A Quick(-ly Written) Guide To Combining Mathematics With English

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Disclaimer

The English grammar/syntax terminology here is largely what I remember from school days, and may not be exactly the same terminology that you would find in a grammar textbook. I've placed an asterisk (*) by terminology that is particularly unikely to be found in a textbook.

By the same token, I have chosen terminology for fragments of mathematics that facilitates an understanding of their roles in English sentences, rather than carefully describing mathematical syntax.

1 Some Naive English Grammar

Here is a naive, partial account of English syntax/grammar:

- The simplest unit in English above the level of individual words is the *phrase*.
  - A phrase is a sequence of one or more words.
  - Although there are many kinds of phrases in English, the three that will concern us most here are *noun phrases*, *descriptor* phrases*, and *verb phrases*. For our purposes,
    - A noun phrase consists of a noun together with zero or more modifiers (e.g. adjectives and adverbs), for example

      dog
      brown dog
      the big shaggy brown dog
    - A descriptor* phrase consists of one or more adjectives and adverbs.

      brown
      big and shaggy
      tired
      really tired
      lazy
      dead
    - A verb phrase consists of a verb together with zero or more modifiers (e.g. adverbs), followed by zero or more noun phrases or descriptor phrases, for example

ate
ate hungrily
hungrily ate
ate the meat
ate the spoiled meat
is
is my mother
is brown

- The next simplest unit in English is the *clause*. According to Wikipedia, a clause is the smallest grammatical unit that can express a complete proposition.

  A common, simple type of clause consists of a noun phrase and a verb phrase. The noun phrase is the *subject* of the sentence, and the verb phrase is the *predicate*, describing the action or state of the subject. If the verb is transitive, the noun phrase inside the verb phrase will be the *object* or recipient of the action of the verb, for example:

$$\underbrace{\textit{The dog}}_{\text{subject}} \quad \underbrace{\underbrace{\textit{ate}}_{\text{transitive verb}} \quad \underbrace{\textit{the meat.}}_{\text{object}}}_{\text{predicate}} \tag{1}$$

  If the verb is intransitive, the following words may be a descriptor phrase describing some feature(s) of the subject, like this:

$$\underbrace{\textit{The dog}}_{\text{subject}} \quad \underbrace{\underbrace{\textit{is}}_{\text{intransitive verb}} \quad \underbrace{\textit{brown.}}_{\text{descriptor}^*}}_{\text{predicate}} \tag{2}$$

  or a noun phrase that is equivalent to the subject, like this:

$$\underbrace{\textit{That dog}}_{\text{subject}} \quad \underbrace{\underbrace{\textit{is}}_{\text{intransitive verb}} \quad \underbrace{\textit{my mother.}}_{\text{equivalent noun phrase}^*}}_{\text{predicate}} \tag{3}$$

  or there may be no words following, if the verb adequately describes the state or action of the subject all by itself:

$$\underbrace{\textit{The dog}}_{\text{subject}} \quad \underbrace{\underbrace{\textit{ate.}}_{\text{intransitive verb}}}_{\text{predicate}} \tag{4}$$

  NOTES:

  1. There are many more kinds of clauses and phrases than this, but this will be enough for now.
  2. The grammatical category that a word or phrase or other unit of English belongs to usually depends on its function or context in the sentence in which it appears, rather than any inherent quality of that unit. For example sentences (1) and (4) show that the verb "ate" can be transitive or intransitive. As another example, we will see below in sentences (8) and (9) that the same sequence of words can be a clause in one sentence but not in another.

- A *sentence* consists of one or more clauses. Clauses may be introduced by special words called prepositions, conjunctions, conjunctive adverbs, etc., and are often set off by punctuation marks such as periods, commas, colons, semicolons, etc. (Many of these punctuation marks may be used to set off other units [words, phrases, etc.] as well). Examples (1) through (4) are all examples of one-clause sentences. Here are some examples with one, two or three clauses:

$$\underbrace{\textit{Before}}\ \underbrace{\textit{I left Philadelphia,}}_{\text{Clause 1}}\ \underbrace{\textit{I packed my suitcase.}}_{\text{Clause 2}} \tag{5}$$

$$\underbrace{\textit{First}\ \underbrace{\textit{I do the research,}}_{\text{Clause 1}}\ \textit{then}\ \underbrace{\textit{I figure out how}\ \underbrace{\textit{I can organize it into a paper.}}_{\text{Clause 3}}}_{\text{Clause 2}}}_{} \tag{6}$$

$$\underbrace{\textit{The car has four wheels,}}_{\text{Clause 1}}\ \textit{and}\ \underbrace{\textit{it goes fast.}}_{\text{Clause 2}} \tag{7}$$

$$\underbrace{\textit{Since}}\ \underbrace{\textit{the dog ate the meat,}}_{\text{Clause 1}}\ \underbrace{\textit{there's none left for me.}}_{\text{Clause 2}} \tag{8}$$

$$\underbrace{\underbrace{\textit{The sentence}\ \underbrace{\textit{“The dog ate the meat.”}}_{\text{Not a clause}^1}}_{\text{noun phrase (subject)}}\ \underbrace{\underbrace{\textit{has}}_{\text{verb}}\ \underbrace{\textit{only one clause.}}_{\text{noun phrase}}}_{\text{verb phrase (predicate)}}}_{\text{A single clause}} \tag{9}$$

Sentences (5) and (6) are just examples of typical multiple-clause sentences in English. Sentence (5) is a fairly simple linear joining of two clauses, with Clause 1 providing temporal context for Clause 2. Sentence (6) shows that clauses can also be nested within other clauses.

Sentences (7), (8) and (9) are examples that will be models for some of the ways of including mathematical expressions in English sentences. As we go forward, imagine replacing tne word “car” in sentence (7), the clause “The dog ate the meat” in sentence (8), and the sequence of words “The dog ate the meat” in sentence (9), with various kinds of mathematical expressions. What kind(s) of expressions fit in each of the sentences when you read them aloud? What kind(s) don't fit?

2 A Naive & Incomplete Taxonomy of Mathematical Expressions

Mathematical expressions can be arbitrarily complex, and stand for quite intricate objects, operations and relations. For the purpose of embedding mathematical expressions in English sentences however, there are really only two classes of expressions. The following is not quite standard terminology, but it will do for this little note:

[1] Instead, “The dog ate the meat.” functions as one of the two nouns in the compound noun “The sentence ‘The dog ate the meat.’”. It plays the same role as the word “Joe” in the compound noun “The boy Joe”.

- *Terms* are labels for objects. There can be *simple terms* like variable names, function names, constants, etc., such as

$$a,\ A,\ \mathcal{B},\ x,\ \alpha_{ij}^{+},\ 23,\ f(\cdot),\ \ldots$$

and there can be *compound terms* that combine simple terms with operators, grouping symbols, relations, etc., again to label single objects, such as

$$E[Y],\ P[Z > 2], \{x \in \mathcal{R} : |x| < 2\},\ a + b,\ \int_3^{\infty} 1/t^2 dt,\ \frac{d}{dx} x \sin(x^2),\ \beta^5,\ f(27.5),\ \ldots$$

Terms, whether simple or compound, play the role of nouns and noun phrases in English.

- *Relationships* combine simple or complex terms with mathematical relations. Examples include

$$\begin{aligned} 2+2 &= 4 \\ \int_0^1 x^2 dx &= \frac{1}{3} \\ a+b &\leq c \\ x &\in \mathcal{R} \\ A &\subseteq S \\ p &\Rightarrow q \\ &\text{etc.} \end{aligned}$$

Relations play the role of clauses that involve the verb "to be", in English.

Inequalities and *equations* are special cases of relations that assert inequality or identity, respectively, between terms.

3 Mathematical Expressions within English

As you might guess from the brief treatment of mathematical expressions above, there are really only two kinds of expressions to worry about, and they have a simple relationship with units of English sentences:

In mathematics		in English
Terms	correspond to	noun phrases
Relationships	correspond to	"is" clauses

So, the simple prescription is, incorporate each mathematical expression into an English sentence as you would any other noun phrase or clause, using punctuation and/or special words (conjunctions, prepositions, conjunctive adjectives, etc.) to glue everything together. We will see some examples below.

Before we see the examples, though, there are two caveats to keep in mind:

1. A mathematical expression should never be a sentence on its own. It must always be a noun phrase or clause within a larger English sentenve.

2. A mathematical expression should never be the very beginning of an English sentence.

Both caveats have to do with reader expectations: readers expect sentences to start with English and not mathematics (and so, in fact, caveat #2 subsumes caveat #1); in addition, you can use the English in the sentence to remind the reader of how to interpret the mathematics, to give an argument involving many mathematical expressionos some "flow", to give the reader mnemonics for remembering certain expressions, etc. If it is most natural to put the mathematical expression first in the sentence, at the very least we can make the mathematical expression be the second noun in a compound noun.

Examples: Terms as nouns and noun phrases

- Single term (noun/noun phrase), displayed on a separate line:

  We see that in order to evaluate

  $$\int \log x \, dx \ ,$$

  we must understand something of freshman calculus.

- Two terms (noun phrases), connected by "is":

  The value of $(a+b)^2$ *is 4.*

- Compound noun to avoid beginning a sentence with math:

  The expression $(a+b)^2$ *is non-negative.*

Examples: Relationships as "is" clauses

- In-line clauses:

  Since $(a+b)^2 \geq 0$*, it must follow that* $ab \leq (a^2+b^2)/2$*.*

- Clause as a display "equation":

  Taking the square root of the double integral, we see that

  $$\int_{-\infty}^{\infty} e^{-x^2} \, dx = \sqrt{2\pi} \ .$$

  Note that punctuation is carried along to displayed expressions, just as if they were in-line.

Examples: Miscellaneous combinations

- Relationship as inline clause (like sentence (8) above):

  Since $(a+b)^2 > 0$*, the proof is complete.*

- Relationship functioning as a noun phrase rather than as a clause (like sentence (9) above):

  The inequality $(a+b)^2 \geq 0$ *is true for all* $a, b \in \mathcal{R}$*.*

  It would in some sense be clearer to write

  The inequality "$(a+b)^2 \geq 0$*" is true for all* $a, b \in \mathcal{R}$*.*

  just as we did in sentence (9), but writing conventions in mathematics disfavor that (presumably because in more complex expressions, the quotes might be a distraction).