Elements of (Proof) Style

1. **Proof is in the eye of the beholder.** The burden of communication lies entirely on the prover. Picture, if you will, a very skeptical (if not actively hostile) jury whom you must convince of the truth of your argument. The logic must be airtight. The language must be crystal clear. There is no benefit of the doubt.

2. **Never 'Mind the gap!'** The truth of each statement must follow directly from the previous one(s), with sufficient explanation to justify the transition.

3. **What you say is what you mean.** Mathematics is a very precise language. Once you know *what* you want to say, you must then figure out exactly *how* to say it. If you want to communicate that A is a subset of B but write $A \in B$, then what you have (mistakenly) said is that A is an element of B, which means an entirely different thing.

4. **Follow the outline.** There is a standard methodology for many types of proofs. Learn them. Use them. Love them.

5. **Use your words.** A proof is much closer to a literature essay than a computer program. Like essays, a proof must be written in clear, complete, grammatically correct sentences. English words should predominate. Mathematical symbols should be used when they are more concise, such as for equations (like $a^3 = b^{-1}$), inequalities (like $m < 5$) and other relations (such as $5 \mid x^2$ or $a \in \mathbb{Z}$), but never at the expense of clarity.

6. **Connect the dots.** Use words like "therefore," "thus," "consequently", or "hence" to communicate conclusions. Use words like "assume," "suppose," or "let" to communicate assumptions. Use "if. . . , then. . . " or ". . . if and only if . . . " to directly link assumptions and conclusions. Use words like "because," "since," or "by" to communicate justifications.

7. **Stake your claim.** Every proof is preceeded by a clear statement of the proposition to be shown. Label it accordingly.

8. **Lay the foundation.** Every proof begins with the assumptions, most often in the form of "Let. . . ."

9. **Tie it up with a bow.** Every proof ends with a clear statement of the proposition that has been shown. QED.

10. **Name, rank, and serial number.** Every new mathematical object needs a proper introduction. It must have a name such as "δ" or "A." It must be clearly quantified when specifying its properties, as in "Let A be a nonempty subset of $\mathbb{R}$." (universal quantification over all nonempty sets of real numbers) or "Choose $\delta = \sqrt{\epsilon + 2}$." (existential quantification of δ depending on a previously introduced ϵ). Once introduced, its name, quantification, and properties are fixed for the rest of the proof.

11. **Order matters.** Write strings of (in)equalities to communicate their purpose.

12. **Be conventional.** Proofs in mathematics adhere to many standards such as:

    - Never start a sentence with a symbol.
    - Logical quantifiers and logical connectives are used only when discussing (basic) logic.
    - Capital letters designate sets, while lower case ones represent elements. Sets of sets often use script like $\mathcal{P}(A)$ for the power set of a set A.
    - Functions are usually denoted f, g, h. Integers are usually denoted i, j, k when indexing and m, n when fixed. The letters x, y, z are general variables.

    Pay attention to the proofs in the book, in class, and online to learn these and other conventions.

13. **Formatting matters.** Clearly label claims, subclaims, etc. as well as the beginning and end of their proofs. Break a proof into short paragraphs. Don't string together to many mathematical symbols in a row. Give long equations their own line. (Seriously consider learning to use LaTeX.)

14. **Don't get caught in a loop.** No step in your proof can depend on the conclusion (in any form). Also, be very careful using the results from other problems in the same assignment so that you don't set up a circular dependency.

15. **Give credit where credit is due.** Cite any outside resource (website, textbook, other homework assignment, ...) which contributed to your proof. Acknowledge any person who communicated with you about the proof.

16. **Prove with examples only when existential.** Examples (and counter-examples) are crucial to understanding mathematics; every mathematician has a stable full of her favorites. However, they can only be used *to prove* existential statements, such as the negation of a universal. When giving an example or counterexample, it must be completely specified. It is not sufficient to say merely "the function f could be one-to-one but not onto." You must give the domain, range, and mapping which defines f and show that it is one-to-one but not onto. You must also prove why this makes the existential statement true.

17. **Use only the tools on hand.** When told to prove something "from the definition," that is what you *must* do — even if there may be an alternative way. In general, you may only use results which have been proved in the book, in class, or in previously assigned problems. All other results must be proved first before you can use them.

18. **"Writing without revising is the literary equivalent of waltzing gaily out of the house in your underwear." – P. Fuller** Finding a proof is a almost always a long, messy process, full of false starts, dead ends, and unproductive detours. *None* of that belongs in a proof. A proof is the final, polished version of your understanding, *not* a record of your stream of consciousness. (This is why typesetting your proofs saves time and energy in the long run.)

19. **Don't get lost in the maze.** Writing a proof is like giving directions for how to get through a maze; it must start at the beginning, and walk step by step through until the end. It doesn't matter how you figured out the solution to the maze — whether you started at the end and worked back to the begining, or from the middle with lots of backtracking, or .... It only matters whether your reader gets to the end safely.

Inspired by the inimitable Struck & White, and loosely based on "Elements of Style" by Anders Hendrickson, Concordia College.