Suppose a longitudinal rating design to be analyzed with the HRM has M time points with ideal and observed ratings at time m nested within each θ_{im}, where θ_{im} is the trait for examinee i at time point m, $(m = 1, \ldots, M)$. Now, instead of assuming some normal distribution for the traits, we model them using a longitudinal model, which is part trend and part autoregressive time series model of order 1, meaning that each time point is modeled using information only from the most previous time point. The other levels are the same but now we have a set of ideal ratings and observed ratings at each timepoint m. Item parameters and rater parameters remain the same over time.

$$\begin{array}{rcl} \theta_{im} & \sim & \text{longitudinal model [AR(1) + trend]} \\ \xi_{imj} \mid \theta_{im} & \sim & \text{polytomous IRT model, } j = 1, \ldots, J, \text{ for each } i,\, m \\ X_{imjr} \mid \xi_{ijm} & \sim & \text{polytomous signal detection model, } r = 1, \ldots, R, \text{ for each } i,\, j,\, m. \end{array} \tag{4}$$

In order to estimate θ_{im}, let the trait for subject i at time point m be a function of two quantities:

$$\theta_{im} = \delta_m + Z_{im} \tag{5}$$

Here, δ_m is the trend in θ_{im} at time m and Z_{im} is the time series component of the model. We can decompose δ_m into a function of the parameter g for overall growth, depending on the type of trend. A linear trend is a straightforward and reasonable approach to modeling linear relationships (whether positive or negative). If a linear trend, for example, then $\delta_m = g \times (m-1)/(M-1)$. Z_{im} decomposes into $Z_{im} = \rho Z_{i(m-1)} + \left(\sqrt{1-\rho^2}\right) \varepsilon_{im}$, where $Z_{i(m-1)}$ the lagged value of Z_{im} is weighted by ρ the autocorrelation parameter. In the second term the random noise for examinee i at time point m denoted is distributed as and is weighted by a function of the autocorrelation $\sqrt{1-\rho^2}$. Weighting ε_{im} assures stationary variance of the noise (and therefore the resultant traits) across M time points. Together, the sum of the two weighted quantities in Z_{im} makes up the component of θ_{im} that incorporates the individual's trait information; the other component incorporates an average trend for all N examinees.