

A Quick(-ly Written) Guide To Combining Mathematics With English

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Disclaimer

The English grammar/syntax terminology here is largely what I remember from school days, and may not be exactly the same terminology that you would find in a grammar textbook. I've placed an asterisk (*) by terminology that is particularly unlikely to be found in a textbook.

By the same token, I have chosen terminology for fragments of mathematics that facilitates an understanding of their roles in English sentences, rather than carefully describing mathematical syntax.

1 Some Naive English Grammar

Here is a naive, partial account of English syntax/grammar:

- The simplest unit in English above the level of individual words is the phrase.
 - A phrase is a sequence of one or more words.
 - Although there are many kinds of phrases in English, the three that will concern us most here are noun phrases, descriptor* phrases, and verb phrases. For our purposes,
 - * A noun phrase consists of a noun together with zero or more modifiers (e.g. adjectives and adverbs), for example
 - dog*
 - brown dog*
 - the big shaggy brown dog*
 - * A descriptor* phrase consists of one or more adjectives and adverbs.
 - brown*
 - big and shaggy*
 - tired*
 - really tired*
 - lazy*
 - dead*
 - * A verb phrase consists of a verb together with zero or more modifiers (e.g. adverbs), followed by zero or more noun phrases or descriptor phrases, for example

ate
ate hungrily
hungrily ate
ate the meat
ate the spoiled meat
is
is my mother
is brown

- The next simplest unit in English is the clause. According to Wikipedia, a clause is the smallest grammatical unit that can express a complete proposition.

A common, simple type of clause consists of a noun phrase and a verb phrase. The noun phrase is the subject of the sentence, and the verb phrase is the predicate, describing the action or state of the subject. If the verb is transitive, the noun phrase inside the verb phrase will be the object or recipient of the action of the verb, for example:

$$\begin{array}{ccccc}
 \underbrace{\textit{The dog}} & & \underbrace{\textit{ate}} & & \underbrace{\textit{the meat.}} \\
 \text{subject} & & \text{transitive verb} & & \text{object} \\
 & & \underbrace{\hspace{1.5cm}} & & \\
 & & \text{predicate} & &
 \end{array} \tag{1}$$

If the verb is intransitive, the following words may be a descriptor phrase describing some feature(s) of the subject, like this:

$$\begin{array}{ccccc}
 \underbrace{\textit{The dog}} & & \underbrace{\textit{is}} & & \underbrace{\textit{brown.}} \\
 \text{subject} & & \text{intransitive verb} & & \text{descriptor*} \\
 & & \underbrace{\hspace{1.5cm}} & & \\
 & & \text{predicate} & &
 \end{array} \tag{2}$$

or a noun phrase that is equivalent to the subject, like this:

$$\begin{array}{ccccc}
 \underbrace{\textit{That dog}} & & \underbrace{\textit{is}} & & \underbrace{\textit{my mother.}} \\
 \text{subject} & & \text{intransitive verb} & & \text{equivalent noun phrase*} \\
 & & \underbrace{\hspace{1.5cm}} & & \\
 & & \text{predicate} & &
 \end{array} \tag{3}$$

or there may be no words following, if the verb adequately describes the state or action of the subject all by itself:

$$\begin{array}{ccccc}
 \underbrace{\textit{The dog}} & & \underbrace{\textit{ate.}} & & \\
 \text{subject} & & \text{intransitive verb} & & \\
 & & \underbrace{\hspace{1.5cm}} & & \\
 & & \text{predicate} & &
 \end{array} \tag{4}$$

NOTES:

1. There are many more kinds of clauses and phrases than this, but this will be enough for now.
2. The grammatical category that a word or phrase or other unit of English belongs to usually depends on its function or context in the sentence in which it appears, rather than any inherent quality of that unit. For example sentences (1) and (4) show that the verb “ate” can be transitive or intransitive. As another example, we will see below in sentences (8) and (9) that the same sequence of words can be a clause in one sentence but not in another.

- Terms are labels for objects. There can be simple terms like variable names, function names, constants, etc., such as

$$a, A, \mathcal{B}, x, \alpha_{ij}^+, 23, f(\cdot), \dots$$

and there can be compound terms that combine simple terms with operators, grouping symbols, relations, etc., again to label single objects, such as

$$E[Y], P[Z > 2], \{x \in \mathcal{R} : |x| < 2\}, a + b, \int_3^\infty 1/t^2 dt, \frac{d}{dx} x \sin(x^2), \beta^5, f(27.5), \dots$$

Terms, whether simple or compound, play the role of nouns and noun phrases in English.

- Relationships combine simple or complex terms with mathematical relations. Examples include

$$\begin{aligned} 2 + 2 &= 4 \\ \int_0^1 x^2 dx &= \frac{1}{2} \\ a + b &\leq c \\ x &\in \mathcal{R} \\ A &\subseteq S \\ p &\Rightarrow q \\ \text{etc.} \end{aligned}$$

Relations play the role of clauses that involve the verb “to be”, in English.

Inequalities and equations are special cases of relations that assert inequality or identity, respectively, between terms.

3 Mathematical Expressions within English

As you might guess from the brief treatment of mathematical expressions above, there are really only two kinds of expressions to worry about, and they have a simple relationship with units of English sentences:

In mathematics		in English
Terms	correspond to	noun phrases
Relationships	correspond to	“is” clauses

So, the simple prescription is, incorporate each mathematical expression into an English sentence as you would any other noun phrase or clause, using punctuation and/or special words (conjunctions, prepositions, conjunctive adjectives, etc.) to glue everything together. We will see some examples below.

Before we see the examples, though, there are two caveats to keep in mind:

1. A mathematical expression should never be a sentence on its own. It must always be a noun phrase or clause within a larger English sentence.
2. A mathematical expression should never be the very beginning of an English sentence.

Both caveats have to do with reader expectations: readers expect sentences to start with English and not mathematics (and so, in fact, caveat #2 subsumes caveat #1); in addition, you can use the English in the sentence to remind the reader of how to interpret the mathematics, to give an argument involving many mathematical expressions some “flow”, to give the reader mnemonics for remembering certain expressions, etc. If it is most natural to put the mathematical expression first in the sentence, at the very least we can make the mathematical expression be the second noun in a compound noun.

Examples: Terms as nouns and noun phrases

- Single term (noun/noun phrase), displayed on a separate line:

We see that in order to evaluate

$$\int \log x \, dx ,$$

we must understand something of freshman calculus.

- Two terms (noun phrases), connected by “is”:

The value of $(a + b)^2$ is 4.

- Compound noun to avoid beginning a sentence with math:

The expression $(a + b)^2$ is non-negative.

Examples: Relationships as “is” clauses

- In-line clauses:

Since $(a + b)^2 \geq 0$, it must follow that $ab \leq (a^2 + b^2)/2$.

- Clause as a display “equation”:

Taking the square root of the double integral, we see that

$$\int_{-\infty}^{\infty} e^{-x^2} \, dx = \sqrt{2\pi} .$$

Note that punctuation is carried along to displayed expressions, just as if they were in-line.

Examples: Miscellaneous combinations

- Relationship as inline clause (like sentence (8) above):

Since $(a + b)^2 > 0$, the proof is complete.

- Relationship functioning as a noun phrase rather than as a clause (like sentence (9) above):

The inequality $(a + b)^2 \geq 0$ is true for all $a, b \in \mathcal{R}$.

It would in some sense be clearer to write

The inequality “ $(a + b)^2 \geq 0$ ” is true for all $a, b \in \mathcal{R}$.

just as we did in sentence (9), but writing conventions in mathematics disfavor that (presumably because in more complex expressions, the quotes might be a distraction).