

THE USE OF THE MULTIPLICITY SAMPLING PROCEDURE
IN A STUDY OF MIGRANT AGRICULTURAL WOMEN

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ABSTRACT

The study of health problems among childbearing-age women in the families of migrant workers presents sampling difficulties. The difficulty lies in the identification of the total population of childbearing-age migrant women. This paper presents Sirken's multiplicity sampling procedure as a solution to this problem, and examines the effectiveness of the method. This paper also estimates the average number of live births for childbearing-age women in the families of Wisconsin migrant workers from data collected with the multiplicity method of sampling.

I. Introduction

Health problems and living conditions of migrant workers and their family members are often ignored, partly because they are numerically a small number, partly because they are not considered residents of any one state, and partly because they frequently change their geographic location. For these, and other reasons, there is much difficulty in making a comprehensive study of the workers. Thus, migrant workers and their family members may be considered a group of people who need serious attention and help from the public and government agencies. One group for which literally no comprehensive data exist is migrant women. The purpose of the Wisconsin Migrant Agricultural Women Study is to obtain basic information about health problems and living conditions of childbearing-age women in the families of migrant workers.

To study a representative sample of migrant women of childbearing ages involves even greater difficulty than a study of migrant workers themselves, since we can identify no total population of childbearing-age migrant women to sample. In such a situation Sirken proposes the "multiplicity" sampling method. In this paper we shall, first, explain the multiplicity method in sampling; second, describe the Wisconsin Migrant Women Study as an example of the application of this method; and finally, evaluate the method.

II. Multiplicity Method in Sampling

In many situations a researcher does not have the framework of a population about which he tries to estimate some properties. The lack of information about the framework of the population makes it impossible to create a systematic sampling procedure of the elements in the population. In such situations substitutions are often made by using a framework directly or indirectly related to the elements in the population. For example, in order to estimate the number of diabetics in a nation, households can be selected as a sample frame. A researcher may randomly select households, and then collect information from diabetics found in the selected households or from diabetics who are related to the selected households (Sirken et al., 1975). In order to estimate the number of people with neurological problems, clinics and hospitals may be selected as a sample frame (Sirken, 1975). In these examples, households, clinics, or hospitals are called "enumeration units" in sample surveys, while diabetics and people with neurological problems are called "population elements" whose parameters are being estimated. The multiplicity method in sampling can be used when enumeration units in sample surveys are not identical to the population elements (Sirken, 1974). When different units are used to select population elements, it is essential to link the two distribu-

tions. In other words, it becomes essential to estimate the probabilities of selecting population elements from the probabilities of selected enumeration units.

One way of estimating the probabilities of selecting population elements is to assume that the probability of selecting one enumeration unit is the same as the probability of selecting all the population elements found in the enumeration unit. For example, suppose one out of ten households is selected to estimate the number of diabetics. Then, if we find two diabetics in one household, the probability of selecting two diabetics from the population is assumed to be the same as the probability of selecting the household, namely, one tenth. This method is called a conventional method, which limits the diabetics in each household to be enumerated only once at his de jure residence (Sirken, et al., 1975). However, this method tends to have a large coverage error, since the selection of households limits the degree of coverage in finding diabetics. Consequently, it tends to have a large sampling error.

The multiplicity method in sampling is a method to correct the limitation of coverage in the conventional method. By expanding the source of information beyond enumeration units and by creating a network of information on the basis of enumerated units, the multiplicity method tries to expand the

coverage of sampling and to reduce the sampling error. For example, instead of simply finding the number of diabetics in each selected household, a question can be asked how many siblings the head of a household has, and whether his siblings are diabetics. By expanding the network of information from the residents of each household to the siblings of the head of the household, the study can increase the coverage of information. However, by doing so the same diabetic person can be enumerated at more than one household and thus he may be enumerated more than once at the same household (Sirken et al., 1975). If we can assume that the siblings of the head of a selected household are also the head of other households, each diabetic is eligible to be counted more than once. This situation is called multiplicity. The multiplicity of a person is defined as the total number of times a person is eligible to be enumerated (Sirken et al., 1975:659). In other words, the probability of selecting each diabetic differs depending on the different situation of each household. Such probabilities can be calculated by specifying rules for multiplicity and linking population elements to enumeration units. The inverse of the probabilities of selecting each population element can be used as weights, and unbiased estimators of population parameters can be obtained by appropriately weighting the selected population

elements (Sirken, 1974). The multiplicity estimator is unbiased if the sum of the weights assigned to the multiple linked elements is equal to unity (Sirken, 1974).

Estimation of a population parameter by using a multiplicity method in sampling involves three steps: 1) selecting a probability sample of enumeration units, 2) counting the number of times population elements are eligible to be counted in the same enumeration units, and 3) weighting each population element by the inverse of its chance of being selected in the sample.

In the first step, sampling weights (W_s) are determined as the inverse of the probabilities of selecting enumeration units. In the second step, weights are determined by creating a counting rule that identifies the number of times a population element is eligible to be counted. The objective of the counting rule strategy is to select the optimum rule for linking population elements to enumeration units and to assign the optimum sets of weights to enumeration units that are linked to the same population elements (Sirken, 1974). Since the counting rule specifies the conditions for linking population elements to enumeration units, the distribution of the population elements among the enumeration units is a function of the rule. Changing the counting rule modifies the population distribution and thereby alters the sampling distribution and the sampling variables of survey estimates

(Sirken, 1974). On the basis of the counting rule, counting rule weights (W_c) are determined. Counting rule weights assigned to a person depend upon the particular counting rule adopted in the survey. Since the weights are usually unknown, information collected in the survey is used to estimate the counting rule weights. Then, in the third step, weights (W) of selected population elements can be calculated as the product of the sampling weight and the counting rule weight. Namely,

$$W = W_s \times W_c$$

In other words, W is the inverse of the probability of each selected population element derived from the probabilities of selecting enumeration elements. By using W as a weighting factor, the multiplicity method in sampling can aid in estimating population parameters from samples of enumeration units (Sirken et al., 1975).

For example, in the study of diabetics mentioned above, the first step is to select a random sample of households. Suppose one out of ten households is selected--then the probability of selecting each household is $1/10$. Consequently, the sampling weight, which is the inverse of the probability of selecting each household, becomes 10. The second step is to count persons with diabetes in the sample households. The counting rule weight (W_c) is as follows:

$$W_c = \frac{\text{\# of times the person is eligible to be enumerated in the household}}{\text{the multiplicity of the person}}$$

Suppose the head of a selected household, Person A, has three brothers, Person B, Person C, and Person D. Among these four people, only Person B is reported to be a diabetic. In this situation W_c becomes $1/4$, since Person B is eligible to be enumerated only once in the household, while the multiplicity of the person is four times. To explain in a different manner, Person A will indicate that Person B is a diabetic. Person B, if his household is selected, will answer that he is a diabetic. Similarly, Person C and Person D, if they are selected, will answer that Person B is a diabetic among his siblings. In other words, Person B is eligible to be mentioned four times, while he should be counted only once as a sample. Thus, by weighting $W_c = 1/4$ the problem of multiplicity can be corrected. In the third step, W can be calculated by multiplying W_s and W_c , resulting in $W = 10 \times 1/4 = 10/4$. In other words, the probability of selecting Person B relative to the sampling scheme of households with the multiplicity method is calculated to be $4/10$. Using W as weights applied to each selected diabetic, the population of diabetics can be estimated.

Therefore, the multiplicity method in sampling allows us to estimate population parameters for a wide range of coverage

of population elements for a rare event, with fewer sampling points and a lower sampling error, when only enumeration units are available as a means of selecting population elements.

III. The Multiplicity Method in Sampling as Used in the Wisconsin Migrant Women Study

Data for the Wisconsin Migrant Women Study were collected as a sub-sample of the Wisconsin Migrant Worker Study. The purpose of the former study is to estimate some properties of childbearing-age migrant women, while the purpose of the latter study is to estimate some properties of migrant workers. Even though the framework of the population of migrant workers in Wisconsin could be established by obtaining lists of every employed migrant worker from each employer, to establish a sampling framework of childbearing-age migrant women was impossible. Since the Wisconsin Migrant Worker Study can establish a systematic sampling framework, the Wisconsin Migrant Women Study uses the sampling elements of the Wisconsin Migrant Worker Study as enumeration units, and establishes a multiplicity method of sampling in order to find the properties of childbearing-age migrant women in the families of migrant workers. Thus, in this study enumeration units are migrant workers, and population elements are childbearing-age migrant women found in the households of migrant workers.

Estimation of a population parameter by using a multiplicity sampling method involves three steps: 1) selecting a probability sample of enumeration units, 2) counting the number of times a population element is eligible to be counted in the same enumeration unit, and 3) weighting each population

element by the inverse of its chance of being enumerated in the sample. We shall explain each step in the Wisconsin Migrant Women Study.

1) Selecting a Probability Sample of Enumeration Units

A ten percent stratified random sample of all migrant agricultural workers age sixteen or older was planned to be sampled as enumeration units. Employers were used as strata. The lists of employers of migrant workers in Wisconsin were obtained from the Job Service unit of the Department of Industry, Labor and Human Relations, and the lists of migrant workers were obtained from each employer. In addition, the area supervisors of Job Service were contacted. These persons knew their specific areas very well, and were able to provide additional local information especially about farms that employed very small numbers of migrant workers. From the obtained list of migrant workers, one out of ten migrant workers were randomly selected in each camp. In cases where an employer had less than ten migrant workers, several camps were combined to obtain 1/10 sampling probability.

As the result, 408 workers were selected, representing about 4,080 migrant workers in Wisconsin in the 1978 season. Of those selected, 262 were interviewed, eight of the remainder refused to be interviewed, and 138 left the camp between the time their names were selected and the time of the inter-

view. This resulted in a response rate of 64%. Among the 262 migrant workers 46 were identified as childbearing-age migrant women.

Since we knew the names, sex, location, and job activity of non-respondents, on the basis of this information, we decided to weigh each interview in each county by a factor which inflated the number of completed interviews to the number of workers sampled. In this process it is assumed that those who are in the county have the same probability of responding to the questions, and that the probability of sample selection is independent of the probability of responding to questions. The weighting factor was determined as follows:

$$\text{Weighting Factor For Each County (Wsc)} = \frac{\text{\# of workers sampled in county}}{\text{\# of workers interviewed in county}}$$

The weighting factor was 1.0 in 12 counties, between 1.1 and 2.0 in 11 counties, and between 2.0 and 3.0 in 7 counties.

Workers were located in 30 counties in the 1978 season, but the 262 workers who were interviewed lived in 27 counties. After consultation with the Bureau of Migrant and Rural Service as to the crop on which the migrants were working and their family structure, it was decided to use a neighboring county with the largest number of respondents to represent workers in these counties. Thus, interviews were obtained from 262 workers in 27 counties, but the final weighted sample represents 408 workers in 30 counties.

Thus, the sampling probability of migrant workers in Wisconsin Migrant Worker Study is 1/10, and the weights derived from the sampling probability is $Wss = 1/(1/10) = 10$. However, since county weights were used to inflate the number of completed interviews to the number of workers sampled, the weights have to be included in sampling weights for the multiplicity method. Even though in a strict sense county weights (Wsc) are not a part of sampling weights, Ws is determined to be the product of Wss and Wsc. Namely,

$$Ws = 10 \times Wsc$$

Thus, Ws is used as the sampling weights.

2) Counting Rule

For the Wisconsin Migrant Women Study, only women under fifty years old who were married or living in a married state ("juntado") are included. Of the 262 workers interviewed in the Wisconsin Migrant Worker Study, 97 were women, but only 46 were married women in the childbearing years. In order to increase the coverage of sampling of childbearing-age migrant women, the multiplicity method of sampling was used. Each respondent in Wisconsin Migrant Worker Study was requested to list members living in his or her household. When women under fifty years old who were married or living in a married state were listed, they were interviewed for the Wisconsin

Migrant Women Study. For example, suppose that one sampled male migrant worker has a household with eight members, two of whom are married women of childbearing-age. The interviewers collected information from these two migrant women. In this process 94 migrant women responded to interviews in addition to the 46 childbearing-age migrant workers sampled as enumeration units. However, because of the multiplicity method of sampling, these two women may be eligible to be enumerated more than once. Suppose in the same household we can find two migrant workers (one in addition to the one who is sampled), then the multiplicity of the women enumerated in the household is counted to be twice. In other words, the sampled migrant worker lists the two migrant women, while the other non-sampled but eligible migrant worker can also indicate the same two migrant women. The two migrant women are eligible to be enumerated twice, although they should be enumerated only once. Thus, the counting rule weight for each of the two women becomes 1/2. Since the sum of multiplicity in the household, $1/2 + 1/2$, is a unity, this method allows us to readjust the probability of selecting childbearing-age women relative to selected migrant workers, and to derive an unbiased estimator of the property of childbearing-age women. However, the verbal responses of family members as to whether they are migrant workers may differ from the obtained list of migrant workers, and this discrepancy may create biases in establishing counting rule weights. Because of difficulty in checking the correspondence between the verbal responses and

the obtained list, we assumed that the verbal response was a reliable source for estimating counting rule weights. Thus, the counting rule weight for each childbearing-age woman in a household is determined to be:

$$W_c = \frac{1}{\begin{array}{l} \text{\# of migrant workers eligible to be included as} \\ \text{a sample of Wisconsin Migrant Worker Study} \end{array}}$$

3. Weighting Population Elements

Since the weighting factor for each population element in the multiplicity sampling method is a product of a sampling weight and a counting rule weight, weights for each sample of childbearing-age migrant women can be calculated to be

$$W = W_s \times W_c.$$

When this weight is used, it is possible to derive an unbiased estimator of properties of childbearing-age migrant women.

Therefore, by using the multiplicity sampling method the Wisconsin Migrant Women Study may establish better estimates of the properties of childbearing-age migrant women in Wisconsin than other methods of sampling.

In order to prove this point, we shall compare the estimates of the average number of children per childbearing-age migrant woman obtained by the multiplicity method of sampling and by the conventional method of sampling.

IV. Comparison Between Multiplicity and Conventional Method

A conventional method of sampling childbearing women in the Wisconsin Migrant Women Study would be to randomly sample workers, and interview those who are childbearing-age women. Thus, due to the lack of sampling framework of the population of childbearing-age women, the most effective and easily available enumeration units may be the lists of migrant workers prepared by every employer of migrant workers in Wisconsin.

Since a conventional method does not expand the coverage of information collection beyond the enumeration units themselves, samples valid to be used for a conventional method are childbearing-age working women found in the samples of migrant workers. Wisconsin Migrant Worker Study found 46 childbearing-age women among 262 sampled migrant workers. In contrast, a multiplicity method collected 145 childbearing-age women by expanding the networks of information gathered from enumeration units to the members of households of migrant workers.

In this section we shall examine the efficiency of the two methods by comparing the standard errors of estimates in one measure, Number of Live Births. Since the conventional method covers only childbearing-age migrant women workers (namely, full-time, part-time, and occasional workers), we shall also select samples of full-time, part-time, and occasional workers in Wisconsin Migrant Women Study for the sake of comparability of statistics. This resulted in selecting 133

women out of 145. Furthermore, we shall not use county weights as a part of sampling weights, partly because county weights are not a part of the sampling procedure, and partly because county weights may obscure the comparability of the conventional and multiplicity methods. We shall first present equations used to calculate standard errors of estimates, and then present the results.

A. Equations

The estimated number of total childbearing-age women and live births are calculated using the probability of a sample being selected in the sampling procedure. The estimated number of total childbearing-age women ($\hat{X}$) is calculated as follows (Raj, 1968):

$$\hat{X} = \frac{1}{n} \sum_{i=1}^n \frac{x_i}{p_i/n} \tag{Formula 1}$$

and the estimated total number of live births for childbearing-age women ($\hat{Y}$) is calculated as follows:

$$\hat{Y} = \frac{1}{n} \sum_{i=1}^n \frac{y_i}{p_i/n} \tag{Formula 2}$$

where

- $\hat{X}$: Estimated Number of Total Childbearing-Age Women
- $\hat{Y}$: Estimated Number of Total Live Births

x_i : Number of Childbearing-Age Women (always $x_i = 1$)
 y_i : Number of Live Births per Childbearing-Age Woman
 p_i : Probability of A Sample Being Selected
 n : Number of Samples.

Similarly, the variance of $\hat{Y}$ is calculated as follows (Raj, 1969):

$$V(\hat{Y}) = \frac{1}{n} \sum_{i=1}^n \frac{p_i}{n} (\hat{Y}_i - \hat{Y})^2 \tag{Formula 3}$$

where

$$\hat{Y}_i = \frac{y_i}{p_i/n}$$

and $V(\hat{Y})$: Variance of the Estimated Number of Live Births.

Since the major interest of this paper is to estimate the average number of live births per childbearing-age migrant woman, a ratio $\hat{R}$ is estimated as follows:

$$\hat{R} = \frac{\hat{Y}}{\hat{X}} \tag{Formula 4}$$

and its variance $V(\hat{R})$ is estimated to be (Raj, 1969):

$$V(\hat{R}) = \frac{V(\hat{U})}{\hat{X}^2} \tag{Formula 5}$$

where $V(\hat{U})$ is estimated to be:

$$\hat{U}_i = \frac{1}{p_i} (y_i - \hat{R} x_i)$$
$$\hat{U} = \frac{1}{n} \sum_{i=1}^n \hat{U}_i$$

and

$$V(\hat{U}) = \frac{\sum_{i=1}^n (\hat{U}_i - \hat{U})^2}{(n-1) n}$$

Using these formula, now we shall compare the efficiency of the two models.

B. Comparison Between Conventional and Multiplicity Methods

Table 1 shows the comparison between conventional and multiplicity methods of sampling. Even though the estimated numbers of total live births and total childbearing-age women in this table do not show the estimated numbers in Wisconsin because county weights are excluded from the calculations, the multiplicity method ($\hat{Y} = 2069$) estimates a higher total number of live births than the conventional method ($\hat{Y} = 1740$). The former method ($\hat{X} = 527$) also estimates a higher total number of childbearing-age migrant women than the latter method ($\hat{X} = 460$). The higher estimated values may be due to discrepancies between the verbal responses of family members and the obtained list of migrant workers. However, we find little difference in the expected average number of live births per childbearing-age women ($\hat{R} = 3.783$ for the conventional method and 3.925 for the multiplicity method). This finding may suggest that in order to know the average number of live births, either method may estimate a similar

value. However, in order to prove this statement, we must examine the standard errors of estimates.

The efficiency of the multiplicity method is indicated by the variances of $\hat{Y}$ and $\hat{R}$. The standard error of estimate, which is the square root of variance, reflects the extent to which the estimated population values tends to be in error (Hays and Winkler, 1971). A smaller standard error of estimate indicates stronger confidence that the estimated value is closer to the population. Comparing the conventional and multiplicity methods, we find that the conventional method tends to be about 1.6 times the size of the multiplicity method variance ($83.44/50.17 = 1.66$) and 1.8 times the size of the multiplicity method rel-variance in estimating the number of childbearing-age women, and 1.9 times the variance ($.19/.10 = 1.9$) and 2.0 times the rel-variance in estimating the average number of live births per childbearing-age woman. This evidence clearly supports Sirken's argument that the multiplicity method, when a counting rule is considered optimal, shows more efficient and better estimations of a population whose sampling frame cannot be established than does a conventional method.

C. Estimations of Number of Live Births in Wisconsin Migrant Women

In the previous section we have found that the multiplicity method of sampling is quite effective in estimating the parameters of a population about which a direct sampling process

is not available. Using the multiplicity method, we have estimated some parameters about the number of live births in Wisconsin childbearing-age migrant women. For calculating sampling weights, both original sampling weights and county weights are used, and for weighting observed scores both sampling weights and counting rule weights are used. Table 2 presents the results.

According to the table, the estimated number of total childbearing-age migrant women in Wisconsin is 921, and the estimated number of live births borne by these women is 4034. The average number of live births per childbearing-age migrant woman is estimated to be 4.38 in Wisconsin. The variance of the expected number of live births is calculated to be 129.50, and that of the average number of births is estimated to be .15. In comparison with findings in Table 1, we find that the introduction of county weights into estimation procedure increases the variances of both $\hat{Y}$ and $\hat{R}$, resulting in increased error in the estimation procedure. However, this procedure seems to be inevitable to obtain some estimates about the number of live births, since 138 workers out of 408 selected could not be interviewed.

Even though the incompleteness of the sampling may increase error terms in the estimation, the estimates presented here are the best estimates derived from the existing data. Considering the difficulty of research about migrant workers,

these estimates may be considered one of a few existing reliable figures.

V. Comparison of Estimates Derived from Multiplicity Method and Simple Random Method

In social science research sampling procedures are often ignored in the stage of analyses, and data collected with non-random methods are often treated as if they are randomly selected. In this section we shall inquire whether simple-random analysis of data collected by the multiplicity method may produce different estimates in the average number of live births than the estimates derived from the multiplicity method.

Table 3 shows the comparison of estimates derived from the multiplicity method and the simple random method. According to the table, the simple random method without any weight ($\hat{R} = 4.317$) shows the closest estimation of the average number of live births to the value derived from the multiplicity method with county weights ($\hat{R} = 4.380$), while the multiplicity method without county weights ($\hat{R} = 4.127$) underestimates the average value and the simple random method with county weights ($\hat{R} = 4.514$) overestimates the value. If we assume that the multiplicity method with county weights is the best estimator among the four methods, we can say that the simple random method without county weights can estimate quite closely to the multiplicity method with county weights. Utilization of either county weights or multiplicity

weights seems to underestimate or overestimate the average number of live births. The finding that the estimated value in the multiplicity method with county weights indicates a value in between the extreme values of the multiplicity method without county weights and the simple random method with county weights may suggest that the best choice is to use the multiplicity method with county weights to treat the data.

If we compare the magnitude of the variance of $\hat{R}$ in the four methods, we find that the magnitude of variances increases from the simple random method without any weight (.08), to the multiplicity method without county weights (.10) and the simple random method with county weights (.10), and to the multiplicity method with county weights (.15). Weight factors appear to function to expand the variance of $\hat{R}$: The highest value for the variance of $\hat{R}$ is found when the two weights are used, while the lowest value is found when no weight is used. Both county weights and multiplicity weights seem to have a similar effect to the variance of $\hat{R}$, since the multiplicity method without county weights and the simple random method with county weights show the same value in the variance. However, since the difference of weighting procedure actually suggests different processes of sampling, and since all methods other than the multiplicity method with county weights may be considered arbitrary, the variances of $\hat{R}$ do not seem to have any comparability among them.

Therefore, we find that the multiplicity method with county weights provides the most modest estimator for the average number of live births. We also find that the simple random method without any weight shows an estimate similar to the multiplicity method with county weights, despite the misapplication of the former method to the sample selected with the multiplicity method. The latter finding may suggest the robustness of the analytical method.

VI. Conclusion

When a researcher does not have the framework of the population about which he tries to estimate some properties, and the lack of information about the framework makes it impossible to create a systematic sampling procedure of the elements in the population, the multiplicity method of sampling is considered to be the most effective and applicable method. The Wisconsin Migrant Women Study collected data using the multiplicity method of sampling. Using the collected data, this paper compared statistics based on a conventional method and on the multiplicity method. The results clearly indicate the greater effectiveness of the multiplicity method than the conventional method. Then, following the rules of the multiplicity method, we estimated the number of live births among Wisconsin childbearing-age migrant women.

Often the sampling procedure is ignored in the process of analyzing data, treating them as if they are collected with the simple random sampling method. In the last section of this paper, we examined whether such mis-treatment of data influences the estimation of the average number of live births. We find that the simple random method without any weight shows an estimate similar to the multiplicity method with county weights, suggesting the robustness of the analytical method.

Table 1: Comparison Between Conventional and Multiplicity Methods

	Conventional Method	Multiplicity Method
Sample Size (n)	46	133
Estimated Number of Childbearing-Age Women ($\hat{X}$)	460	527
Estimated Number of Live Births ($\hat{Y}$)	1740	2069
Variance of $\hat{Y}$	83.44	50.17
	$(\sqrt{\text{Var}(\hat{Y})} = 9.13)$	$(\sqrt{\text{Var}(\hat{Y})} = 7.08)$
Rel-variance	0.18	0.10
$\hat{Y} - \sqrt{\hat{Y}} \leq \hat{Y} \leq \hat{Y} + \sqrt{\hat{Y}}$	$1730.84 \leq \hat{Y} \leq 1749.44$	$2061.92 \leq \hat{Y} \leq 2076.08$
Estimated Average Number of Live Births Per Childbearing-Age Women ($\hat{R}$)	3.783	3.925
Variance of $\hat{R}$	0.19	0.10
	$(\sqrt{\text{Var}(\hat{R})} = .44)$	$(\sqrt{\text{Var}(\hat{R})} = .31)$
Rel-variance	0.0004	0.0002
$\hat{R} - \sqrt{\hat{R}} \leq \hat{R} \leq \hat{R} + \sqrt{\hat{R}}$	$3.34 \leq \hat{R} \leq 4.22$	$3.62 \leq \hat{R} \leq 4.23$

Note: 1) County weights are not included as a part of sampling weights.
 2) Childbearing-age women who are full-time, part-time, or occasional workers are selected for the multiplicity method.

Table 2: Estimations of Number of Live Births in Wisconsin Migrant Women

	Estimated Values
Sample Size (n)	145
Estimated Number of Childbearing-Age Women ($\hat{X}$)	921
Estimated Number of Live Births ($\hat{Y}$)	4034
Variance of $\hat{Y}$	129.50 ($\sqrt{\text{Var}(\hat{Y})} = 11.38$)
$\hat{Y} - \sqrt{\hat{Y}} \leq \hat{Y} \leq \hat{Y} + \sqrt{\hat{Y}}$	$4022.62 \leq \hat{Y} \leq 4045.38$
Estimated Average Number of Live Births Per Childbearing-Age Women ($\hat{R}$)	4.38
Variance of $\hat{R}$	0.15 ($\sqrt{\text{Var}(\hat{R})} = .39$)
$\hat{R} - \sqrt{\hat{R}} \leq \hat{R} \leq \hat{R} + \sqrt{\hat{R}}$	$3.99 \leq \hat{R} \leq 4.77$

Note: 1) County weights are included as a part of sampling weights.

Table 3: Comparison of Estimation Derived from Multiplicity Method and from Simple Random Method with and without County Weights

	Multi- plicity Method With County Weights	Multi- plicity Method Without County Weights	Simple Random Method Without Any Weight	Simple Random Method With County Weights
Estimated Average Number of Live Births ($\hat{R}$)	4.380	4.127	4.317	4.514
Variance of $\hat{R}$	0.15 ($\sqrt{\text{Var}(\hat{R})} = .39$)	0.10 ($\sqrt{\text{Var}(\hat{R})} = .32$)	0.08 ($\sqrt{\text{Var}(\hat{R})} = .28$)	0.10 ($\sqrt{\text{Var}(\hat{R})} = .32$)
$\hat{R} - \sqrt{\hat{\text{Var}}(\hat{R})} \leq \hat{R} \leq \hat{R} + \sqrt{\hat{\text{Var}}(\hat{R})}$	$3.99 \leq \hat{R} \leq 4.770$	$3.807 \leq \hat{R} \leq 4.447$	$4.037 \leq \hat{R} \leq 4.597$	$4.194 \leq \hat{R} \leq 4.834$

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