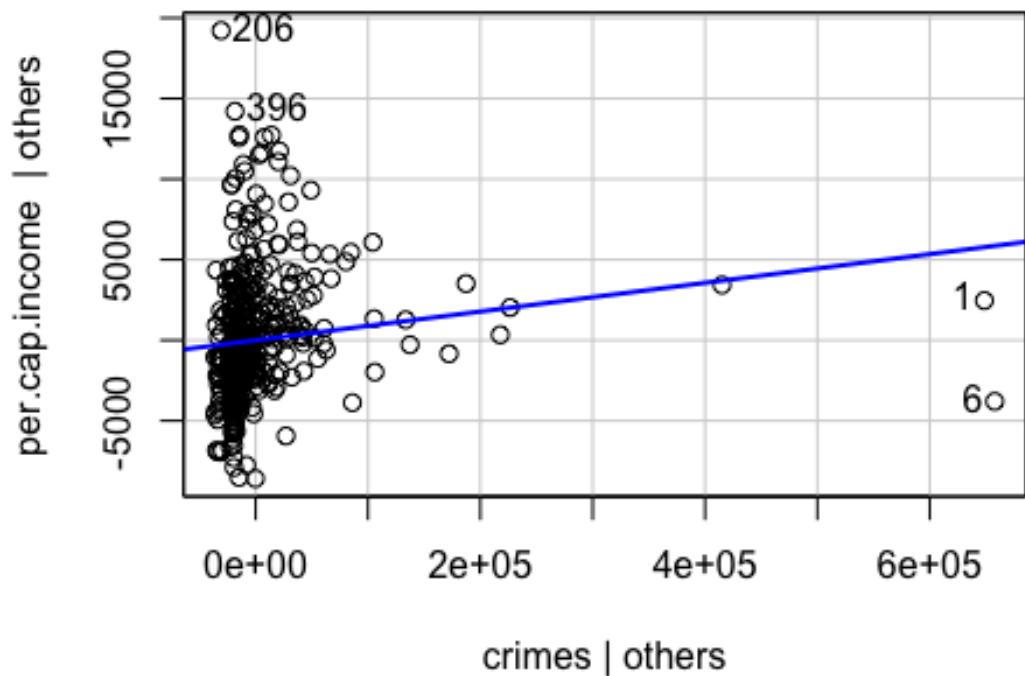
```
mmmps(lm1,layout=c(1,2))  
## Warning in mmmps(lm1, layout = c(1, 2)): Interactions and/or factors skippe  
d
```

## Marginal Model Plots



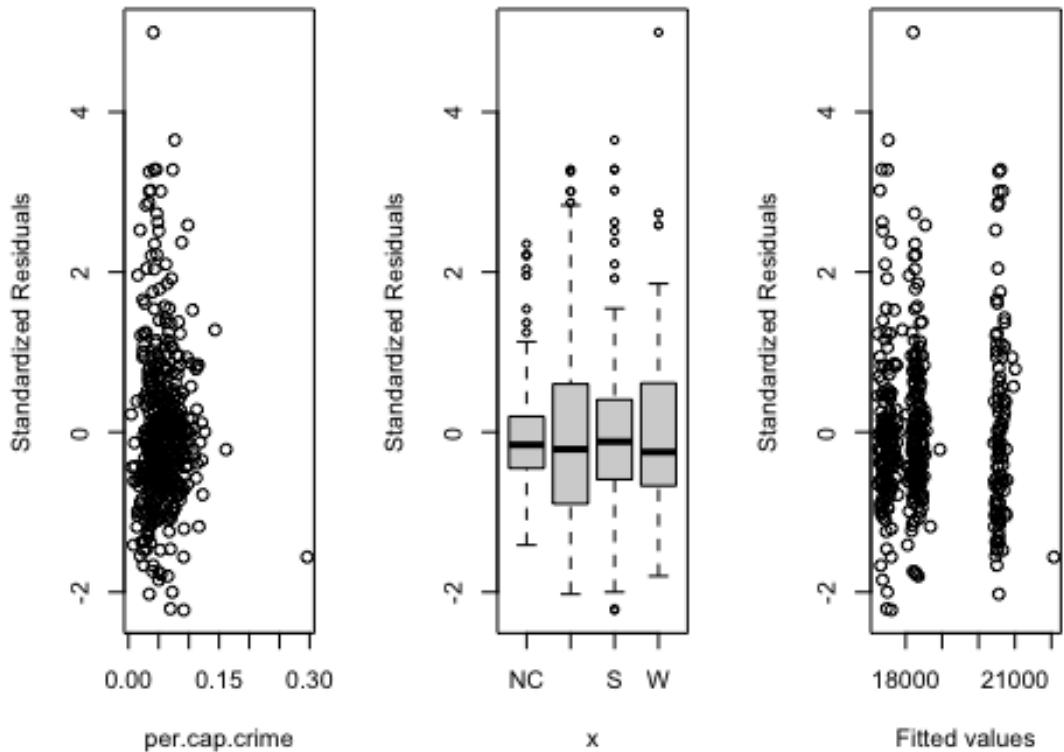
*Marginal model plots for model lm1*

```
library(car)
par(mfrow=c(1,1))
avPlot(lm1,variable=crimes,ask=FALSE, main="")
```



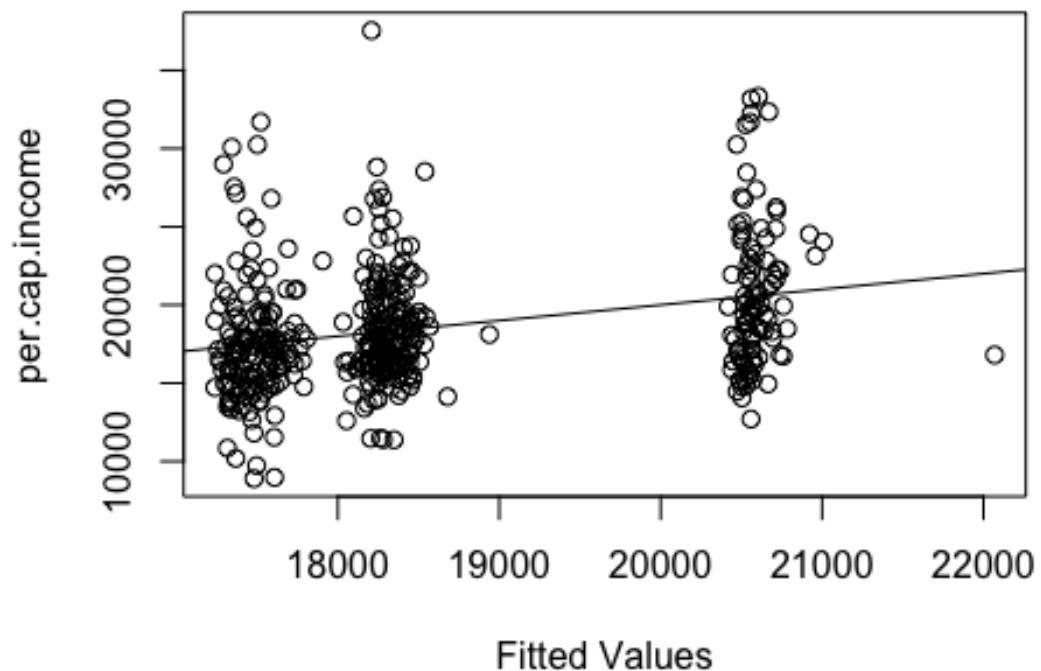
*Added-variable plots for model lm1*

```
StanRes2 <- rstandard(lm2)
par(mfrow=c(1,3))
plot(per.cap.crime, StanRes2, ylab="Standardized Residuals")
plot(region, StanRes2, ylab="Standardized Residuals")
plot(lm2$fitted.values, StanRes2, ylab="Standardized Residuals", xlab="Fitted values")
```



Plots of the standardized residuals from model lm2

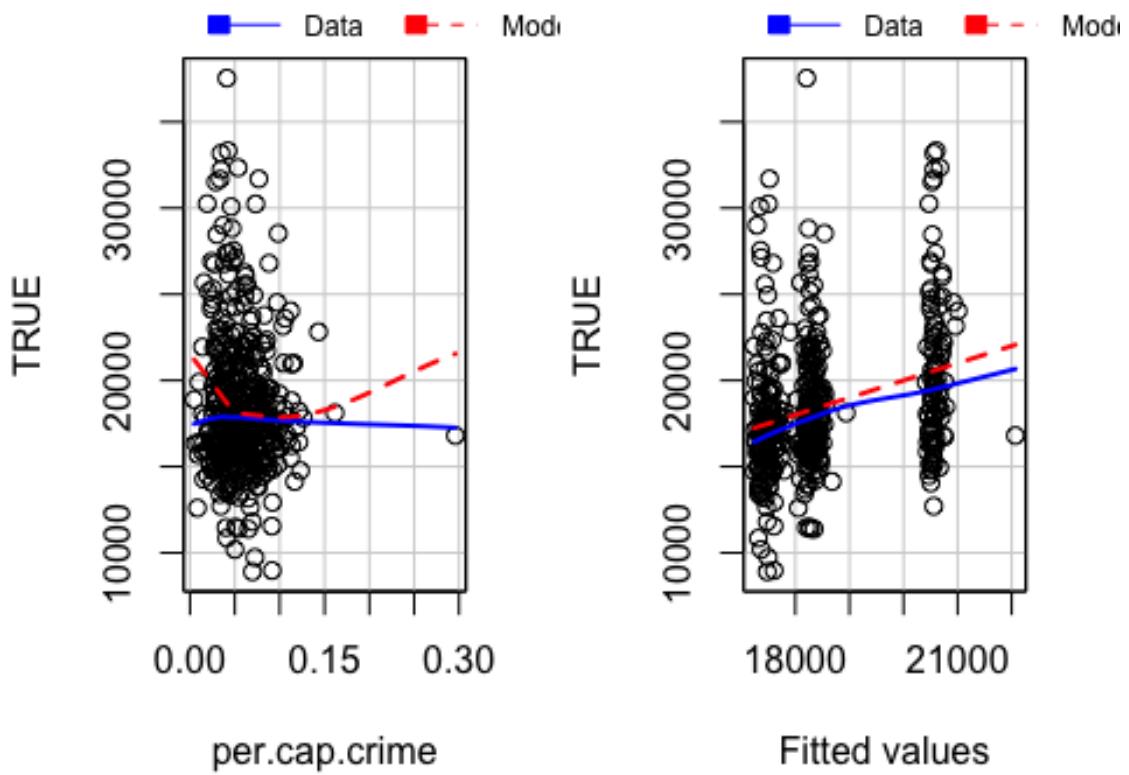
```
par(mfrow=c(1,1))
plot(lm2$fitted.values, per.cap.income, xlab="Fitted Values")
abline(lsfit(lm2$fitted.values, per.cap.income))
```



*A plot of per.cap.income against fitted values*

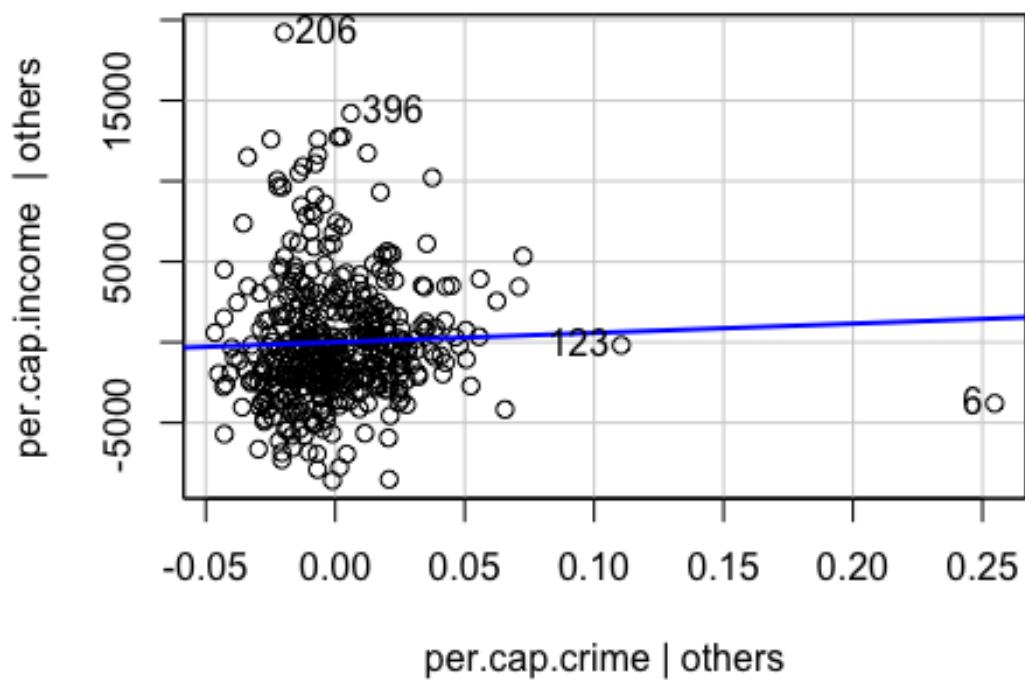
```
mmmps(lm2,layout=c(1,2))  
## Warning in mmmps(lm2, layout = c(1, 2)): Interactions and/or factors skippe  
d
```

## Marginal Model Plots



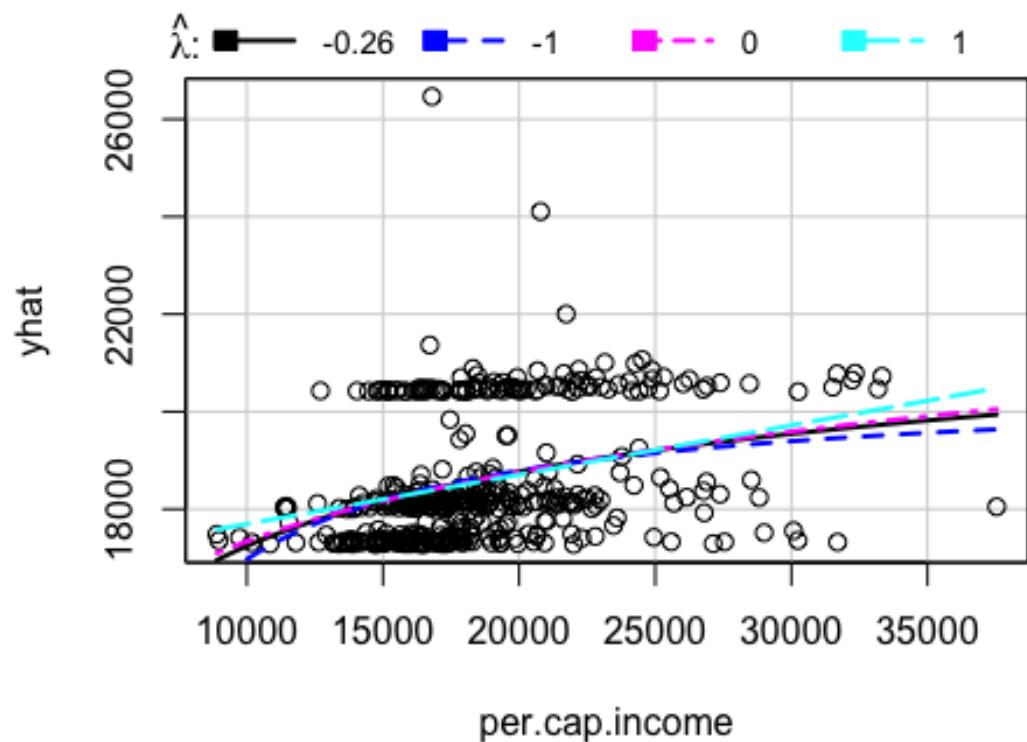
*Marginal model plots for model lm2*

```
library(car)
par(mfrow=c(1,1))
avPlot(lm2,variable=per.cap.crime,ask=FALSE, main="")
```



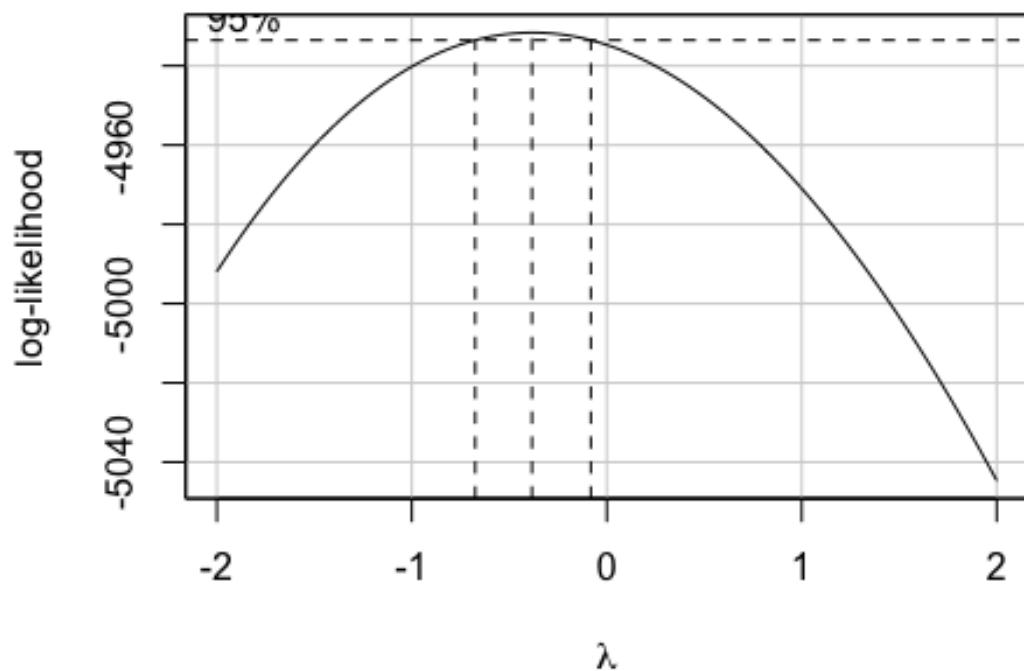
*Added-variable plots for model lm2*

```
inverseResponsePlot(lm1)
```

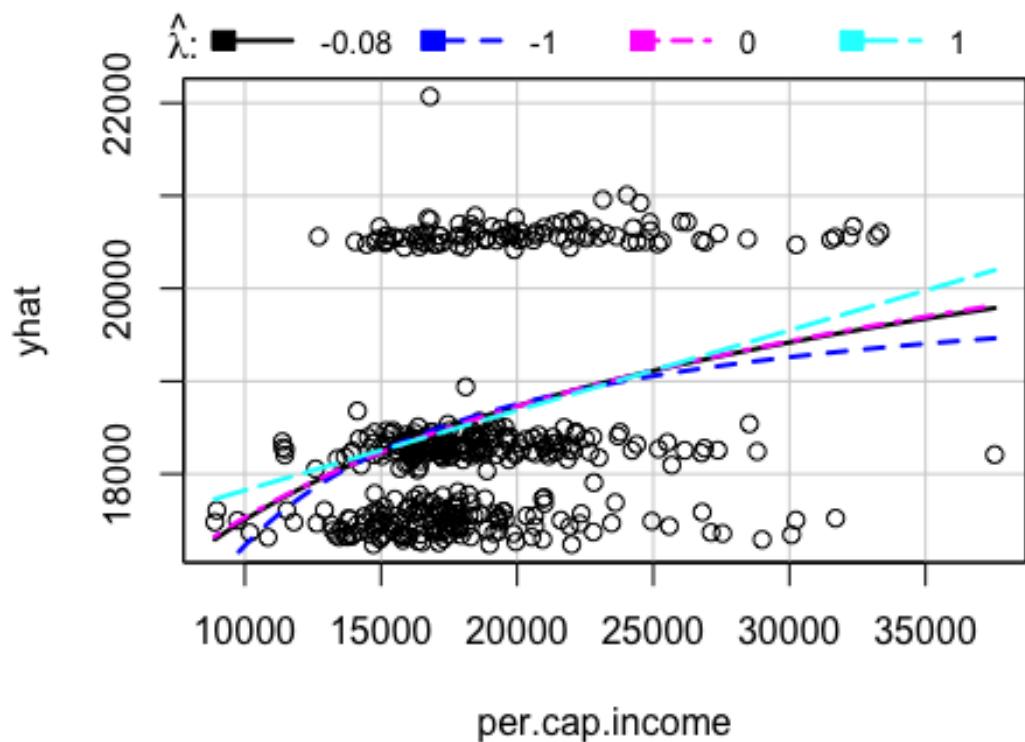


```
##      lambda      RSS
## 1 -0.2581433 652567923
## 2 -1.0000000 654566436
## 3  0.0000000 652800071
## 4  1.0000000 657627602
boxCox(lm1)
```

## Profile Log-likelihood



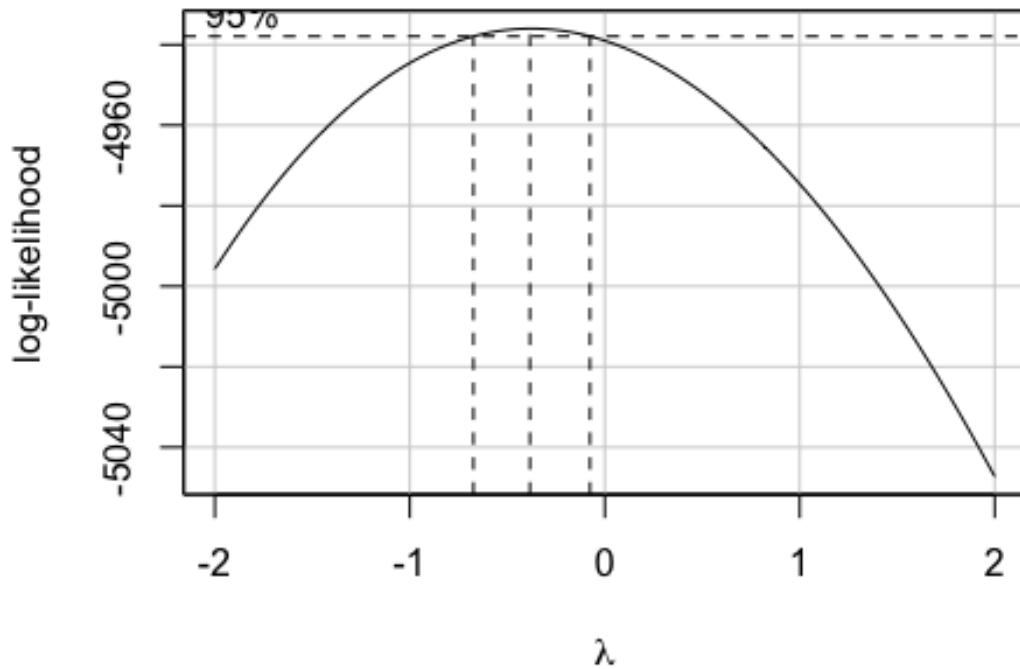
```
inverseResponsePlot(lm2)
```



```
##      lambda      RSS
## 1 -0.0774348 567496318
## 2 -1.0000000 569469857
## 3  0.0000000 567509697
## 4  1.0000000 569893885

boxCox(lm2)
```

## Profile Log-likelihood



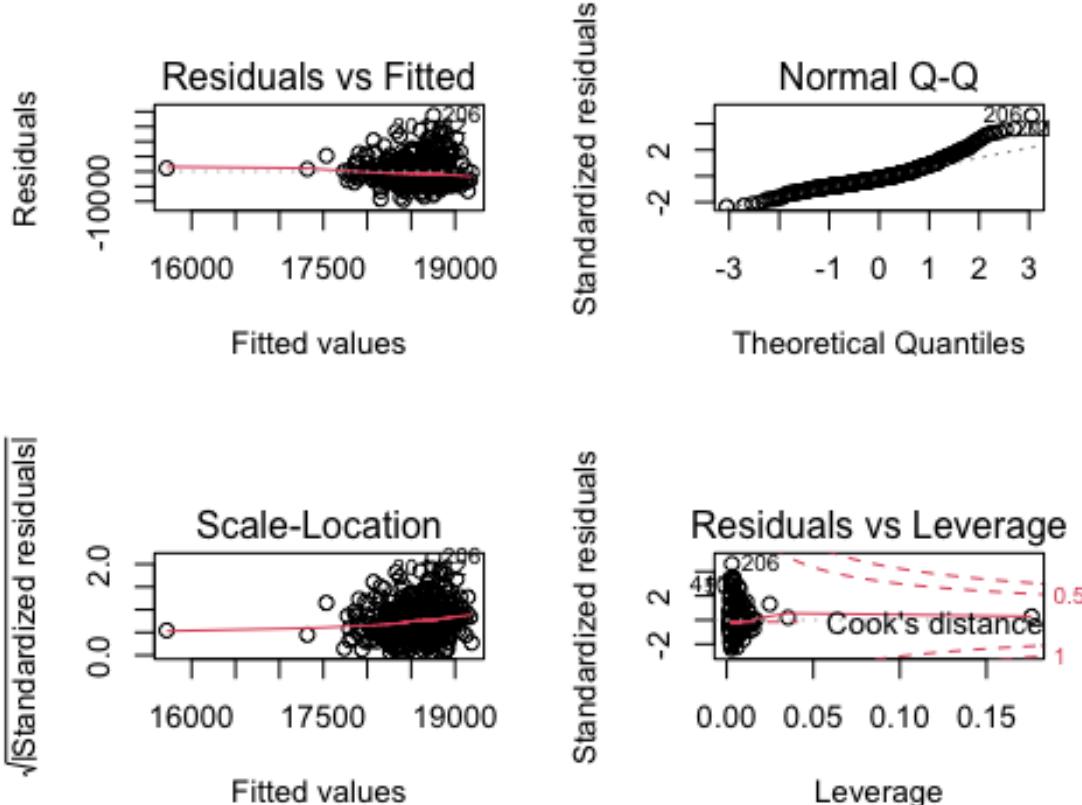
### Interaction

```
lm3 <- lm(per.cap.income ~ per.cap.crime, data=cdi)

summary(lm3)

##
## Call:
## lm(formula = per.cap.income ~ per.cap.crime, data = cdi)
##
## Residuals:
##      Min    1Q   Median    3Q   Max 
## -9515.2 -2568.9 -749.9 1574.9 18786.7 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 19244.3    448.9  42.867 <2e-16 ***
## per.cap.crime -11919.3    7074.5 -1.685  0.0927 .  
## --- 
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 
## 
## Residual standard error: 4051 on 438 degrees of freedom
## Multiple R-squared:  0.006439, Adjusted R-squared:  0.004171 
## F-statistic: 2.839 on 1 and 438 DF, p-value: 0.09274
```

```
par(mfrow=c(2,2))
plot(lm3)
```



```
lm4 <- lm(per.cap.income ~ per.cap.crime*region, data=cdi)

summary(lm4)

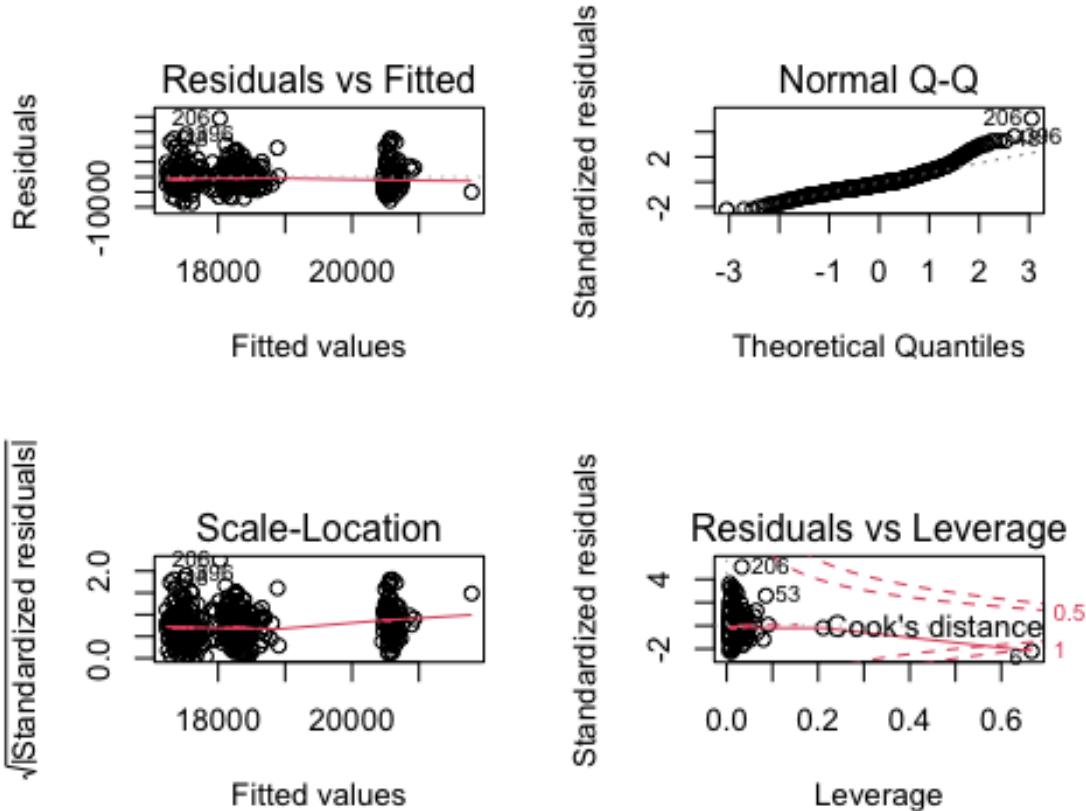
##
## Call:
## lm(formula = per.cap.income ~ per.cap.crime * region, data = cdi)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -8637.7 -2333.9 - 629.5 1759.1 19515.6 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 18077.3    895.2 20.193 <2e-16 ***
## per.cap.crime        4379.1   15893.5  0.276  0.783    
## regionNE          2329.0   1101.4  2.115  0.035 *  
## regions           -1010.4   1323.8 -0.763  0.446    
## regionW            -670.0   1983.9 -0.338  0.736    
## per.cap.crime:regionNE 288.4   20184.7  0.014  0.989
```

```

## per.cap.crime:regions      1558.9      20556.1    0.076    0.940
## per.cap.crime:regionW     10655.5      32322.4    0.330    0.742
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3911 on 432 degrees of freedom
## Multiple R-squared:  0.08648,   Adjusted R-squared:  0.07168
## F-statistic: 5.842 on 7 and 432 DF,  p-value: 1.713e-06

par(mfrow=c(2,2))
plot(lm4)

```



```

anova(lm3, lm4)

## Analysis of Variance Table
##
## Model 1: per.cap.income ~ per.cap.crime
## Model 2: per.cap.income ~ per.cap.crime * region
##   Res.Df       RSS Df Sum of Sq    F    Pr(>F)
## 1    438  7186843542
## 2    432  6607856753  6  578986790 6.3087 2.252e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

```

anova(lm2, lm4)

## Analysis of Variance Table
##
## Model 1: per.cap.income ~ per.cap.crime + region
## Model 2: per.cap.income ~ per.cap.crime * region
##   Res.Df      RSS Df Sum of Sq    F Pr(>F)
## 1     435 6609753963
## 2     432 6607856753  3   1897210 0.0413 0.9888

data.frame(AIC=AIC(lm2, lm3, lm4),
           BIC=BIC(lm2, lm3, lm4))[, -3] %>%
  kbl(booktabs=T, col.names=c("df", "AIC", "BIC")) %>% kable_classic(full_width=F)

df
AIC
BIC
lm2
6
8531.682
8556.203
lm3
3
8562.513
8574.773
lm4
9
8537.556
8574.337

formula(lm2)

## per.cap.income ~ per.cap.crime + region
round(coef(summary(lm2)), 2)

##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 18006.04    537.04   33.53   0.00
## per.cap.crime 5773.20    7520.41    0.77   0.44
## regionNE     2354.70    541.97    4.34   0.00

```

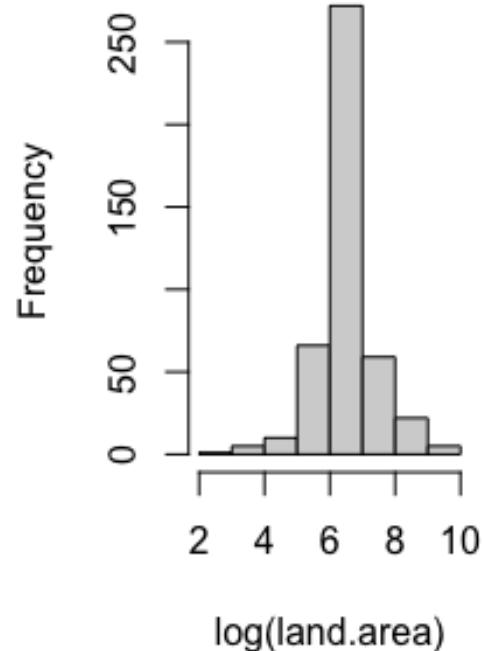
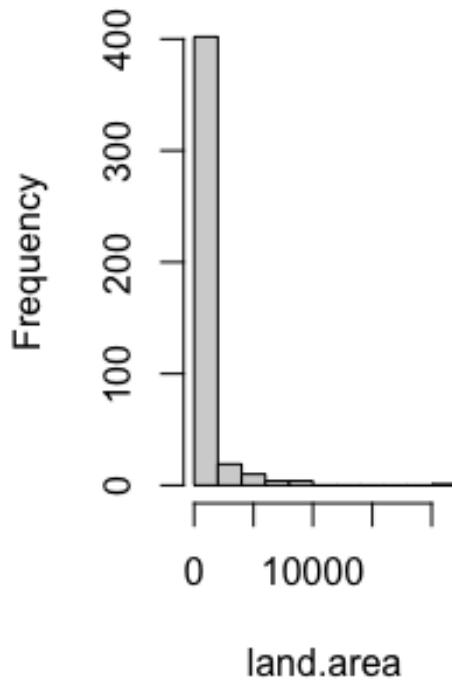
```
## regions          -927.45    512.31   -1.81    0.07  
## regionW         -34.92    586.03   -0.06    0.95
```

## Appendix C

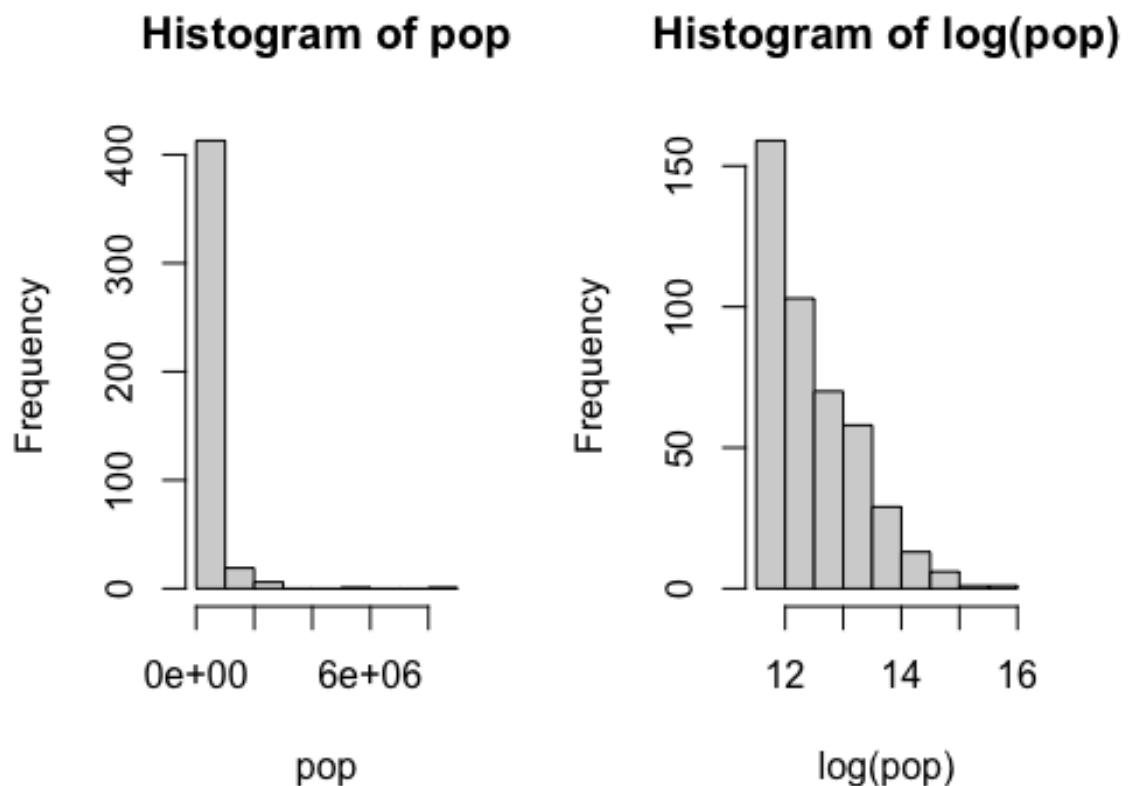
### Transformation

```
par(mfrow=c(1,2))  
hist(land.area)  
hist(log(land.area))
```

Histogram of land.area   Histogram of log(land.area)

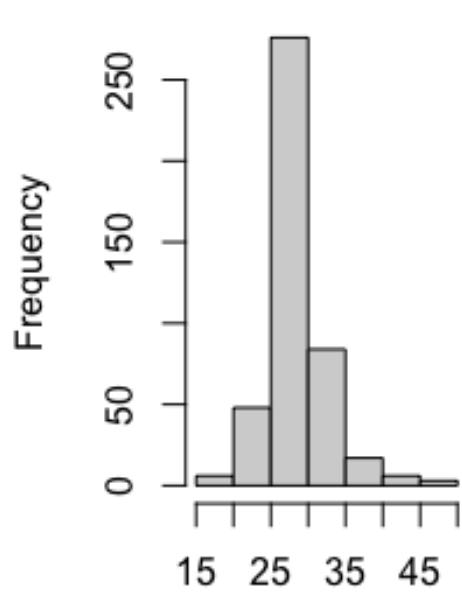


```
par(mfrow=c(1,2))  
hist(pop)  
hist(log(pop))
```

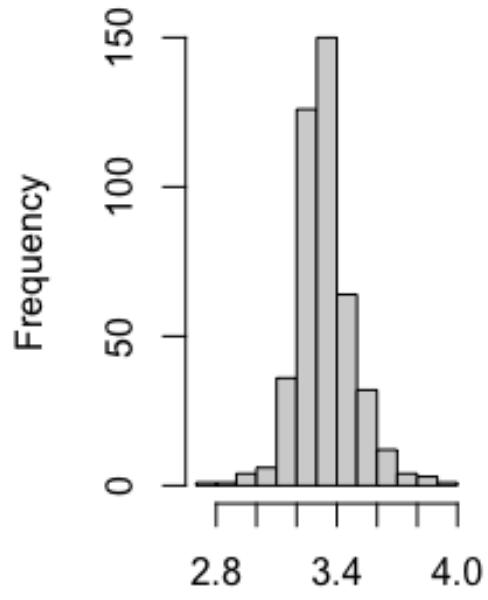


```
par(mfrow=c(1,2))
hist(pop.18_34)
hist(log(pop.18_34))
```

## Histogram of pop.18\_34 Histogram of log(pop.18\_



pop.18\_34

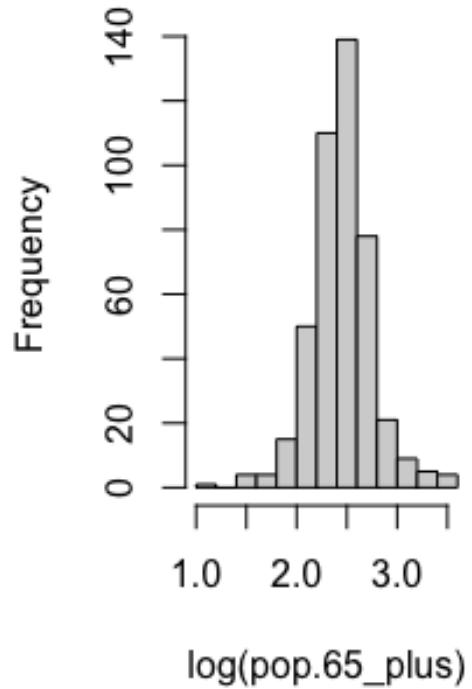
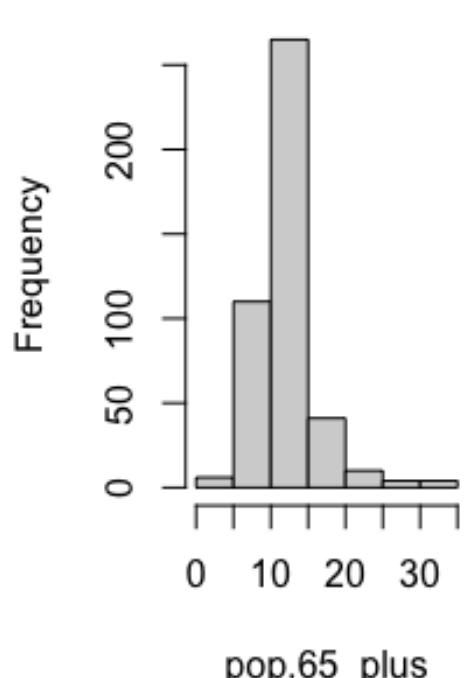


log(pop.18\_34)

```
par(mfrow=c(1,2))
hist(pop.65_plus)
hist(log(pop.65_plus))
```

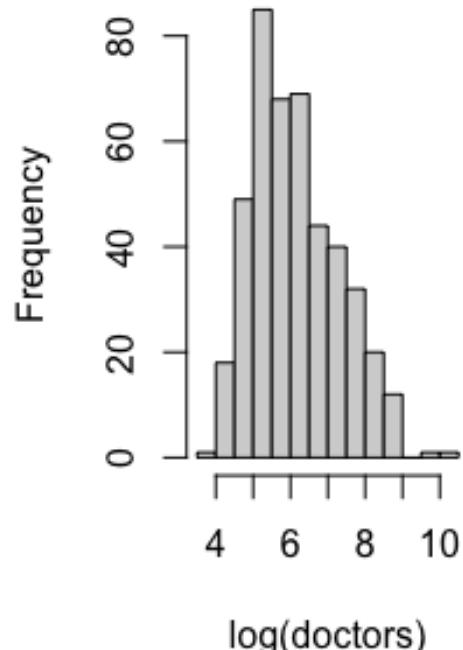
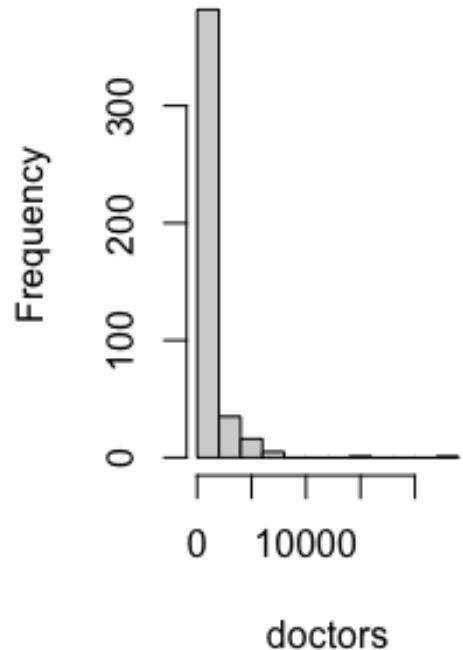
## Histogram of pop.65\_plus

## Histogram of log(pop.65\_plus)



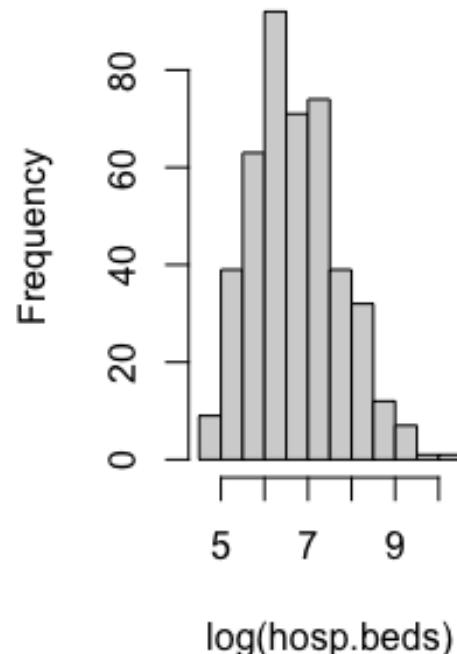
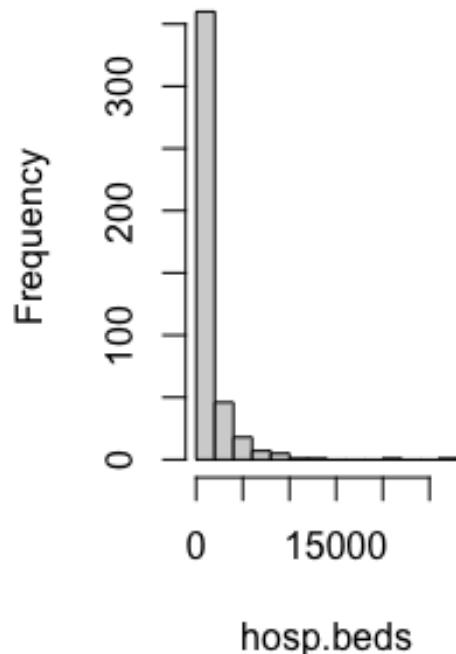
```
par(mfrow=c(1,2))
hist(doctors)
hist(log(doctors))
```

**Histogram of doctors      Histogram of log(doctor)**



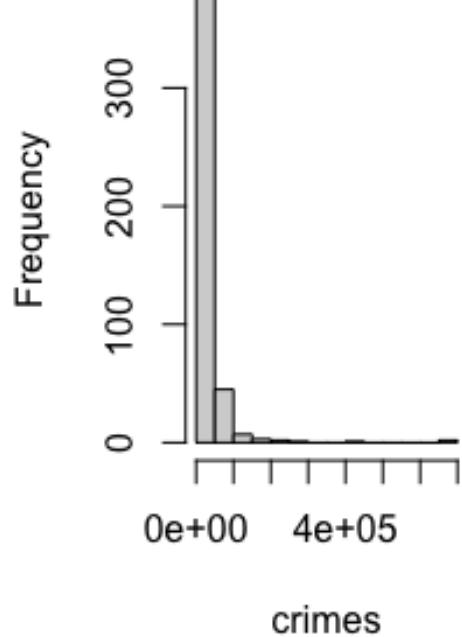
```
par(mfrow=c(1,2))
hist(hosp.beds)
hist(log(hosp.beds))
```

## Histogram of hosp.bed: Histogram of log(hosp.be

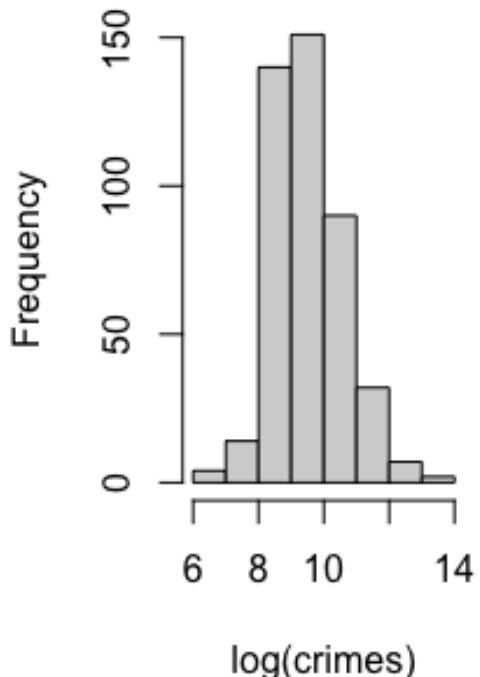


```
par(mfrow=c(1,2))
hist(crimes)
hist(log(crimes))
```

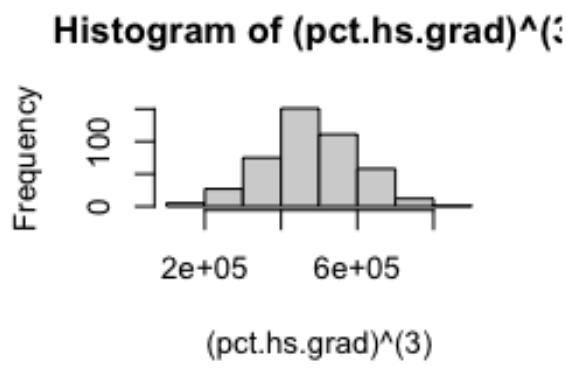
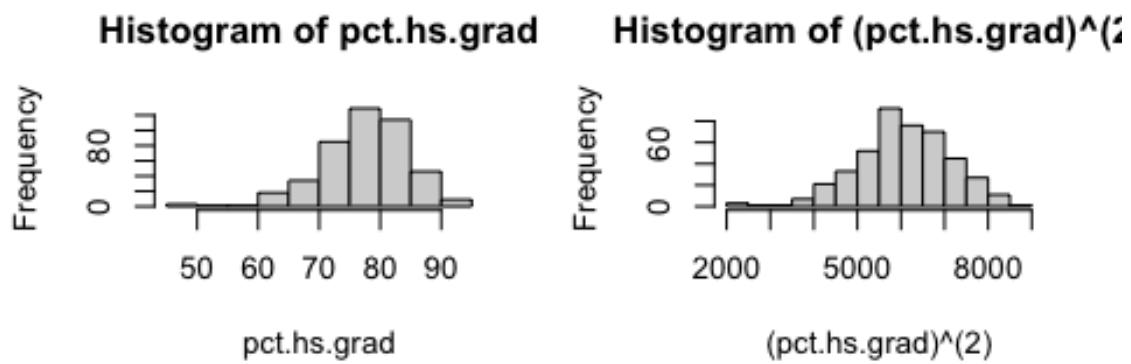
### Histogram of crimes



### Histogram of log(crimes)

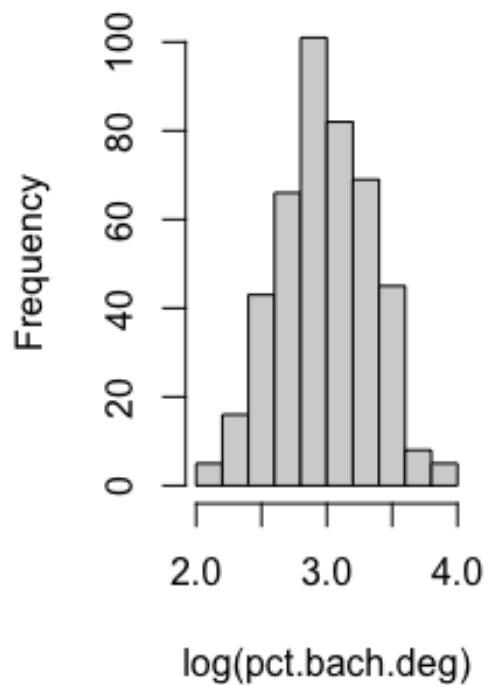
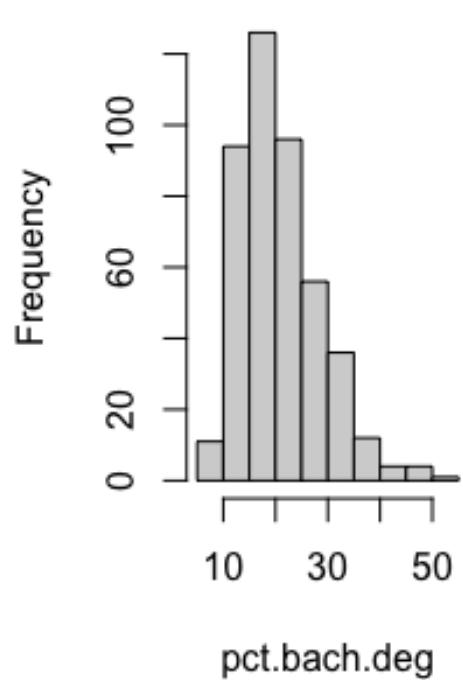


```
par(mfrow=c(2,2))
hist(pct.hs.grad)
hist((pct.hs.grad)^(2))
hist((pct.hs.grad)^(3))
```



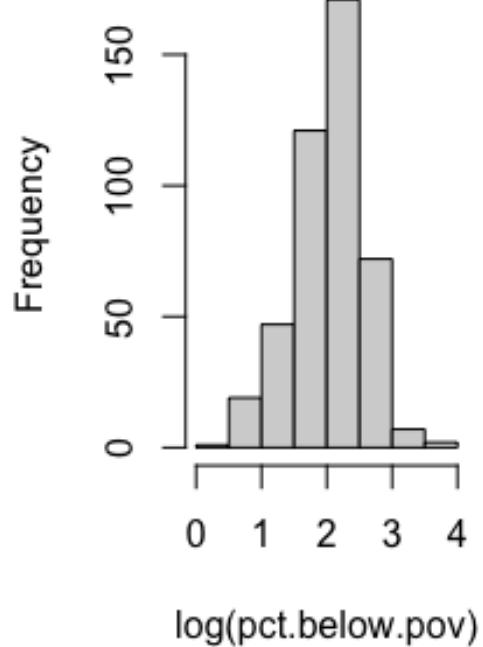
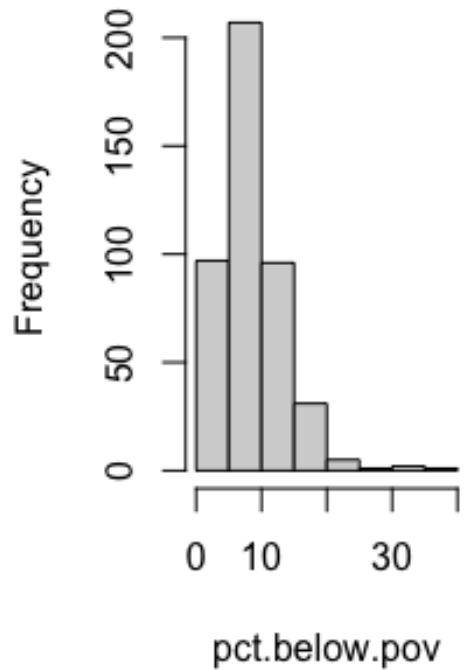
```
par(mfrow=c(1,2))
hist(pct.bach.deg)
hist(log(pct.bach.deg))
```

## Histogram of pct.bach.dHistogram of log(pct.bach.



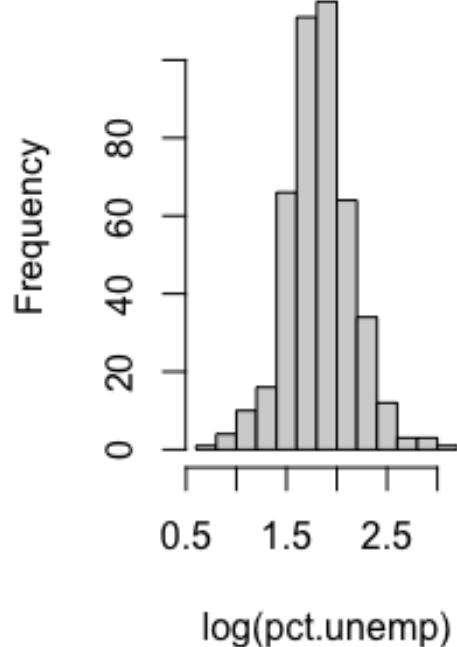
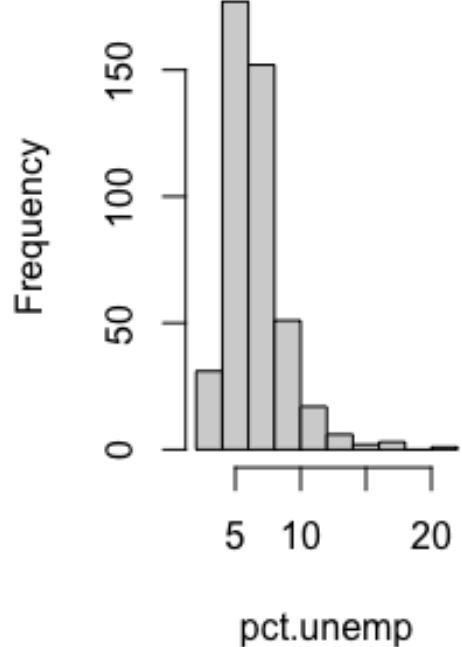
```
par(mfrow=c(1,2))
hist(pct.below.pov)
hist(log(pct.below.pov))
```

## Histogram of pct.below.pov histogram of log(pct.below.pov)



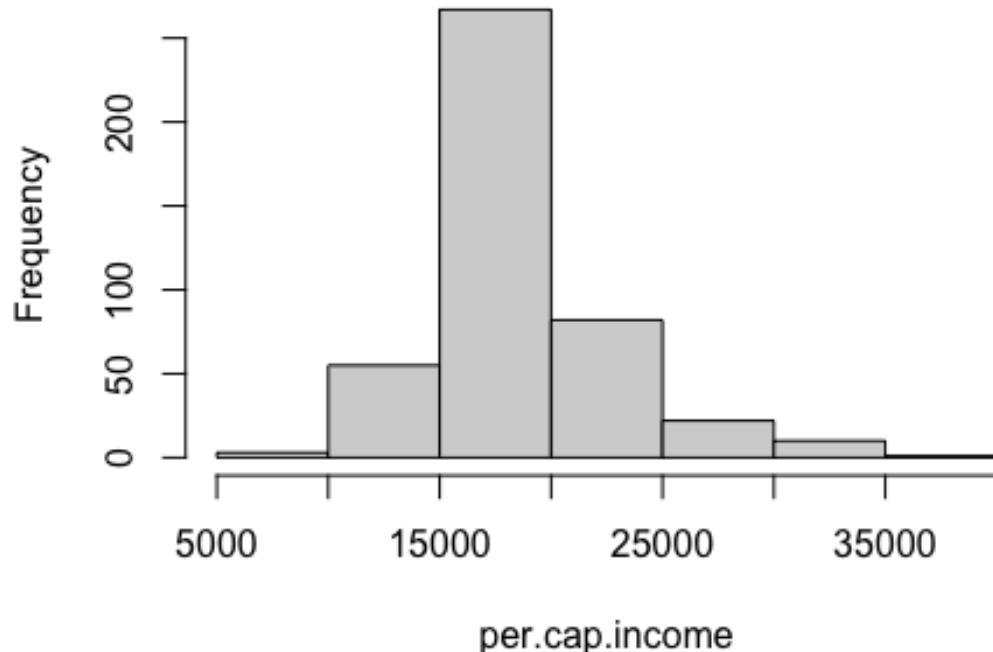
```
par(mfrow=c(1,2))
hist(pct.unemp)
hist(log(pct.unemp))
```

## Histogram of pct.unemp | Histogram of log(pct.unemp)



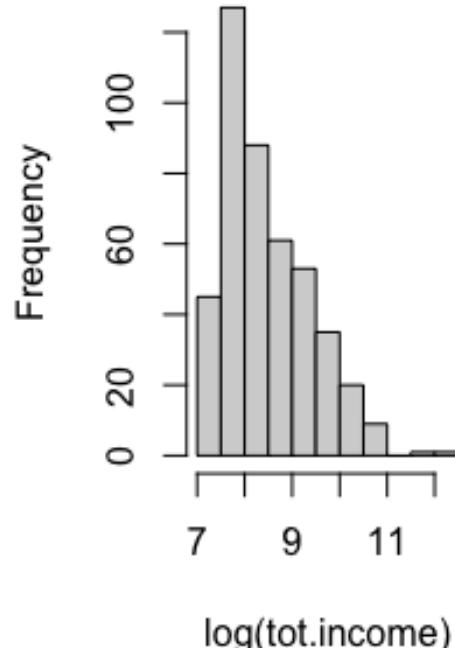
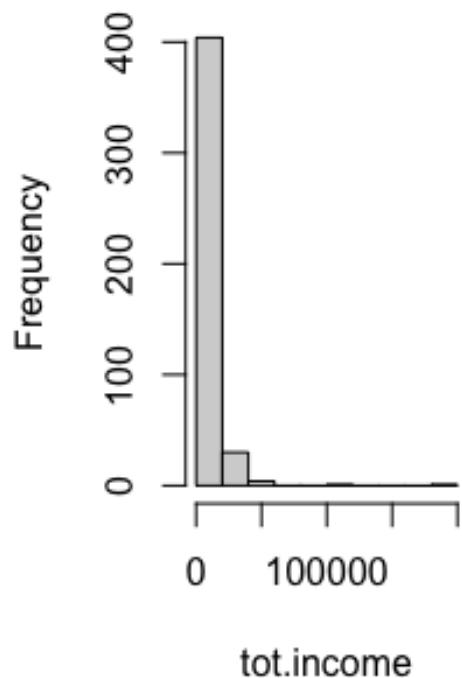
```
hist(per.cap.income)
```

## Histogram of per.cap.income



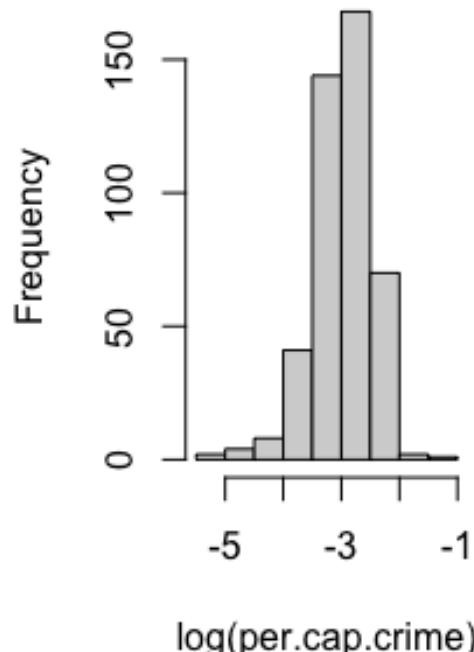
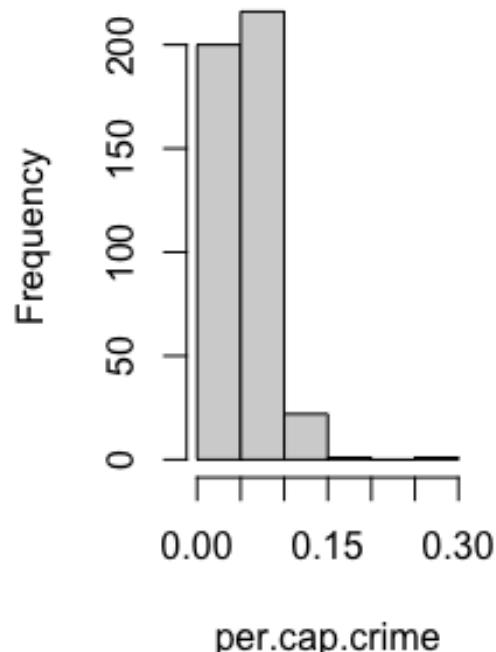
```
par(mfrow=c(1,2))
hist(tot.income)
hist(log(tot.income))
```

## Histogram of tot.incom Histogram of log(tot.inco)



```
par(mfrow=c(1,2))
hist(per.cap.crime)
hist(log(per.cap.crime))
```

## Histogram of per.cap.criHistogram of log(per.cap.cri



### Variable Selection

```
fullmodel <- lm(per.cap.income ~ log(land.area) + log(per.cap.crime)
+ log(pct.unemp) + log(pct.below.pov)
+ log(pct.bach.deg) + (pct.hs.grad)^3
+ log(hosp.beds) + log(doctors) + log(pop.65_plus)
+ log(pop.18_34), data = cdi)

summary(fullmodel)

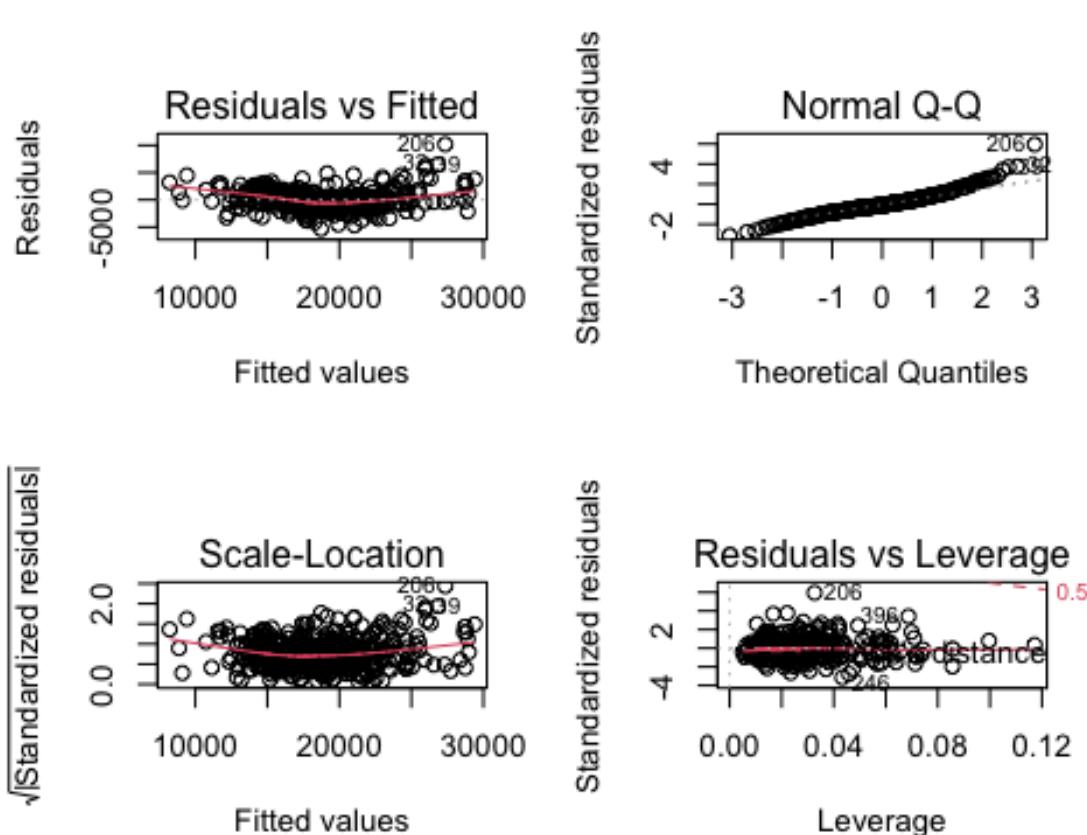
##
## Call:
## lm(formula = per.cap.income ~ log(land.area) + log(per.cap.crime) +
##     log(pct.unemp) + log(pct.below.pov) + log(pct.bach.deg) +
##     (pct.hs.grad)^3 + log(hosp.beds) + log(doctors) + log(pop.65_plus) +
##     log(pop.18_34), data = cdi)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -5338.8  -996.2  -221.6   822.1 10182.2 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 28683.34    4024.88   7.127 4.39e-12 ***
##
```

```

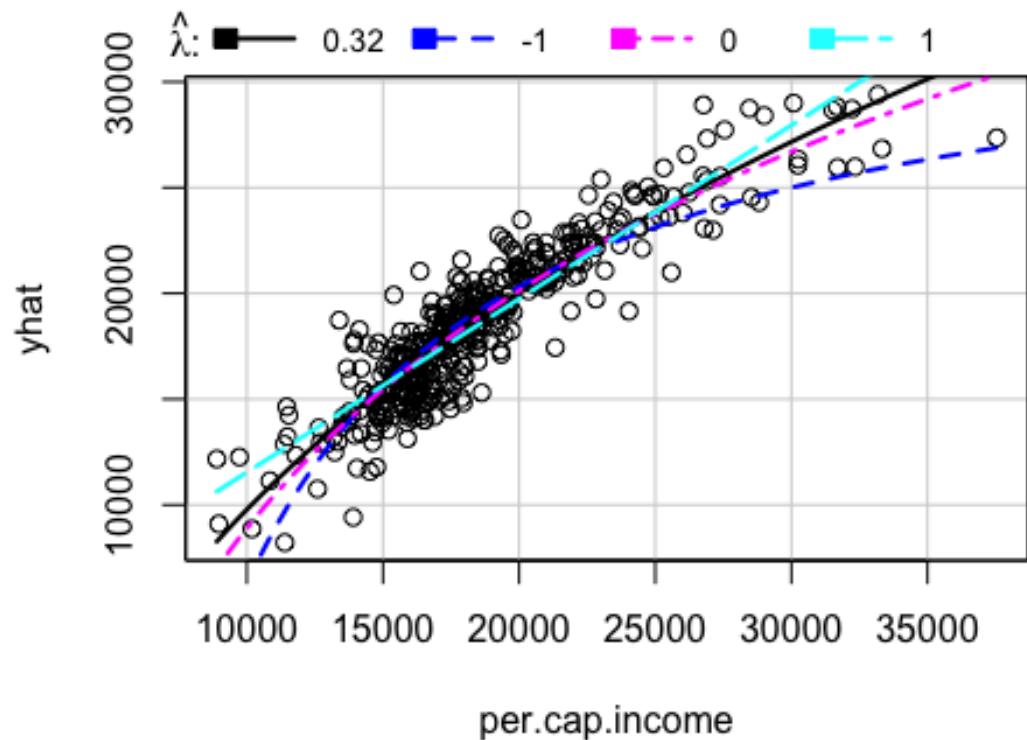
## log(land.area)      -653.02    101.85   -6.412 3.80e-10 ***
## log(per.cap.crime) 282.98     217.27    1.302 0.193456
## log(pct.unemp)     1688.07    340.03    4.964 9.96e-07 ***
## log(pct.below.pov) -4308.60   263.22   -16.369 < 2e-16 ***
## log(pct.bach.deg)  6067.05    513.56   11.814 < 2e-16 ***
## pct.hs.grad        -81.56     21.73   -3.754 0.000198 ***
## log(hosp.beds)     -123.18    253.19   -0.486 0.626865
## log(doctors)       1121.19    239.96   4.672 3.99e-06 ***
## log(pop.65_plus)   840.56     406.68   2.067 0.039342 *
## log(pop.18_34)     -5769.21   909.72   -6.342 5.77e-10 ***
## ...
## Signif. codes:  0 '****' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1740 on 429 degrees of freedom
## Multiple R-squared:  0.8205, Adjusted R-squared:  0.8163
## F-statistic: 196.1 on 10 and 429 DF,  p-value: < 2.2e-16

par(mfrow=c(2,2))
plot(fullmodel)

```



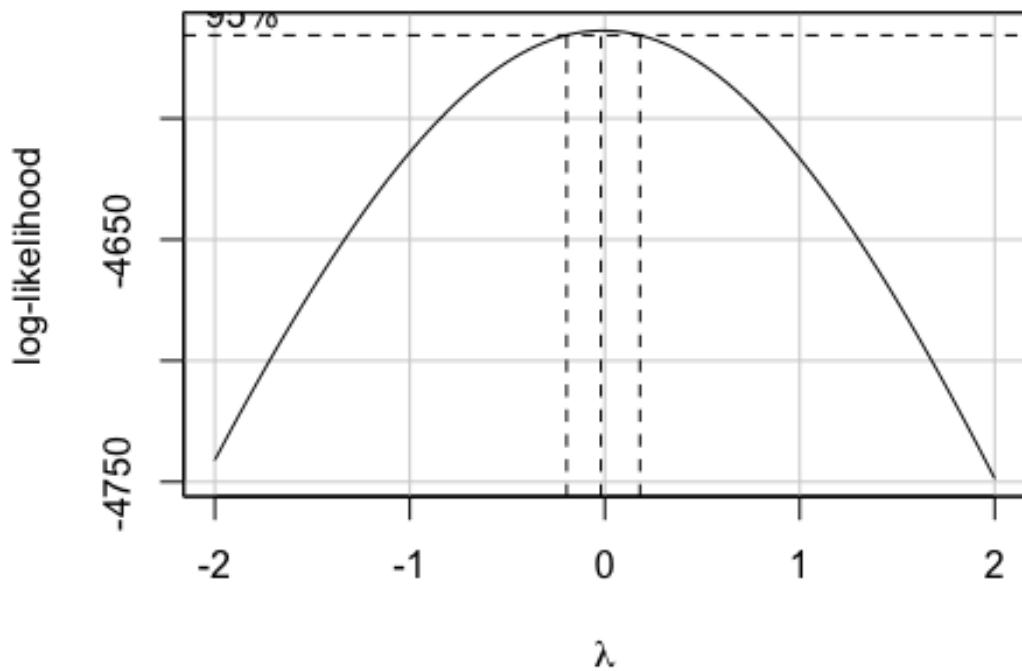
```
inverseResponsePlot(fullmodel)
```



```
##          lambda      RSS
## 1  0.3240419 986682935
## 2 -1.0000000 1320831767
## 3  0.0000000 1006059816
## 4  1.0000000 1065325734

boxCox(fullmodel)
```

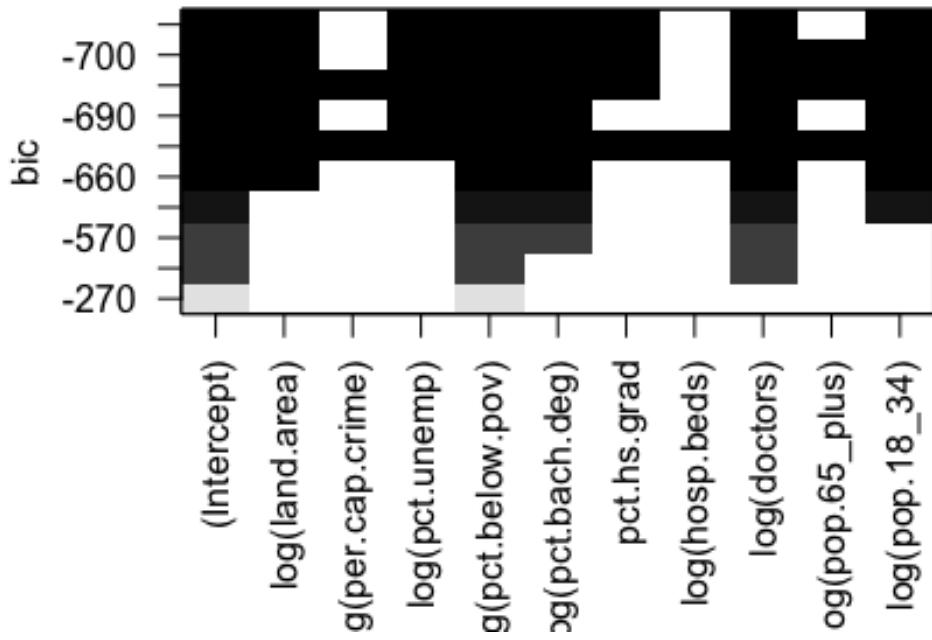
## Profile Log-likelihood



### All Subsets

```
cdi.subsets <- regsubsets(per.cap.income ~ log(land.area) + log(per.cap.crime)
)
+ log(pct.unemp) + log(pct.below.pov)
+ log(pct.bach.deg) + (pct.hs.grad)^3
+ log(hosp.beds) + log(doctors) + log(pop.65_plus)
+ log(pop.18_34) + log(land.area), data = cdi, nvmax=10)

plot(cdi.subsets)
```



```

cdi.subsets.summary <- summary(cdi.subsets)
names(cdi.subsets.summary)

## [1] "which"   "rsq"     "rss"      "adjr2"    "cp"       "bic"      "outmat"   "obj"

cdi.subsets.summary$bic

##  [1] -268.4528 -541.6377 -568.1158 -624.5520 -663.9394 -692.4577 -701.7173
## [8] -699.0234 -694.6304 -688.7863

min(cdi.subsets.summary$bic)

## [1] -701.7173

print(best.model <- which.min(cdi.subsets.summary$bic))

## [1] 7

coef(cdi.subsets, best.model)

##          (Intercept)      log(land.area)      log(pct.unemp)      log(pct.below.pov)
## 32700.92650           -681.96985          1712.85362         -4165.2773
##      log(pct.bach.deg)      pct.hs.grad      log(doctors)      log(pop.18_34)

```

```

)
##       6137.59648          -84.97262           1111.08392        -6863.4662
9

cdi.subsets.final.model <- lm(per.cap.income ~ log(land.area)
+ log(pct.unemp) + log(pct.below.pov)
+ log(pct.bach.deg) + (pct.hs.grad)^3
+ log(doctors) + log(pop.18_34), data = cdi)
summary(cdi.subsets.final.model)

##
## Call:
## lm(formula = per.cap.income ~ log(land.area) + log(pct.unemp) +
##     log(pct.below.pov) + log(pct.bach.deg) + (pct.hs.grad)^3 +
##     log(doctors) + log(pop.18_34), data = cdi)
##
## Residuals:
##    Min      1Q  Median      3Q      Max
## -5884.1  -986.7  -213.2   809.1 10223.2
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 32700.93    2806.52 11.652 < 2e-16 ***
## log(land.area) -681.97    100.57 -6.781 3.93e-11 ***
## log(pct.unemp) 1712.85    339.06  5.052 6.47e-07 ***
## log(pct.below.pov) -4165.28    229.08 -18.183 < 2e-16 ***
## log(pct.bach.deg) 6137.60    485.54 12.641 < 2e-16 ***
## pct.hs.grad -84.97     21.70 -3.916 0.000105 ***
## log(doctors) 1111.08     91.42 12.153 < 2e-16 ***
## log(pop.18_34) -6863.47    729.42 -9.409 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1744 on 432 degrees of freedom
## Multiple R-squared:  0.8183, Adjusted R-squared:  0.8154
## F-statistic:  278 on 7 and 432 DF,  p-value: < 2.2e-16

summary(cdi.subsets.final.model)$coef

##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 32700.92650 2806.52362 11.651755 1.802367e-27
## log(land.area) -681.96985 100.56687 -6.781258 3.929352e-11
## log(pct.unemp) 1712.85362 339.06173  5.051746 6.472157e-07
## log(pct.below.pov) -4165.27735 229.08120 -18.182537 2.831074e-55
## log(pct.bach.deg) 6137.59648 485.53550 12.640881 2.195710e-31
## pct.hs.grad -84.97262 21.70014 -3.915764 1.047011e-04
## log(doctors) 1111.08392 91.42147 12.153425 1.948148e-29
## log(pop.18_34) -6863.46629 729.41915 -9.409496 2.981996e-19

vif(cdi.subsets.final.model)

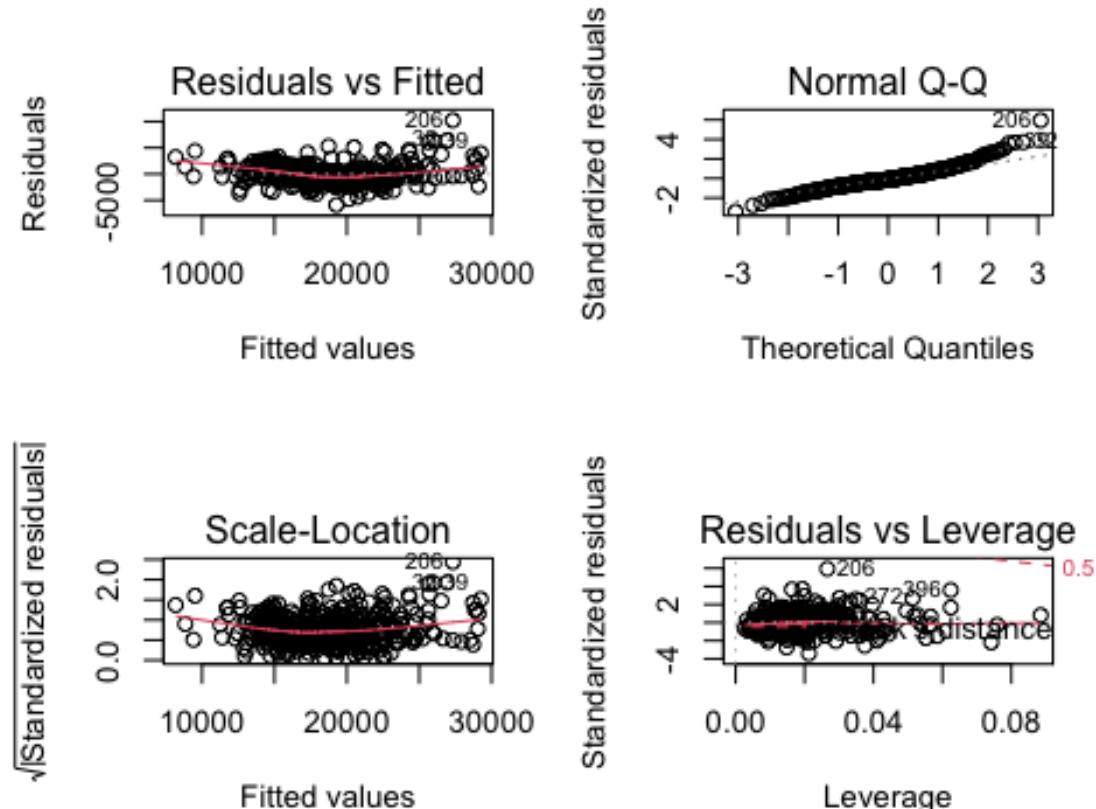
```

```

##      log(land.area)      log(pct.unemp) log(pct.below.pov) log(pct.bach.deg
##
##      1.109011          1.765859        2.131198        4.27391
##
##      pct.hs.grad      log(doctors)   log(pop.18_34)    1.524308
##      3.344148          1.578605

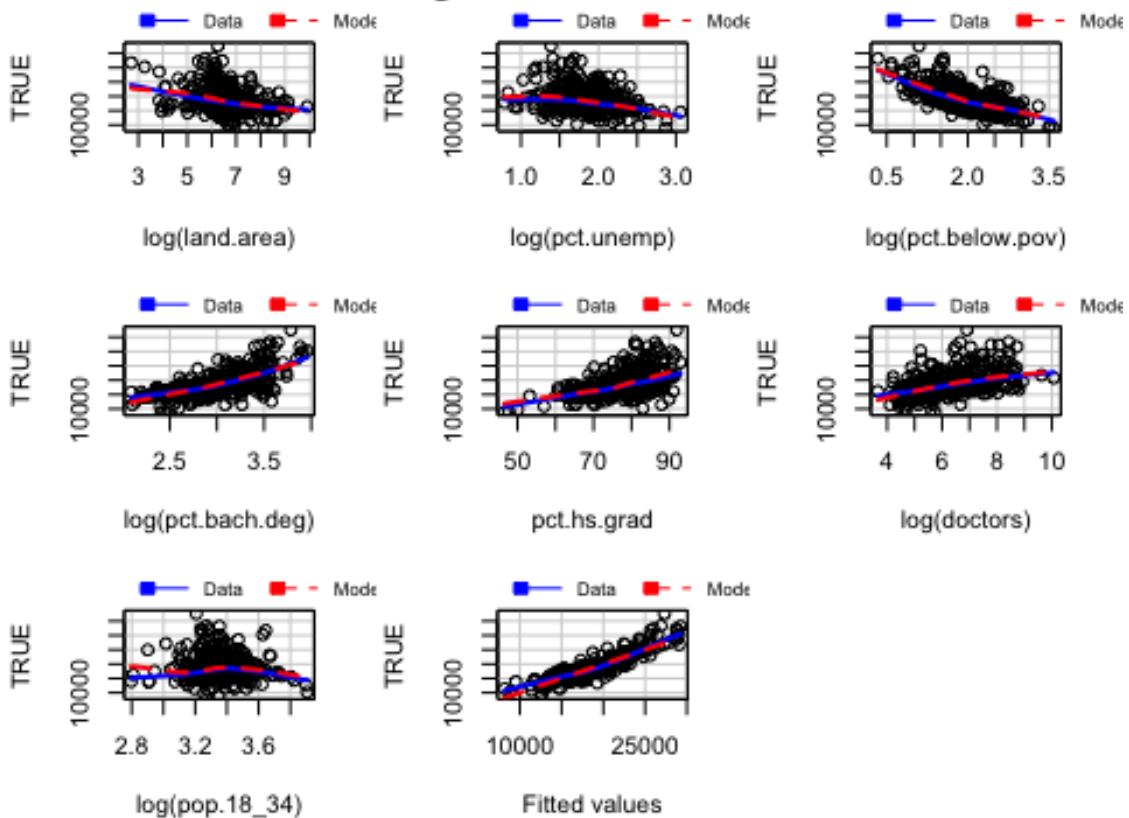
par(mfrow=c(2,2))
plot(cdi.subsets.final.model)

```



```
mmps(cdi.subsets.final.model)
```

## Marginal Model Plots



```

cdisub <- cdi[-c(1:3, 5, 10, 16)]
cdi_all_subsets <- cdisub[c(1:2, 4, 6:11)]

cdi_all_subsets <- cdi_all_subsets %>%
  mutate(land.area = log(land.area)) %>%
  dplyr::rename(log.land.area = land.area) %>%
  mutate(pop.18_34 = log(pop.18_34)) %>%
  dplyr::rename(log.pop.18_34 = pop.18_34) %>%
  mutate(pct.unemp = log(pct.unemp)) %>%
  dplyr::rename(log.pct.unemp = pct.unemp) %>%
  mutate(doctors = log(doctors)) %>%
  dplyr::rename(log.doctors = doctors) %>%
  mutate(pct.bach.deg = log(pct.bach.deg)) %>%
  dplyr::rename(log.pct.bach.deg = pct.bach.deg) %>%
  mutate(pct.below.pov = log(pct.below.pov)) %>%
  dplyr::rename(log.pct.below.pov = pct.below.pov) %>%
  mutate(pct.hs.grad = (pct.hs.grad)^3) %>%
  dplyr::rename(pct.hs.grad.3 = pct.hs.grad)

cdi.subsets.final.model.with.region <- lm(per.cap.income ~ .*region,
                                            data=cdi_all_subsets)
summary(cdi.subsets.final.model.with.region)

```

```

## 
## Call:
## lm(formula = per.cap.income ~ . * region, data = cdi_all_subsets)
## 
## Residuals:
##    Min      1Q  Median      3Q     Max 
## -4751.4   -965.6  -130.4   678.9  9679.8 
## 
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)    
## (Intercept)                2.844e+04  5.480e+03   5.190 3.32e-07 ***
## log.land.area              -4.389e+02  3.216e+02  -1.365 0.173095  
## log.pop.18_34               -7.156e+03  1.792e+03  -3.993 7.75e-05 ***
## log.doctors                 1.017e+03  2.056e+02   4.948 1.10e-06 ***
## pct.hs.grad.3               -6.339e-04  3.889e-03  -0.163 0.870589  
## log.pct.bach.deg            4.657e+03  1.357e+03   3.431 0.000664 ***
## log.pct.below.pov            -3.200e+03  5.521e+02  -5.797 1.36e-08 ***
## log.pct.unemp                1.982e+03  6.685e+02   2.965 0.003207 ** 
## regionNE                   9.776e+03  7.935e+03   1.232 0.218666  
## regions                     1.759e+03  6.890e+03   0.255 0.798592  
## regionW                     6.459e+03  8.667e+03   0.745 0.456519  
## log.land.area:regionNE      -1.492e+02  4.282e+02  -0.348 0.727646  
## log.land.area:regions        -4.326e+02  3.709e+02  -1.166 0.244139  
## log.land.area:regionW       -2.388e+02  3.867e+02  -0.617 0.537271  
## log.pop.18_34:regionNE      -4.511e+03  2.602e+03  -1.733 0.083785 .  
## log.pop.18_34:regions        5.709e+01  2.083e+03   0.027 0.978145  
## log.pop.18_34:regionW       1.794e+03  2.597e+03   0.691 0.489937  
## log.doctors:regionNE        4.744e+01  3.004e+02   0.158 0.874619  
## log.doctors:regionS         -1.956e+02  2.586e+02  -0.756 0.449858  
## log.doctors:regionW         -1.174e+02  2.907e+02  -0.404 0.686638  
## pct.hs.grad.3:regionNE      -7.450e-03  5.315e-03  -1.402 0.161790  
## pct.hs.grad.3:regions        -5.334e-03  4.587e-03  -1.163 0.245531  
## pct.hs.grad.3:regionW       -1.936e-02  5.050e-03  -3.833 0.000146 *** 
## log.pct.bach.deg:regionNE   3.990e+03  1.983e+03   2.012 0.044842 *  
## log.pct.bach.deg:regions     2.485e+03  1.603e+03   1.550 0.121836  
## log.pct.bach.deg:regionW     3.285e+03  1.766e+03   1.860 0.063536 .  
## log.pct.below.pov:regionNE  -7.876e+02  7.768e+02  -1.014 0.311234  
## log.pct.below.pov:regions    -4.538e+02  6.896e+02  -0.658 0.510818  
## log.pct.below.pov:regionW    -3.847e+03  9.403e+02  -4.091 5.18e-05 *** 
## log.pct.unemp:regionNE      -5.526e+02  1.108e+03  -0.499 0.618326  
## log.pct.unemp:regions        -1.343e+03  9.540e+02  -1.408 0.159820  
## log.pct.unemp:regionW       -9.855e+02  9.924e+02  -0.993 0.321281  
## --- 
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 
## 
## Residual standard error: 1642 on 408 degrees of freedom
## Multiple R-squared:  0.848, Adjusted R-squared:  0.8364 
## F-statistic: 73.41 on 31 and 408 DF,  p-value: < 2.2e-16

```

```

cdi.subsets.final.model.with.some.region <- update(cdi.subsets.final.model.with.region,
                                                    . ~ . - region:log.land.ar
ea -
                                                    region:log.doctors -
                                                    region:log.pct.unemp)
summary(cdi.subsets.final.model.with.some.region)

##
## Call:
## lm(formula = per.cap.income ~ log.land.area + log.pop.18_34 +
##      log.doctors + pct.hs.grad.3 + log.pct.bach.deg + log.pct.below.pov +
##      log.pct.unemp + region + log.pop.18_34:region + pct.hs.grad.3:region +
##      log.pct.bach.deg:region + log.pct.below.pov:region, data = cdi_all_sub-
sets)
##
## Residuals:
##    Min      1Q  Median      3Q     Max
## -4787.7  -982.3 -135.6   714.8  9626.3
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)    
## (Intercept)            3.360e+04  4.645e+03   7.235 2.25e-12 ***
## log.land.area          -7.042e+02  1.160e+02  -6.070 2.88e-09 ***
## log.pop.18_34          -7.613e+03  1.676e+03  -4.543 7.28e-06 ***
## log.doctors             9.303e+02  9.476e+01   9.818 < 2e-16 ***
## pct.hs.grad.3          -1.494e-03  3.493e-03  -0.428 0.669130  
## log.pct.bach.deg       4.676e+03  1.022e+03   4.574 6.33e-06 ***
## log.pct.below.pov      -3.094e+03  5.243e+02  -5.901 7.49e-09 ***
## log.pct.unemp           1.264e+03  3.599e+02   3.513 0.000491 ***
## regionNE                6.507e+03  6.535e+03   0.996 0.319934  
## regions                 -7.750e+03  5.118e+03  -1.514 0.130705  
## regionW                  8.801e+02  7.303e+03   0.120 0.904145  
## log.pop.18_34:regionNE -4.386e+03  2.371e+03  -1.849 0.065105 . 
## log.pop.18_34:regions   1.143e+03  1.926e+03   0.594 0.553079  
## log.pop.18_34:regionW   2.167e+03  2.495e+03   0.869 0.385500  
## pct.hs.grad.3:regionNE -6.806e-03  4.468e-03  -1.523 0.128469  
## pct.hs.grad.3:regions   -4.696e-03  4.207e-03  -1.116 0.264941  
## pct.hs.grad.3:regionW   -1.791e-02  4.387e-03  -4.082 5.34e-05 ***
## log.pct.bach.deg:regionNE 4.292e+03  1.285e+03   3.340 0.000913 ***
## log.pct.bach.deg:regionS 2.482e+03  1.147e+03   2.164 0.030997 *  
## log.pct.bach.deg:regionW 3.229e+03  1.374e+03   2.350 0.019231 *  
## log.pct.below.pov:regionNE -7.567e+02  7.117e+02  -1.063 0.288320  
## log.pct.below.pov:regionS -8.122e+02  6.451e+02  -1.259 0.208756  
## log.pct.below.pov:regionW -3.949e+03  9.132e+02  -4.324 1.92e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1635 on 417 degrees of freedom

```

```

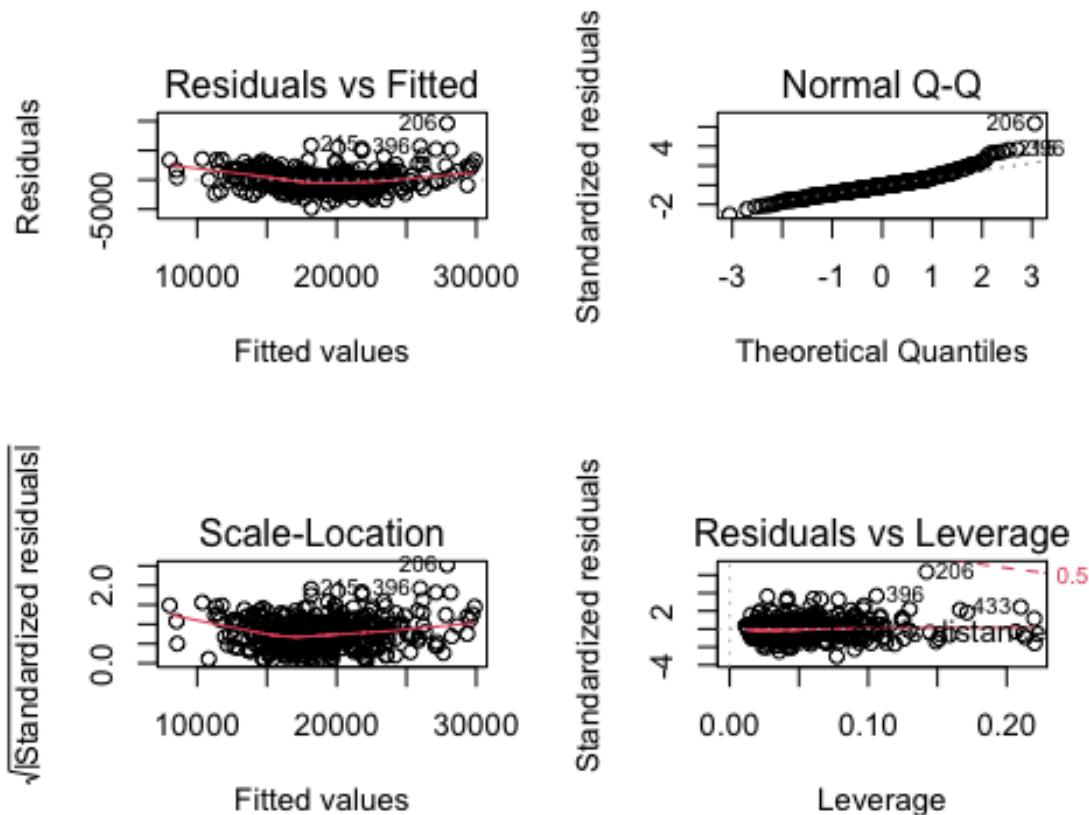
## Multiple R-squared:  0.846, Adjusted R-squared:  0.8378
## F-statistic: 104.1 on 22 and 417 DF,  p-value: < 2.2e-16

vif(cdi.subsets.final.model.with.some.region)

##                                     GVIF Df GVIF^(1/(2*Df))
## log.land.area           1.680366e+00  1     1.296289
## log.pop.18_34            9.161875e+00  1     3.026859
## log.doctors              1.931049e+00  1     1.389622
## pct.hs.grad.3            2.948870e+01  1     5.430350
## log.pct.bach.deg         2.157800e+01  1     4.645212
## log.pct.below.pov        1.271309e+01  1     3.565542
## log.pct.unemp             2.265215e+00  1     1.505063
## region                   4.593495e+08  3     27.777333
## log.pop.18_34:region     1.183970e+09  3     32.525461
## pct.hs.grad.3:region      6.001590e+05  3     9.184265
## log.pct.bach.deg:region   2.276982e+07  3     16.835523
## log.pct.below.pov:region  1.275326e+05  3     7.094747

par(mfrow=c(2,2))
plot(cdi.subsets.final.model.with.some.region)

```



```

anova(cdi.subsets.final.model, cdi.subsets.final.model.with.some.region)

```

```

## Analysis of Variance Table
##
## Model 1: per.cap.income ~ log(land.area) + log(pct.unemp) + log(pct.below.pov) +
##           log(pct.bach.deg) + (pct.hs.grad)^3 + log(doctors) + log(pop.18_34)
## Model 2: per.cap.income ~ log.land.area + log.pop.18_34 + log.doctors +
##           pct.hs.grad.3 + log.pct.bach.deg + log.pct.below.pov + log.pct.unemp +
##           region + log.pop.18_34:region + pct.hs.grad.3:region + log.pct.bach.deg:region +
##           log.pct.below.pov:region
##   Res.Df      RSS Df Sum of Sq    F    Pr(>F)
## 1     432 1314204306
## 2     417 1114159243 15 200045063 4.9914 4.626e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

AIC(cdi.subsets.final.model, cdi.subsets.final.model.with.some.region)

##                                df      AIC
## cdi.subsets.final.model          9 7826.944
## cdi.subsets.final.model.with.some.region 24 7784.286

BIC(cdi.subsets.final.model, cdi.subsets.final.model.with.some.region)

##                                df      BIC
## cdi.subsets.final.model          9 7863.725
## cdi.subsets.final.model.with.some.region 24 7882.369

formula(cdi.subsets.final.model.with.some.region)

## per.cap.income ~ log.land.area + log.pop.18_34 + log.doctors +
##           pct.hs.grad.3 + log.pct.bach.deg + log.pct.below.pov + log.pct.unemp +
##           region + log.pop.18_34:region + pct.hs.grad.3:region + log.pct.bach.deg:region +
##           log.pct.below.pov:region

round(summary(cdi.subsets.final.model.with.some.region)$coef, 2)

##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)            33603.04    4644.53    7.23    0.00
## log.land.area         -704.21     116.01   -6.07    0.00
## log.pop.18_34        -7613.16    1675.90   -4.54    0.00
## log.doctors           930.31      94.76    9.82    0.00
## pct.hs.grad.3         0.00       0.00   -0.43    0.67
## log.pct.bach.deg     4676.09    1022.42    4.57    0.00
## log.pct.below.pov   -3094.10     524.35   -5.90    0.00
## log.pct.unemp         1264.35    359.89    3.51    0.00
## regionNE              6507.45    6535.02    1.00    0.32
## regions               -7750.28    5118.03   -1.51    0.13
## regionW                880.06    7303.42    0.12    0.90

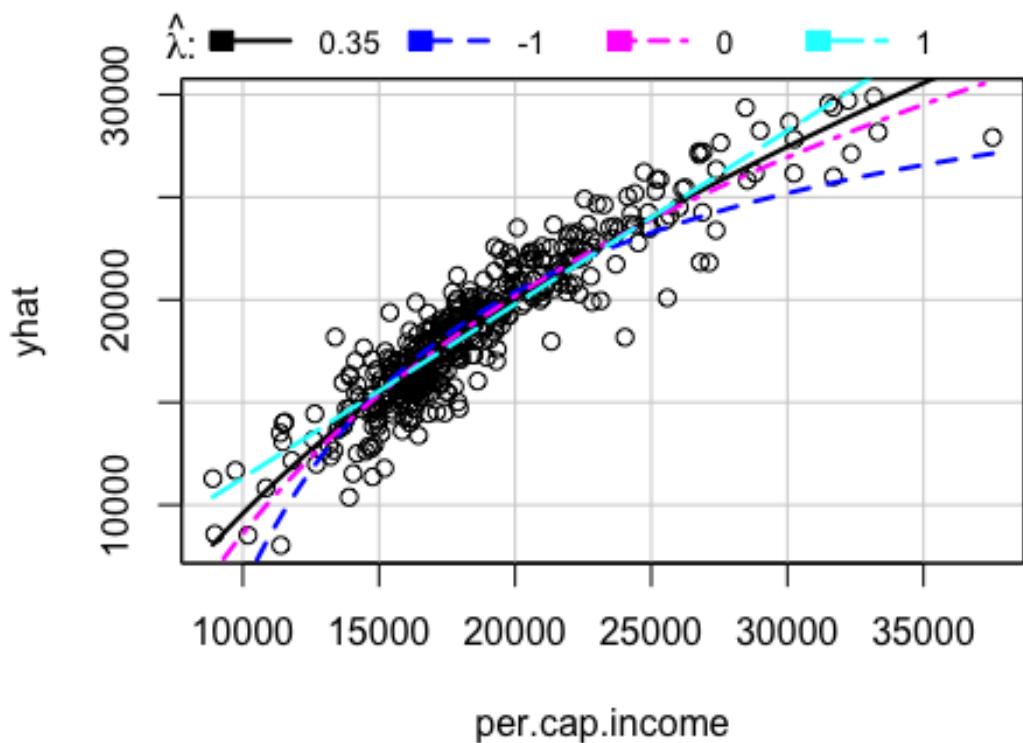
```

```

## log.pop.18_34:regionNE      -4385.78    2371.44   -1.85    0.07
## log.pop.18_34:regions       1143.33    1925.98    0.59    0.55
## log.pop.18_34:regionW       2167.35    2494.87    0.87    0.39
## pct.hs.grad.3:regionNE     -0.01      0.00     -1.52    0.13
## pct.hs.grad.3:regions       0.00      0.00     -1.12    0.26
## pct.hs.grad.3:regionW      -0.02      0.00     -4.08    0.00
## log.pct.bach.deg:regionNE  4292.42    1285.09    3.34    0.00
## log.pct.bach.deg:regions    2481.75    1146.60    2.16    0.03
## log.pct.bach.deg:regionW    3229.20    1374.04    2.35    0.02
## log.pct.below.pov:regionNE -756.68     711.72   -1.06    0.29
## log.pct.below.pov:regions   -812.19     645.14   -1.26    0.21
## log.pct.below.pov:regionW   -3948.57    913.22   -4.32    0.00

inverseResponsePlot(cdi.subsets.final.model.with.some.region)

```



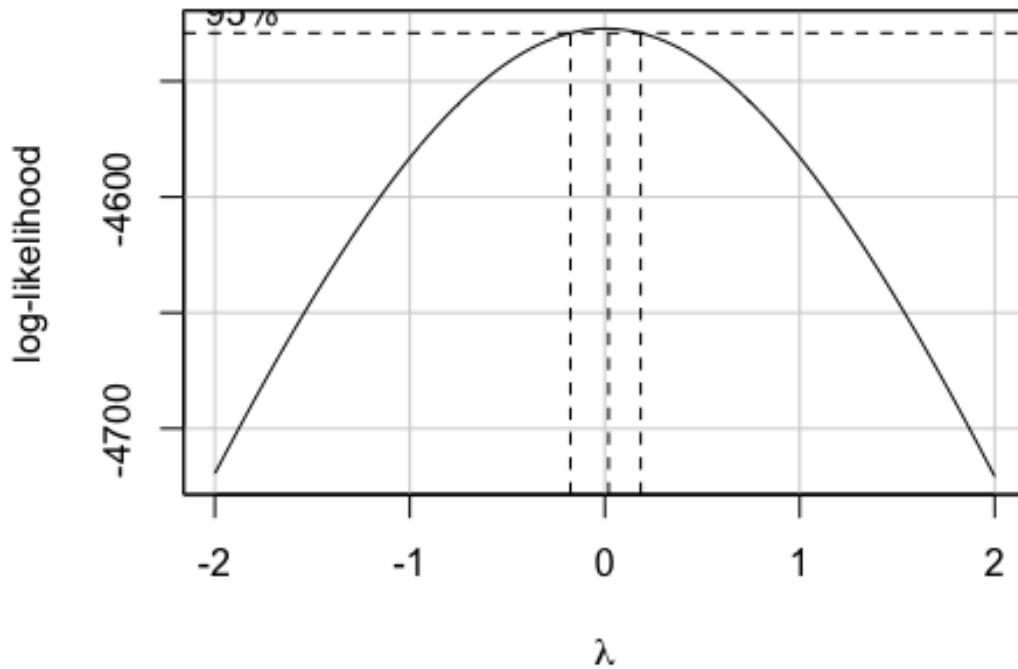
```

##      lambda      RSS
## 1  0.3502333 867768343
## 2 -1.0000000 1225709535
## 3  0.0000000 891058518
## 4  1.0000000 942545990

boxCox(cdi.subsets.final.model.with.some.region)

```

## Profile Log-likelihood



### Appendix D

```

cdi.cont <- data.frame(cdi_all_subsets, state=cdi$state)
cdi.cont <- cdi.cont[-c(9)]

final.model.with.state <- lm(per.cap.income ~ ., data=cdi.cont)
summary(final.model.with.state)

##
## Call:
## lm(formula = per.cap.income ~ ., data = cdi.cont)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -4005.2  -822.5   -42.7   603.6  8704.6 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 3.459e+04 2.834e+03 12.207 < 2e-16 ***
## log.land.area -7.550e+02 1.290e+02 -5.855 1.02e-08 ***
## log.pop.18_34 -8.270e+03 7.511e+02 -11.010 < 2e-16 ***
## log.doctors    9.530e+02 9.203e+01 10.356 < 2e-16 ***
## pct.hs.grad.3 -4.417e-03 1.672e-03 -2.641 0.008596 ** 
## log.pct.bach.deg 6.195e+03 5.792e+02 10.696 < 2e-16 ***

```

```

## log.pct.below.pov -3.487e+03 2.912e+02 -11.976 < 2e-16 ***
## log.pct.unemp      5.698e+02 4.983e+02   1.143 0.253543
## stateAR          -7.594e+02 1.276e+03  -0.595 0.552103
## stateAZ          -3.213e+02 9.770e+02  -0.329 0.742457
## stateCA          1.928e+03 7.022e+02   2.746 0.006316 **
## stateCO          4.889e+02 8.326e+02   0.587 0.557410
## stateCT          1.577e+03 8.746e+02   1.803 0.072168 .
## stateDC          1.226e+03 1.753e+03   0.700 0.484632
## stateDE          3.999e+02 1.290e+03   0.310 0.756811
## stateFL          -5.530e+02 6.950e+02  -0.796 0.426667
## stateGA          4.355e+02 8.204e+02   0.531 0.595814
## stateHI          6.876e+02 1.141e+03   0.602 0.547280
## stateID          -1.686e+02 1.722e+03  -0.098 0.922070
## stateIL          9.757e+02 7.451e+02   1.309 0.191192
## stateIN          -4.633e+01 7.606e+02  -0.061 0.951458
## stateKS          3.070e+02 1.032e+03   0.297 0.766311
## stateKY          -3.701e+02 1.110e+03  -0.333 0.739056
## stateLA          -1.874e+02 8.090e+02  -0.232 0.816929
## stateMA          6.244e+02 8.551e+02   0.730 0.465765
## stateMD          2.070e+02 8.325e+02   0.249 0.803753
## stateME          1.815e+02 9.541e+02   0.190 0.849209
## stateMI          1.338e+03 7.681e+02   1.742 0.082269 .
## stateMN          -9.703e+01 8.915e+02  -0.109 0.913388
## stateMO          5.894e+01 8.480e+02   0.070 0.944623
## stateMS          -9.158e+02 1.102e+03  -0.831 0.406349
## stateMT          4.318e+02 1.721e+03   0.251 0.802026
## stateNC          1.708e+02 7.273e+02   0.235 0.814483
## stateND          5.069e+01 1.747e+03   0.029 0.976869
## stateNE          -9.194e+02 1.171e+03  -0.785 0.432875
## stateNH          -6.326e+01 1.050e+03  -0.060 0.952008
## stateNJ          2.155e+03 7.689e+02   2.803 0.005325 **
## stateNM          -1.455e+03 1.284e+03  -1.133 0.257811
## stateNV          4.176e+03 1.327e+03   3.146 0.001782 **
## stateNY          5.421e+02 7.160e+02   0.757 0.449478
## stateOH          3.382e+02 7.148e+02   0.473 0.636382
## stateOK          -7.892e+02 1.015e+03  -0.777 0.437471
## stateOR          -1.088e+03 9.174e+02  -1.186 0.236496
## statePA          -2.777e+02 7.032e+02  -0.395 0.693151
## stateRI          -1.991e+03 1.148e+03  -1.735 0.083480 .
## stateSC          -2.025e+02 7.720e+02  -0.262 0.793217
## stateSD          3.279e+02 1.745e+03   0.188 0.851042
## stateTN          -3.497e+02 8.269e+02  -0.423 0.672590
## stateTX          -2.591e+01 6.735e+02  -0.038 0.969331
## stateUT          -4.061e+03 1.047e+03  -3.877 0.000124 ***
## stateVA          7.491e+02 8.826e+02   0.849 0.396594
## stateVT          -7.505e+02 1.716e+03  -0.437 0.662016
## stateWA          -1.601e+02 8.325e+02  -0.192 0.847561
## stateWI          2.053e+02 8.102e+02   0.253 0.800128
## stateWV          -5.462e+02 1.702e+03  -0.321 0.748382
## ---
```

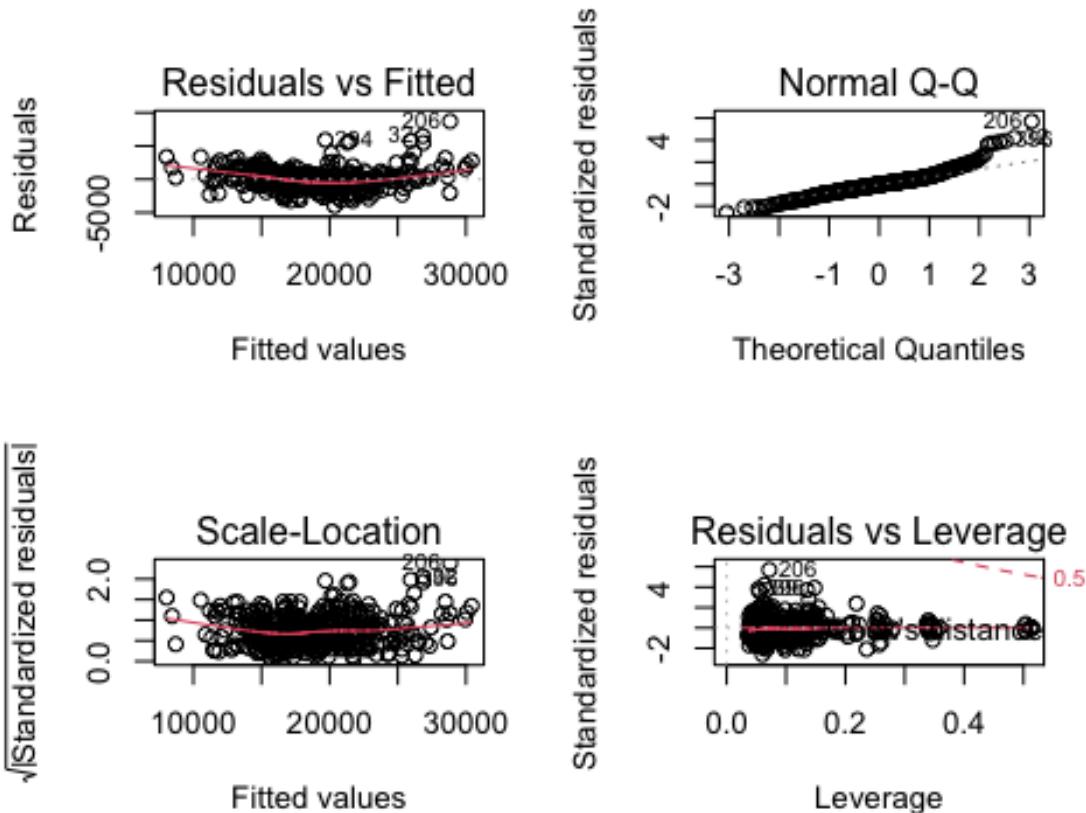
```

## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1585 on 385 degrees of freedom
## Multiple R-squared:  0.8662, Adjusted R-squared:  0.8475
## F-statistic: 46.16 on 54 and 385 DF,  p-value: < 2.2e-16

par(mfrow=c(2,2))
plot(final.model.with.state)

## Warning: not plotting observations with leverage one:
##    73, 232, 233, 339, 356, 388, 429

```



```

final.model.with.state.int <- lm(per.cap.income ~ .*state, data=cdi.cont)
par(mfrow=c(2,2))
plot(final.model.with.state.int)

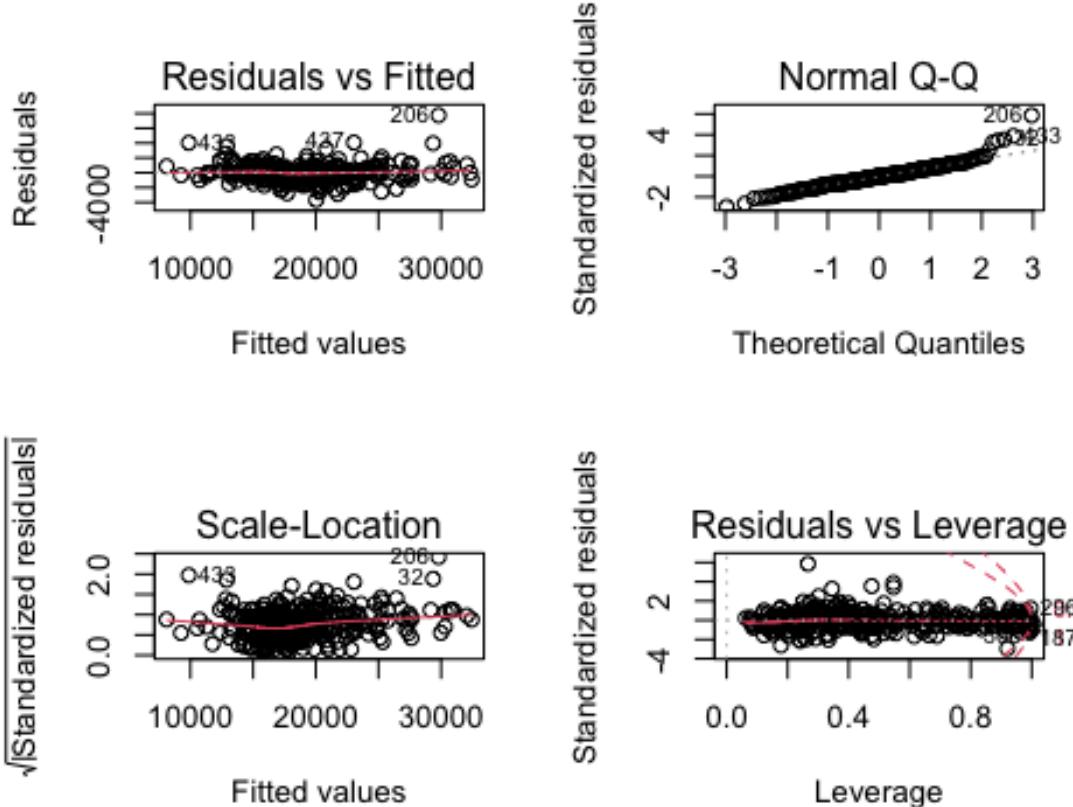
## Warning: not plotting observations with leverage one:
##    7, 27, 28, 35, 37, 39, 40, 43, 49, 52, 65, 66, 68, 71, 73, 74, 75, 78, 9
##    0, 92, 97, 99, 109, 118, 120, 123, 130, 141, 143, 148, 149, 156, 167, 169, 17
##    2, 175, 184, 190, 191, 192, 199, 200, 203, 209, 213, 225, 226, 230, 231, 232,
##    233, 235, 246, 267, 268, 271, 276, 280, 282, 285, 286, 288, 291, 298, 309, 3
##    11, 312, 319, 320, 323, 334, 339, 348, 356, 368, 370, 375, 377, 380, 381, 382
##    , 385, 388, 389, 392, 395, 397, 402, 412, 415, 418, 421, 425, 428, 429, 431,
##    432, 438, 439, 440

```

```

## Warning in sqrt(crit * p * (1 - hh)/hh): NaNs produced
## Warning in sqrt(crit * p * (1 - hh)/hh): NaNs produced

```



```

anova(cdi.subsets.final.model, final.model.with.state, final.model.with.state.int)

## Analysis of Variance Table
##
## Model 1: per.cap.income ~ log(land.area) + log(pct.unemp) + log(pct.below.pov) +
##           log(pct.bach.deg) + (pct.hs.grad)^3 + log(doctors) + log(pop.18_34)
## Model 2: per.cap.income ~ log.land.area + log.pop.18_34 + log.doctors +
##           pct.hs.grad.3 + log.pct.bach.deg + log.pct.below.pov + log.pct.unemp +
##           state
## Model 3: per.cap.income ~ (log.land.area + log.pop.18_34 + log.doctors +
##           pct.hs.grad.3 + log.pct.bach.deg + log.pct.below.pov + log.pct.unemp +
##           state) * state
##      Res.Df       RSS   Df Sum of Sq      F    Pr(>F)
## 1     432 1314204306
## 2     385  967714409   47 346489897 3.0968 3.526e-08 ***

```

```

## 3    180  428500031 205 539214379 1.1049    0.2464
## ---
## Signif. codes:  0 '****' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

AIC(cdi.subsets.final.model, final.model.with.state, final.model.with.state.int)

##                  df      AIC
## cdi.subsets.final.model     9 7826.944
## final.model.with.state     56 7786.282
## final.model.with.state.int 261 7837.838

BIC(cdi.subsets.final.model, final.model.with.state, final.model.with.state.int)

##                  df      BIC
## cdi.subsets.final.model     9 7863.725
## final.model.with.state     56 8015.141
## final.model.with.state.int 261 8904.486

formula(final.model.with.state)

## per.cap.income ~ log.land.area + log.pop.18_34 + log.doctors +
##       pct.hs.grad.3 + log.pct.bach.deg + log.pct.below.pov + log.pct.unemp +
##       state

round(summary(final.model.with.state)$coef, 2)

##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) 34590.94   2833.74   12.21   0.00
## log.land.area -755.02    128.95   -5.85   0.00
## log.pop.18_34 -8269.77   751.12  -11.01   0.00
## log.doctors    953.01    92.03   10.36   0.00
## pct.hs.grad.3    0.00     0.00   -2.64   0.01
## log.pct.bach.deg  6195.10   579.20   10.70   0.00
## log.pct.below.pov -3487.25   291.18  -11.98   0.00
## log.pct.unemp     569.76   498.26    1.14   0.25
## stateAR        -759.37  1275.96   -0.60   0.55
## stateAZ        -321.27   976.99   -0.33   0.74
## stateCA        1928.36   702.25    2.75   0.01
## stateCO        488.91   832.60    0.59   0.56
## stateCT        1576.99   874.64    1.80   0.07
## stateDC        1225.97  1752.52    0.70   0.48
## stateDE        399.89  1290.40    0.31   0.76
## stateFL       -553.01   694.96   -0.80   0.43
## stateGA        435.54   820.43    0.53   0.60
## stateHI        687.60  1141.48    0.60   0.55
## stateID       -168.60  1722.38   -0.10   0.92
## stateIL        975.67   745.15    1.31   0.19
## stateIN       -46.33   760.62   -0.06   0.95
## stateKS       306.97  1032.14    0.30   0.77

```

```

## stateKY      -370.07   1110.16  -0.33    0.74
## stateLA     -187.40    808.98  -0.23    0.82
## stateMA      624.35    855.14   0.73    0.47
## stateMD      207.01    832.51   0.25    0.80
## stateME      181.52    954.10   0.19    0.85
## stateMI     1338.13    768.06   1.74    0.08
## stateMN     -97.03    891.51  -0.11    0.91
## stateMO      58.94    847.99   0.07    0.94
## stateMS     -915.79   1101.70  -0.83    0.41
## stateMT      431.82   1721.10   0.25    0.80
## stateNC      170.78    727.32   0.23    0.81
## stateND      50.69    1747.23   0.03    0.98
## stateNE     -919.37   1171.02  -0.79    0.43
## stateNH     -63.26    1050.33  -0.06    0.95
## stateNJ     2154.89    768.88   2.80    0.01
## stateNM     -1455.18   1284.06  -1.13    0.26
## stateNV     4176.17    1327.31   3.15    0.00
## stateNY      542.07    716.02   0.76    0.45
## stateOH      338.22    714.83   0.47    0.64
## stateOK     -789.18    1015.31  -0.78    0.44
## stateOR     -1087.71    917.40  -1.19    0.24
## statePA     -277.69    703.22  -0.39    0.69
## stateRI     -1991.33   1147.52  -1.74    0.08
## stateSC     -202.51    771.99  -0.26    0.79
## stateSD      327.86   1744.72   0.19    0.85
## stateTN     -349.73    826.94  -0.42    0.67
## stateTX     -25.91    673.53  -0.04    0.97
## stateUT     -4060.76   1047.34  -3.88    0.00
## stateVA      749.06    882.63   0.85    0.40
## stateVT     -750.54   1715.64  -0.44    0.66
## stateWA     -160.13    832.45  -0.19    0.85
## stateWI      205.27    810.20   0.25    0.80
## stateWV     -546.25   1701.69  -0.32    0.75

summary(cdi.subsets.final.model.with.some.region)

##
## Call:
## lm(formula = per.cap.income ~ log.land.area + log.pop.18_34 +
##     log.doctors + pct.hs.grad.3 + log.pct.bach.deg + log.pct.below.pov +
##     log.pct.unemp + region + log.pop.18_34:region + pct.hs.grad.3:region +
##     log.pct.bach.deg:region + log.pct.below.pov:region, data = cdi_all_sub-
##     sets)
##
## Residuals:
##      Min    1Q Median    3Q   Max 
## -4787.7 -982.3 -135.6  714.8 9626.3 
##
## Coefficients:

```

```

##                                     Estimate Std. Error t value Pr(>|t|) 
## (Intercept)                  3.360e+04  4.645e+03   7.235 2.25e-12 ***
## log.land.area                 -7.042e+02  1.160e+02  -6.070 2.88e-09 ***
## log.pop.18_34                  -7.613e+03  1.676e+03  -4.543 7.28e-06 ***
## log.doctors                     9.303e+02  9.476e+01   9.818 < 2e-16 ***
## pct.hs.grad.3                  -1.494e-03  3.493e-03  -0.428 0.669130
## log.pct.bach.deg                4.676e+03  1.022e+03   4.574 6.33e-06 ***
## log.pct.below.pov                -3.094e+03  5.243e+02  -5.901 7.49e-09 ***
## log.pct.unemp                   1.264e+03  3.599e+02   3.513 0.000491 ***
## regionNE                        6.507e+03  6.535e+03   0.996 0.319934
## regions                          -7.750e+03  5.118e+03  -1.514 0.130705
## regionW                           8.801e+02  7.303e+03   0.120 0.904145
## log.pop.18_34:regionNE          -4.386e+03  2.371e+03  -1.849 0.065105 .
## log.pop.18_34:regions            1.143e+03  1.926e+03   0.594 0.553079
## log.pop.18_34:regionW            2.167e+03  2.495e+03   0.869 0.385500
## pct.hs.grad.3:regionNE          -6.806e-03  4.468e-03  -1.523 0.128469
## pct.hs.grad.3:regions            -4.696e-03  4.207e-03  -1.116 0.264941
## pct.hs.grad.3:regionW            -1.791e-02  4.387e-03  -4.082 5.34e-05 ***
## log.pct.bach.deg:regionNE       4.292e+03  1.285e+03   3.340 0.000913 ***
## log.pct.bach.deg:regions         2.482e+03  1.147e+03   2.164 0.030997 *
## log.pct.bach.deg:regionW         3.229e+03  1.374e+03   2.350 0.019231 *
## log.pct.below.pov:regionNE      -7.567e+02  7.117e+02  -1.063 0.288320
## log.pct.below.pov:regions        -8.122e+02  6.451e+02  -1.259 0.208756
## log.pct.below.pov:regionW        -3.949e+03  9.132e+02  -4.324 1.92e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 1635 on 417 degrees of freedom
## Multiple R-squared:  0.846, Adjusted R-squared:  0.8378
## F-statistic: 104.1 on 22 and 417 DF,  p-value: < 2.2e-16

round(summary(cdi.subsets.final.model.with.some.region)$coef,2)

##                                     Estimate Std. Error t value Pr(>|t|) 
## (Intercept)                  33603.04    4644.53    7.23    0.00
## log.land.area                 -704.21     116.01   -6.07    0.00
## log.pop.18_34                  -7613.16    1675.90   -4.54    0.00
## log.doctors                     930.31     94.76    9.82    0.00
## pct.hs.grad.3                  0.00      0.00   -0.43    0.67
## log.pct.bach.deg                4676.09    1022.42   4.57    0.00
## log.pct.below.pov                -3094.10    524.35  -5.90    0.00
## log.pct.unemp                   1264.35    359.89   3.51    0.00
## regionNE                        6507.45    6535.02   1.00    0.32
## regions                          -7750.28    5118.03  -1.51    0.13
## regionW                           880.06    7303.42   0.12    0.90
## log.pop.18_34:regionNE          -4385.78    2371.44  -1.85    0.07
## log.pop.18_34:regions            1143.33    1925.98   0.59    0.55
## log.pop.18_34:regionW            2167.35    2494.87   0.87    0.39
## pct.hs.grad.3:regionNE          -0.01      0.00   -1.52    0.13
## pct.hs.grad.3:regions            0.00      0.00   -1.12    0.26

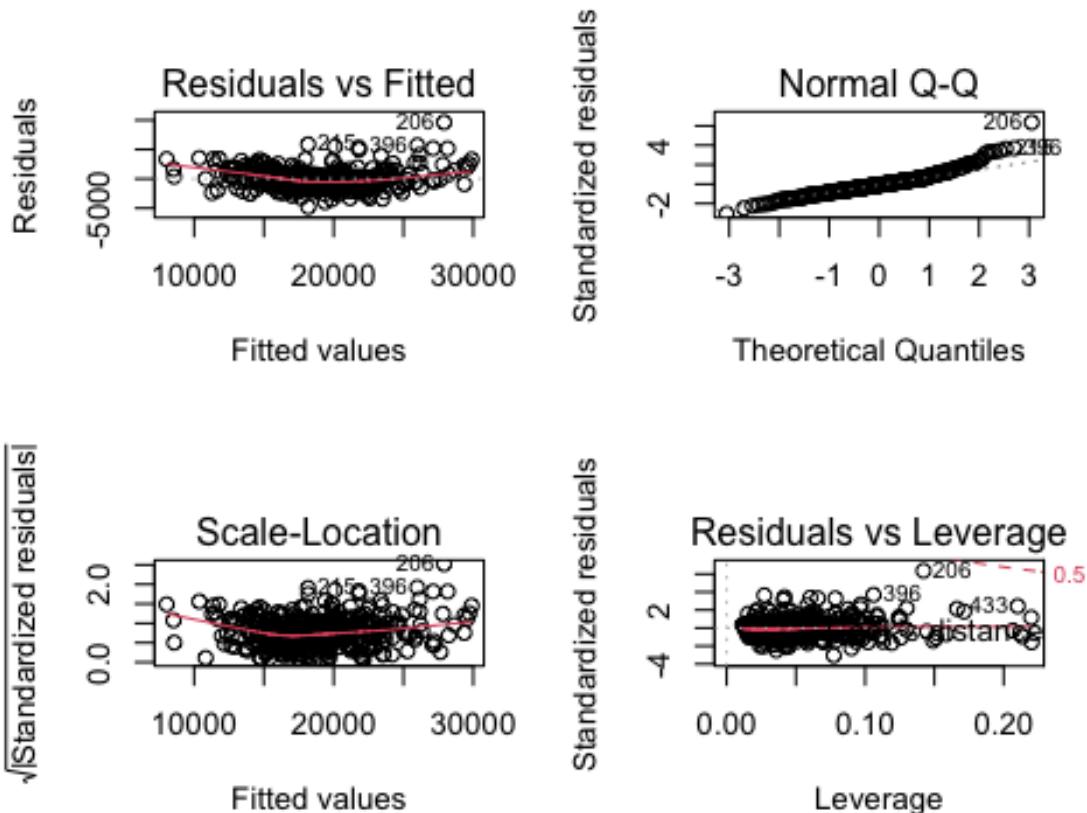
```

```

## pct.hs.grad.3:regionW      -0.02      0.00   -4.08    0.00
## log.pct.bach.deg:regionNE 4292.42  1285.09   3.34    0.00
## log.pct.bach.deg:regionS  2481.75  1146.60   2.16    0.03
## log.pct.bach.deg:regionW  3229.20  1374.04   2.35    0.02
## log.pct.below.pov:regionNE -756.68   711.72  -1.06    0.29
## log.pct.below.pov:regionS  -812.19   645.14  -1.26    0.21
## log.pct.below.pov:regionW  -3948.57  913.22  -4.32    0.00

par(mfrow=c(2,2))
plot(cdi.subsets.final.model.with.some.region)

```

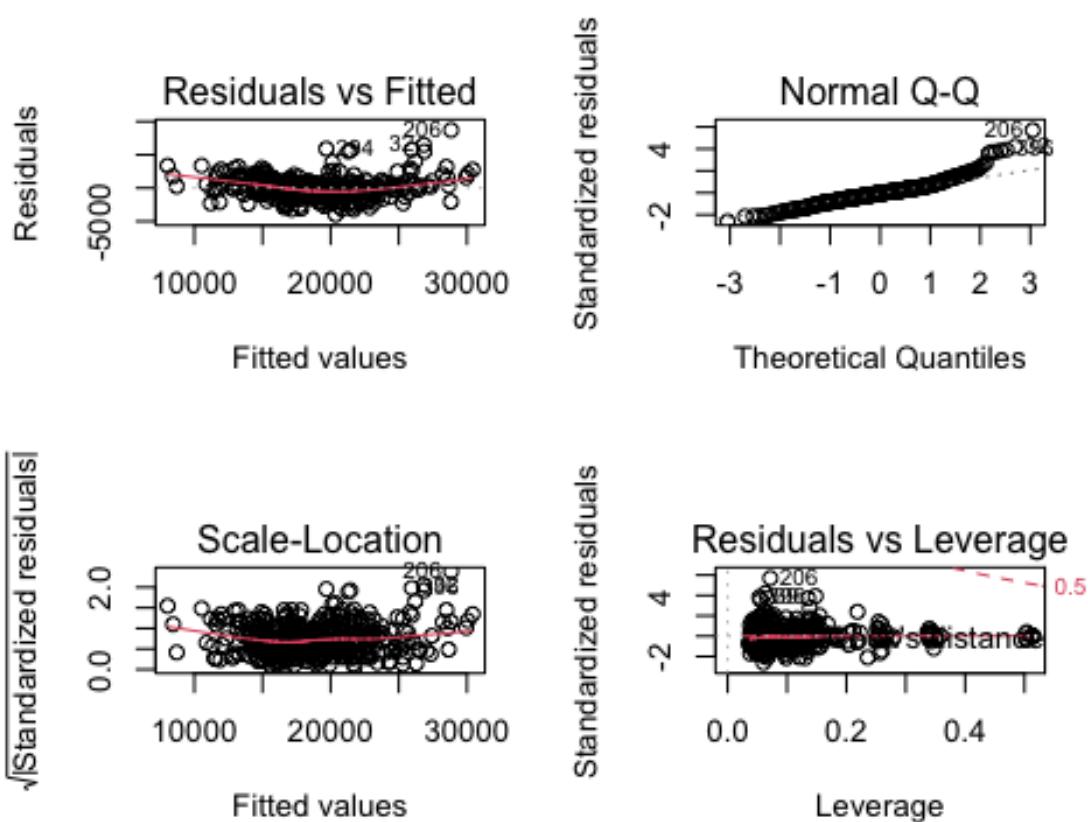


```

plot(final.model.with.state)

## Warning: not plotting observations with leverage one:
##    73, 232, 233, 339, 356, 388, 429

```



```
vif(final.model.with.state)

##                                     GVIF Df GVIF^(1/(2*Df))
## log.land.area      2.206895  1    1.485562
## log.pop.18_34     1.956279  1    1.398671
## log.doctors       1.936019  1    1.391409
## pct.hs.grad.3    7.181496  1    2.679831
## log.pct.bach.deg 7.361002  1    2.713117
## log.pct.below.pov 4.167370  1    2.041414
## log.pct.unemp     4.615294  1    2.148323
## state            32.180767 47   1.037620

vif(cdi.subsets.final.model.with.some.region)

##                                     GVIF Df GVIF^(1/(2*Df))
## log.land.area      1.680366e+00  1    1.296289
## log.pop.18_34      9.161875e+00  1    3.026859
## log.doctors        1.931049e+00  1    1.389622
## pct.hs.grad.3     2.948870e+01  1    5.430350
## log.pct.bach.deg  2.157800e+01  1    4.645212
## log.pct.below.pov 1.271309e+01  1    3.565542
## log.pct.unemp     2.265215e+00  1    1.505063
## region           4.593495e+08   3    27.777333
## log.pop.18_34:region 1.183970e+09  3    32.525461
```

```

## pct.hs.grad.3:region      6.001590e+05  3      9.184265
## log.pct.bach.deg:region  2.276982e+07  3     16.835523
## log.pct.below.pov:region 1.275326e+05  3     7.094747

anova(cdi.subsets.final.model, cdi.subsets.final.model.with.some.region, final.model.with.state)

## Analysis of Variance Table

## Model 1: per.cap.income ~ log(land.area) + log(pct.unemp) + log(pct.below.pov) +
##           log(pct.bach.deg) + (pct.hs.grad)^3 + log(doctors) + log(pop.18_34)
## Model 2: per.cap.income ~ log.land.area + log.pop.18_34 + log.doctors +
##           pct.hs.grad.3 + log.pct.bach.deg + log.pct.below.pov + log.pct.unemp +
##           region + log.pop.18_34:region + pct.hs.grad.3:region + log.pct.bach.deg:region +
##           log.pct.below.pov:region
## Model 3: per.cap.income ~ log.land.area + log.pop.18_34 + log.doctors +
##           pct.hs.grad.3 + log.pct.bach.deg + log.pct.below.pov + log.pct.unemp +
##           state

##   Res.Df       RSS Df Sum of Sq      F    Pr(>F)
## 1    432 1314204306
## 2    417 1114159243 15 200045063 5.3058 1.037e-09 ***
## 3    385  967714409 32 146444834 1.8207  0.005001 **
## ---
## Signif. codes:  0 '****' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

AIC(cdi.subsets.final.model, cdi.subsets.final.model.with.some.region, final.model.with.state)

##                               df      AIC
## cdi.subsets.final.model          9 7826.944
## cdi.subsets.final.model.with.some.region 24 7784.286
## final.model.with.state          56 7786.282

BIC(cdi.subsets.final.model, cdi.subsets.final.model.with.some.region, final.model.with.state)

##                               df      BIC
## cdi.subsets.final.model          9 7863.725
## cdi.subsets.final.model.with.some.region 24 7882.369
## final.model.with.state          56 8015.141

detach(cdi)

```

