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Exploring Prerequisite Relationships between Mathematical Concepts in Intelligent Tutoring Systems

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Meet the Team and Client

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Agenda

- Introduction
- Prerequisite relationships between KCs
  - Data
  - Methods
  - Results
- Prerequisite relationships between workspaces
  - Data
  - Methods
  - Results
  - R Shiny Application
- Next Steps
- Final Recommendation

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Introduction (1)

- Intelligent Tutoring Systems (ITS) is an educational technology that provides students with a virtual learning environment.

- In an ITS, every action of a student is tracked and recorded as a transaction.

- MATHia is an ITS for learning mathematics, developed by Carnegie Learning, that personalizes instruction for middle school students and high school students.

Introduction (2)

The project aims to investigate whether prerequisite relations among math topics can be detected in log data.

Questions to be addressed

- How do we determine whether two math skills are related?
- What metrics and at what granularity of the metric should we use for evaluating learning and performance?
- How do we test whether workspace/knowledge component A is prerequisite for B?

Prerequisite Relations between Knowledge Components (KCs)

Data (1)

- Workspace is a group of problems on the same topic.
- Each workspace has problems, which further has steps. Each step can be solved using some skills in mathematics.
- Whenever a student solves a step (opportunity) correctly, they may learn a Knowledge Component (KC). Knowledge Component (KC), in simple words, is a skill in mathematics.
- For each KC, students are given multiple opportunities (steps) until they demonstrate mastery of that mathematical skill. Different number of opportunities are given to each student based on their performance.

Data (2)

- Datasets on 3 workspaces:
  - A = “Analyzing Models of Two-Step Linear Relationships”
  - B = “Modeling Two-Step Expressions”
  - C = “Using Scale Factor”

- Content in A (and presumably KCs in A) are a prerequisite for (presumably KCs in) B
- C is prior to both A & B, and we have no reason to think that C is a prerequisite for A or B.

Data (3)

Dataset	# of Students	# of Unique Knowledge Components	# of Unique Steps (Opportunities)
A: Analyzing Models of Two-Step	29949	7	7
B: Modeling Two-Step Expressions	27005	9	9
C: Using Scale Factor	19521	4	29

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Method - Overview

Gaussian Graphical Model	Initial Opportunities	GLMER
Understand relationships between KCs through partial correlations.	Use success rate on initial few opportunities to get better indication of student performance.	Better understand and quantify the relationships between pairs of KCs. Explore the direction of correlations.

Methods - Metrics for Student Performance

- Success Rate
  - For a given student and KC, we find the proportion of correct responses for all opportunities.
  - Calculating the mean success rate for the initial opportunities.
  - We apply logit transformation:

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right)$$

    - $p = 0 \rightarrow 0.0001$
    - $p = 1 \rightarrow 0.9999$

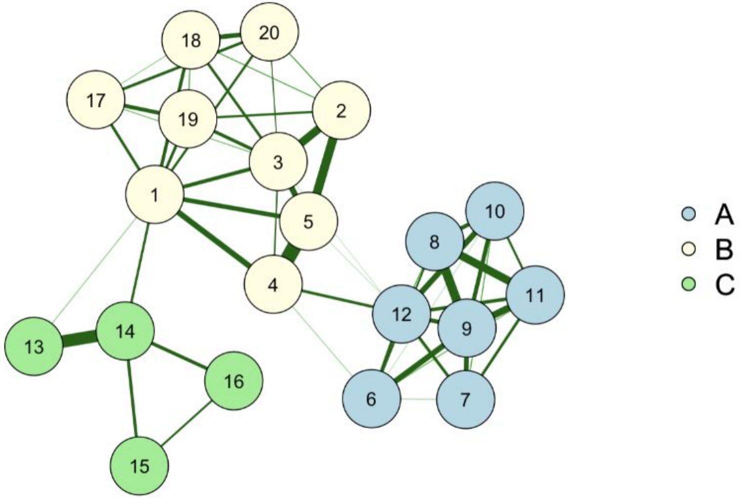
Methods - Gaussian Graphical Models

- Gaussian Graphical Models (GGMs) are an exploratory analysis tool that provides relationships between variables in a study.
- GGM has KCs, depicted by nodes, and edges that connect those nodes which visualize the relationship between KCs.
- The thickness of these edges represent the strength of the relationship. The edge is green in color if the correlation is positive and red otherwise.
- KCs which are strongly correlated are placed spatially close to each other in the plot.

Results - Gaussian Graphical Models (1)

- To obtain GGM, we need to have a correlation matrix. We calculate correlations between KCs where each observation in the data represents the logit transformation of a student's success rate on initial 2 opportunities.

- The correlations are calculated using full information maximum likelihood procedure (FIML).



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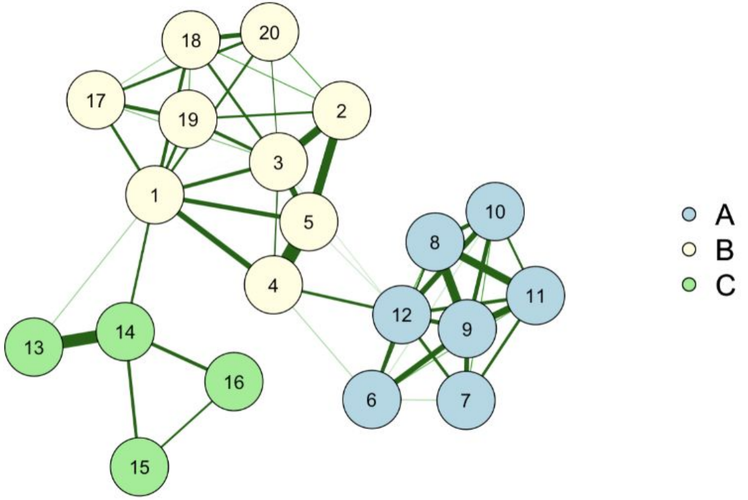
Results - Gaussian Graphical Models (2)

- The thickness of the edges between various KCs represent the strength of relationships and are interpreted as partial correlation coefficients.

- Since relationships estimated in GGM are interpreted as partial correlations, we reduce the risk of finding any spurious correlations.

- The partial correlations are estimated by the GGM using glasso algorithm. This forces the unimportant partial correlation coefficients to zero.



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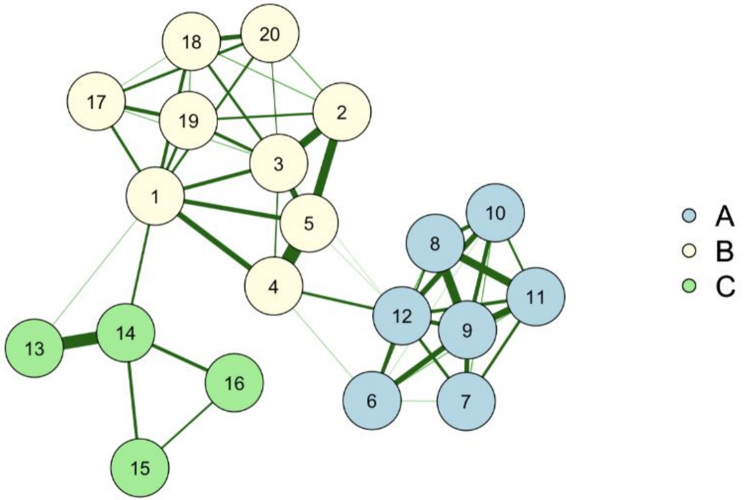
Method - Initial Opportunities

- What is an opportunity?
  - A step (include all attempts a student made to that step)
- Why initial opportunities?
  - For each KC, steps are given until students demonstrate mastery
  - Using all would smooth out differences among students
  - Better indication of performance

Results - Initial Opportunities

- Decide based on the structures of GGMs generated by different cutoffs
- Only partial correlations > 0.05 in absolute

  value are shown
- **Initial 2 opportunities** give the best graph
  - more edges between A and B



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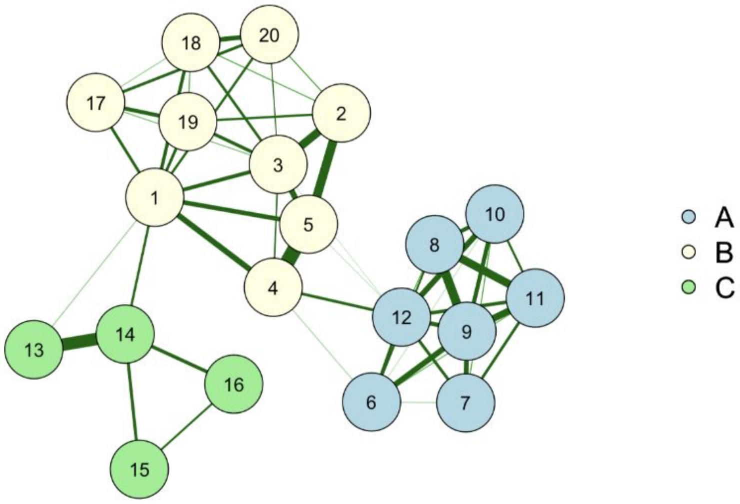
Results - Gaussian Graphical Models (3)

Node (1)	KC (1)	Node (2)	KC (2)	Partial Correlation
4	find y, any form-1	5	identifying units-1	0.1917
13	scale drawings-3 determine unknown measure, complex scale factor	14	scale drawings-3 determine unknown measure, simple scale factor	0.1703
8	match dep expression with description	9	match indep expression with description	0.1376
2	enter given, reading numerals-1	5	identifying units-1	0.1363
2	enter given, reading numerals-1	3	enter given, reading words-1	0.1284
8	match dep expression with description	11	match linear-term expression with description	0.1274
9	match indep expression with description	11	match linear-term expression with description	0.1164
3	enter given, reading words-1	5	identifying units-1	0.1063
1	define variable-1	4	find y, any form-1	0.1052
10	match intercept expression with description	12	match slope expression with description	0.1046



Method - Determine the Direction of Correlations

If KC2 is a prerequisite for KC1, then whether a student knows KC2 would influence their performance of KC1

Logistic regression with mixed effects (glmer)

- KC1 Performance - whether the first attempt was correct
- 1 + KC1 Opportunity + know_kc2 + KC1 Opportunity : know_kc2 + (1|Anon.Student.Id)

Result - Determine the Direction of Correlations

"Prereq" KC	KC	Main Effect	Opportunity	Interaction
1 (define variable)	5 (identifying units)	1.21	0.18	-0.13
5 (identifying units)	1 (define variable)	2.43	0.34	-0.31
4 (find y, any form-1)	5 (identifying units)	1.20	0.10	-0.07
5 (identifying units)	4 (find y, any form-1)	0.50	0.08	-0.04
18 (write expression, negative slope-1)	1 (define variable)	1.45*	0.63	-0.60
1 (define variable)	18 (write expression, negative slope-1)	30.98*	28.63*	-28.90*
18 (write expression, negative slope-1)	5 (identifying units)	1.20	0.12	-0.08
5 (identifying units)	18 (write expression, negative slope-1)	4.00	1.99	-1.64

Prerequisite Relations between Workspaces

Data (1)

- MATHia Course 2 dataset contains a random sample of 500 grade 7 students' performance in the 2019-2020 academic year.
- The dataset contains 427 unique workspaces.
- For determining prerequisite relations, we only use workspaces which are a part of 'shipped' Course 2 curriculum.
- 76 workspaces are a part of 'shipped' Course 2 curriculum (we exclude 'pre launch protocol').
- Majority of these workspaces have been attempted by about 200 students.

Data (2)

Dataset	# of Unique Workspaces	# of Students	# of Unique Knowledge Components	# of Unique Steps (Opportunities)
MATHia Course 2 (Shipped)	76	500	223	65685

Method - Overview

Gaussian Graphical Model → **Retrospective Study**

Gaussian Graphical Model

Understand relationships between workspaces based on partial correlations.

Retrospective Study

We retrospectively study the effect of first attempting ‘prerequisite’ workspace on student performance.

Methods - Gaussian Graphical Models

- Gaussian Graphical Models (GGMs) are an exploratory analysis tool that provides relationships between variables in a study.

- GGM has workspaces, depicted by nodes, and and edges that connect those nodes which visualize the relationship between workspaces.

- The thickness of these edges represent the strength of the relationship. The edge is green in color if the correlation is positive and red otherwise.

- Workspaces which are strongly correlated are placed spatially close to each other in the plot.

Methods - Metrics for Student Performance (1)

- Success Rate
  - Finding the outcome of first attempt for all problems and all steps in that workspace.
  - Calculating proportion of correct responses.

- Assistance Score
  - Assistance score across student’s attempts for all problems and steps in that workspace.

Methods - Metrics for Student Performance (2)

- Success Rate
  - 0 if the first attempt of the step is wrong
  - 1 if the first attempt of the step is correct
  - For each workspace,

    Success Rate = # of correct first attempts / # of steps
  - We apply logit transformation:

    $$\mathrm{logit}(p) = \log(\frac{p}{1-p})$$
    - *p = 0* → *0.0001*
    - *p = 1* → *0.9999*

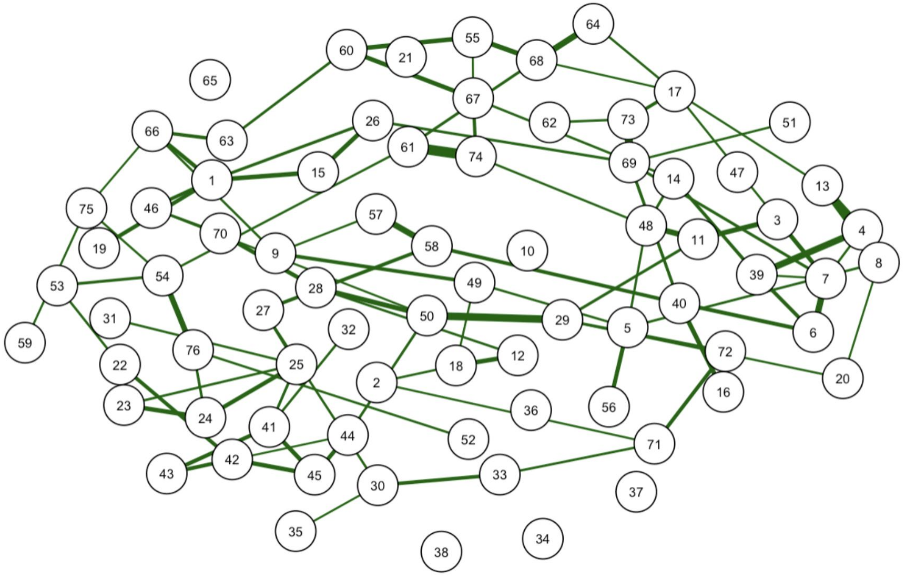
Methods - Metrics for Student Performance (3)

- Assistance Score
  - For a single student, for each workspace
  - Assistance Score = **(# of Wrong Attempts + # of Hints requested)**
  - We apply log transformation
    - Assistance Score = *0* → *0.0001*

Results - Gaussian Graphical Models (1)

Student Performance Metric: Success Rate

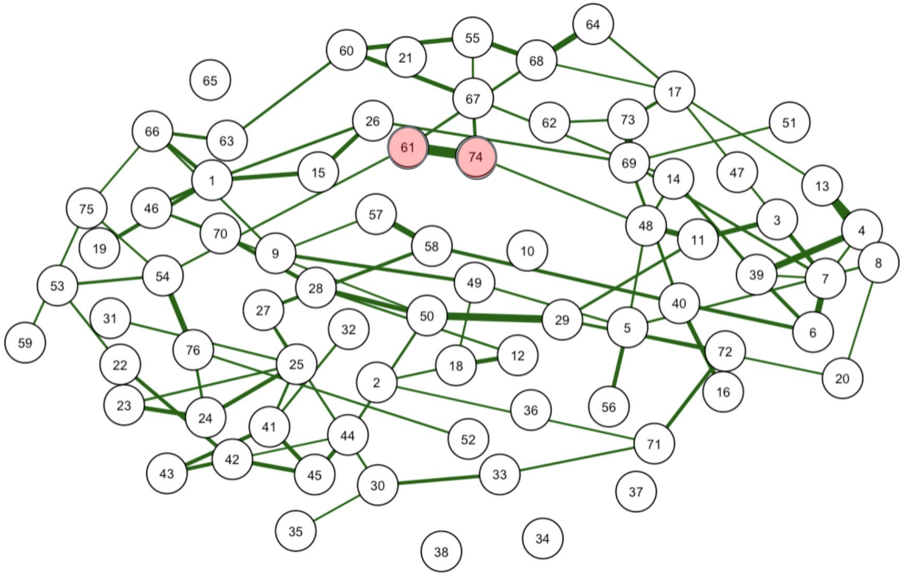
- To obtain GGM, we need to have a correlation matrix.
- We compute pairwise correlations between workspaces and convert the resulting correlation matrix to positive definite using a smoothing technique.
- The thickness of the edges between various KCs represent the strength of relationships and are interpreted as partial correlation coefficients.



Results - Gaussian Graphical Models (2)

Student Performance Metric: Success Rate

Node (1)	Workspace (1)	Node (2)	Workspace (2)	Partial Correlation
61	solving simple percent problems	74	using proportions to solve percent problems	0.4122
69	graphs of equations	73	using graphs to solve equations	0.3293
29	scale drawings 3	50	volume surface area right prism vol-backward	0.3112
8	comparing theoretical and experimental probabilities	13	determining probabilities	0.2875
4	calculating compound probabilities	13	determining probabilities	0.2743
4	calculating compound probabilities	39	simulating compound events	0.2702
6	comparing characteristics of data displays	7	comparing populations using data displays	0.2672
64	converting with fractional percents	68	fractional percent models	0.2587
57	ratio proportion change3	58	ratio proportion change4	0.2556
11	critical attributes of similar figures	48	using scale drawings	0.2357



Results - Gaussian Graphical Models (3)

Student Performance Metric: Success Rate

Node (1)	Workspace (1)	Node (2)	Workspace (2)	Partial Correlation
61	solving simple percent problems	74	using proportions to solve percent problems	0.4122
69	graphs of equations	73	using graphs to solve equations	0.3293
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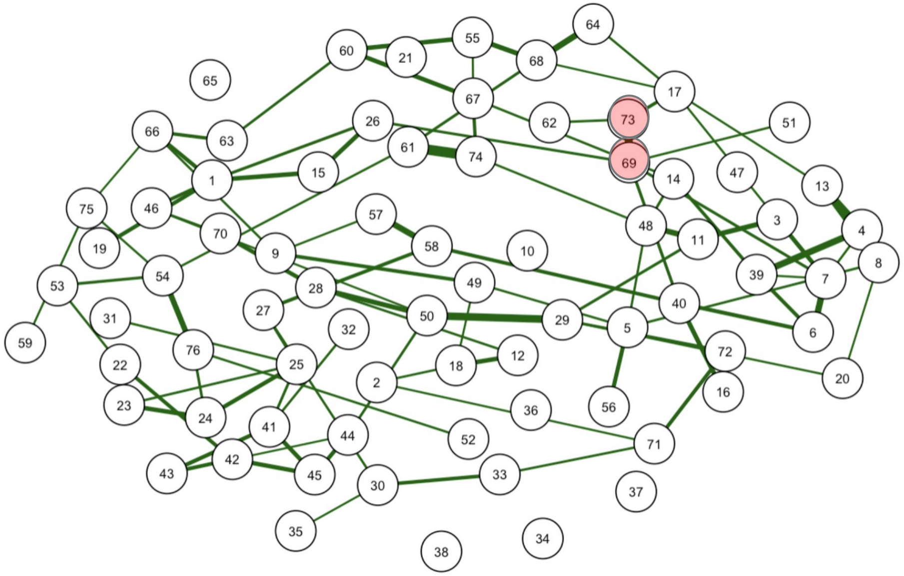

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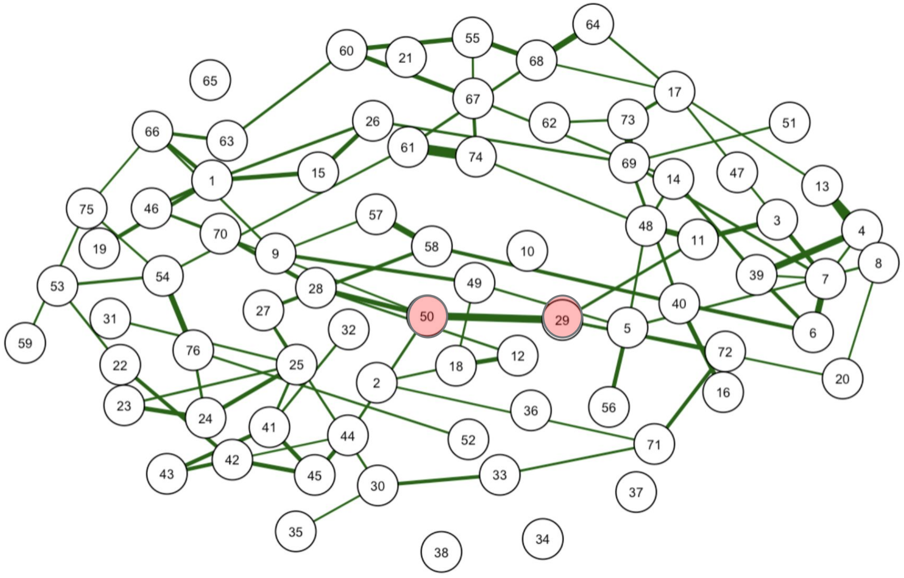
Results - Gaussian Graphical Models (4)

Student Performance Metric: Success Rate

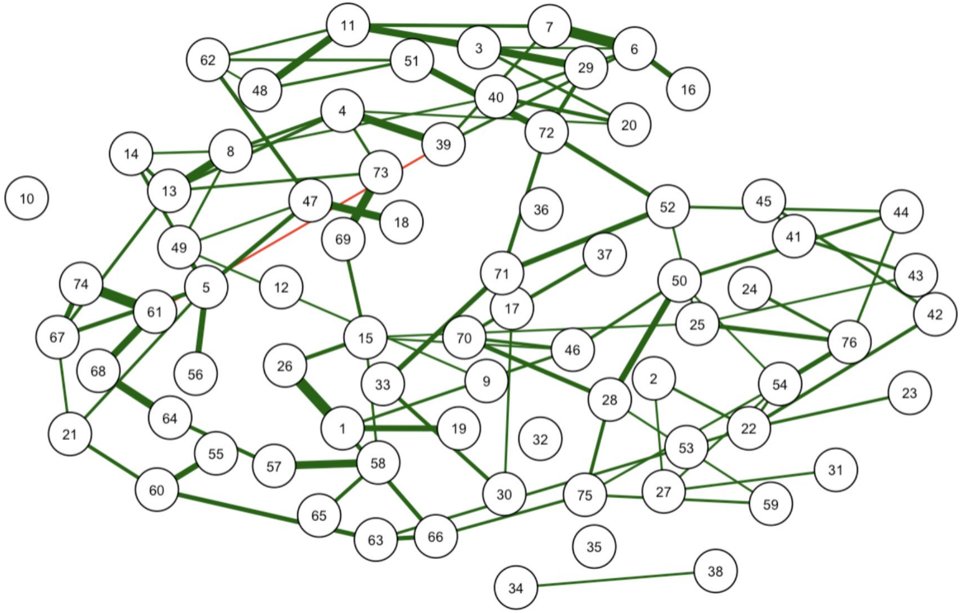
Node (1)	Workspace (1)	Node (2)	Workspace (2)	Partial Correlation
61	solving simple percent problems	74	using proportions to solve percent problems	0.4122
69	graphs of equations	73	using graphs to solve equations	0.3293
29	scale drawings 3	50	volume surface area right prism vol-backward	0.3112
8	comparing theoretical and experimental probabilities	13	determining probabilities	0.2875
4	calculating compound probabilities	13	determining probabilities	0.2743
4	calculating compound probabilities	39	simulating compound events	0.2702
6	comparing characteristics of data displays	7	comparing populations using data displays	0.2672
64	converting with fractional percents	68	fractional percent models	0.2587
57	ratio proportion change3	58	ratio proportion change4	0.2556
11	critical attributes of similar figures	48	using scale drawings	0.2357



Results - Gaussian Graphical Models (5)

Student Performance Metric: Assistance Score

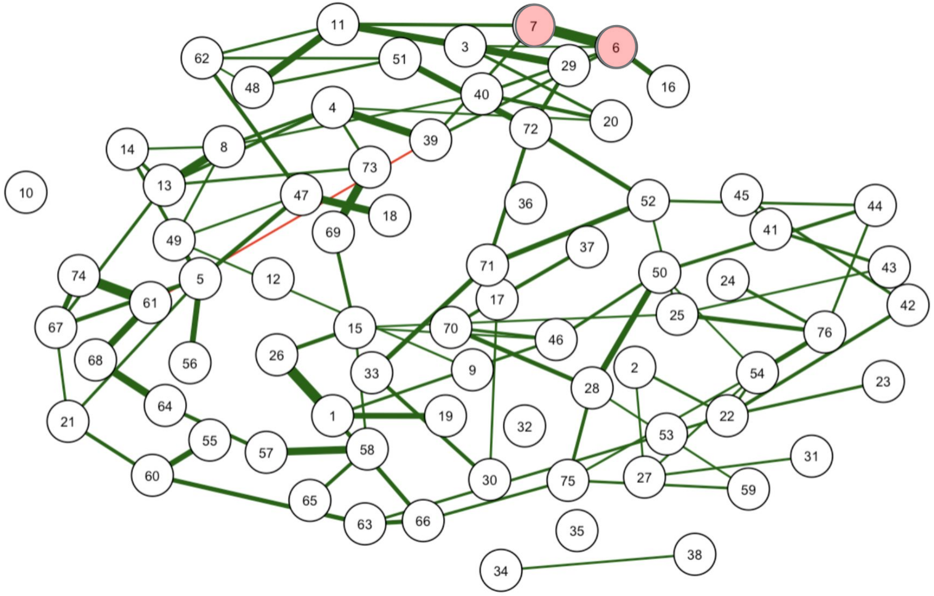
- To obtain GGM, we need to have a correlation matrix.
- We compute pairwise correlations between workspaces and convert the resulting correlation matrix to positive definite using a smoothing technique.
- The thickness of the edges between various KCs represent the strength of relationships and are interpreted as partial correlation coefficients.



Results - Gaussian Graphical Models (6)

Student Performance Metric: Success Rate

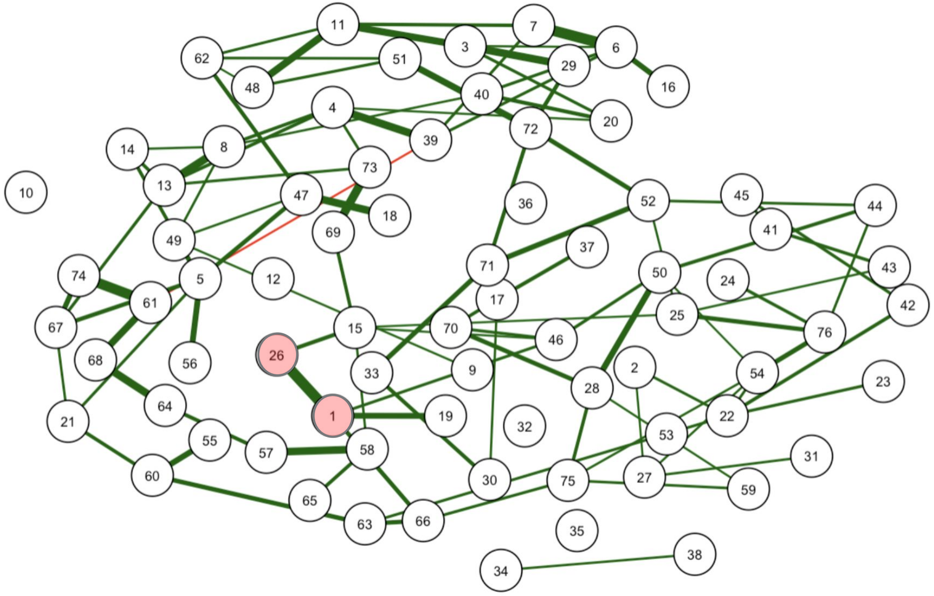
Node (1)	Workspace (1)	Node (2)	Workspace (2)	Partial Correlation
6	comparing characteristics of data displays	7	comparing populations using data displays	0.3527
1	adding and subtracting integers	26	multiplying and dividing integers	0.3512
61	solving simple percent problems	74	using proportions to solve percent problems	0.3369
69	graphs of equations	73	using graphs to solve equations	0.2926
8	comparing theoretical and experimental probabilities	13	determining probabilities	0.2747
4	calculating compound probabilities	39	simulating compound events	0.2677
64	converting with fractional percents	68	fractional percent models	0.2668
3	calculating angles	29	scale drawings 3	0.2646
18	identifying signs of starting values and rates	47	understanding volume of right prisms	0.2644
57	ratio proportion change3	58	ratio proportion change4	0.2534



Results - Gaussian Graphical Models (7)

Student Performance Metric: Success Rate

Node (1)	Workspace (1)	Node (2)	Workspace (2)	Partial Correlation
6	comparing characteristics of data displays	7	comparing populations using data displays	0.3527
1	adding and subtracting integers	26	multiplying and dividing integers	0.3512
61	solving simple percent problems	74	using proportions to solve percent problems	0.3369
69	graphs of equations	73	using graphs to solve equations	0.2926
8	comparing theoretical and experimental probabilities	13	determining probabilities	0.2747
4	calculating compound probabilities	39	simulating compound events	0.2677
64	converting with fractional percents	68	fractional percent models	0.2668
3	calculating angles	29	scale drawings 3	0.2646
18	identifying signs of starting values and rates	47	understanding volume of right prisms	0.2644
57	ratio proportion change3	58	ratio proportion change4	0.2534



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Results - Gaussian Graphical Models (8)

Student Performance Metric: Success Rate

Node (1)	Workspace (1)	Node (2)	Workspace (2)	Partial Correlation
6	comparing characteristics of data displays	7	comparing populations using data displays	0.3527
1	adding and subtracting integers	26	multiplying and dividing integers	0.3512
61	solving simple percent problems	74	using proportions to solve percent problems	0.3369
69	graphs of equations	73	using graphs to solve equations	0.2926
8	comparing theoretical and experimental probabilities	13	determining probabilities	0.2747
4	calculating compound probabilities	39	simulating compound events	0.2677
64	converting with fractional percents	68	fractional percent models	0.2668
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18	identifying signs of starting values and rates	47	understanding volume of right prisms	0.2644
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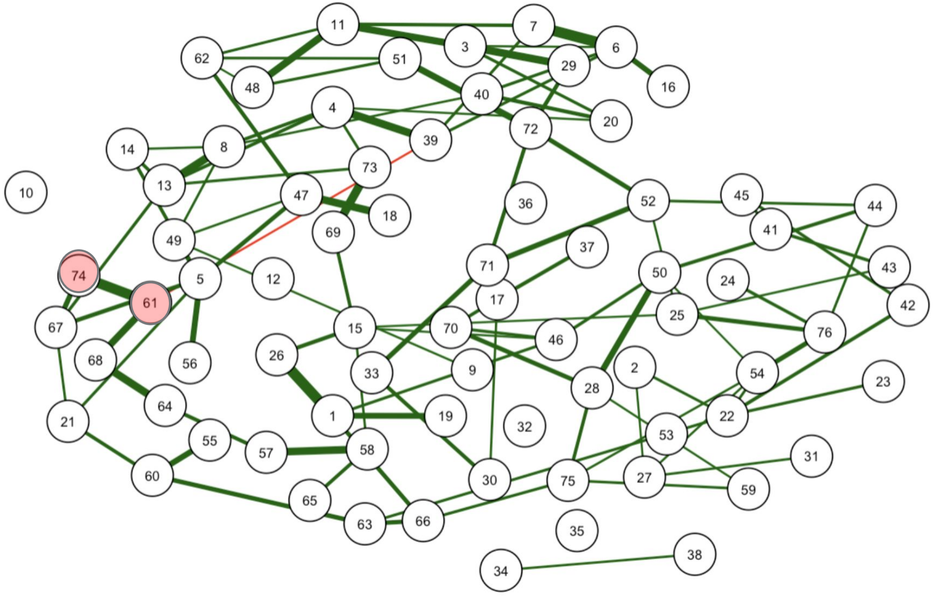

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Methods - Retrospective Study

For a pair of workspaces, there are 4 possible orders in which student work on them:

- Only WS1
- Only WS2
- WS2 before WS1
- WS1 before WS2

Control Group

Treatment Group

We are interested in student's performance on WS2, treatment is WS1.

Two-sample t-test (assuming unequal variances) is used to determine if the difference between treatment and control group is statistically significant.

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Results - Retrospective Study (1)

- Workspace 1: ‘fractional percent models’

  Workspace 2: ‘solving simple percent problems’

- Prove: Workspace 1 is a possible prerequisite of Workspace 2.
- We consider a retrospective study design and test whether student performance on Workspace 2 is greater for students who have first attempted Workspace 1.
- 258 students attempted Workspace 1 first.

  102 students didn’t attempt Workspace 1 first.

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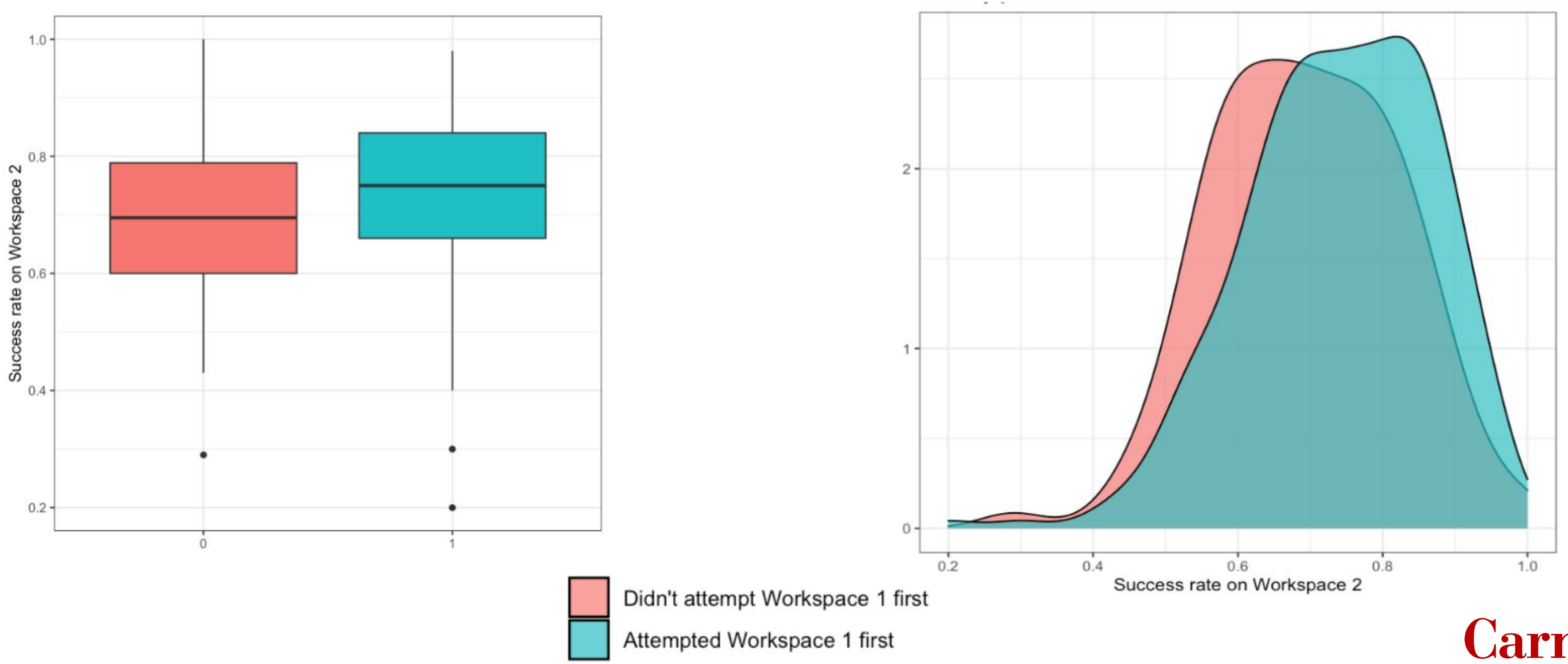
Results - Retrospective Study (2)



Results - Retrospective Study (3)

- The mean success rate on Workspace 2 for students who first attempted Workspace 1 is 0.7388.
- The mean success rate on Workspace 2 for students who didn’t attempt Workspace 1 first is 0.6944.
- We obtain a p-value of 0.0018 after performing the t-test. At 5% level of significance, the mean success rate on Workspace 2 is greater for students who have first attempted Workspace 1.
- Workspace 1 is a possible prerequisite of Workspace 2.

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Workspaces with Prerequisite Relationships

"Prereq" Workspace	Workspace
comparing populations using data displays	graphs of equations
simplify order of ops expression numeric contrast addsub multdiv	simplify order of ops expression variable contrast addsub multdiv
adding and subtracting integers	ratio proportion change3
picture algebra mix variable	worksheet grapher a1 direct variation
worksheet grapher a1 solving 2step int	worksheet grapher a1 direct variation
linear relations 1	simplify order of ops expression numeric mix type complex

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R Shiny Application Demo

Next Steps and Final Recommendations

Next Steps (1)

- Explore other metrics for student performance:
  - Success rate and assistance score are useful, but simple metrics.
  - They may not capture true student performance.

- Glmer models is a good way to quantify the correlation between KCs, but does not imply causal relationships.
  - The model does not take into account the time order in which a student learns KCs.

Next Steps (2)

- Gaussian Graphical Models:
  - Presence of negative partial correlation coefficients.
  - Glasso estimates important partial correlations, not necessarily significant ones
- Retrospective study design:
  - Because there is no randomization here, the study is not unbiased.
  - Since we only have 500 students in our sample, the data for some pairs of workspaces if highly unbalanced.

Final Recommendations

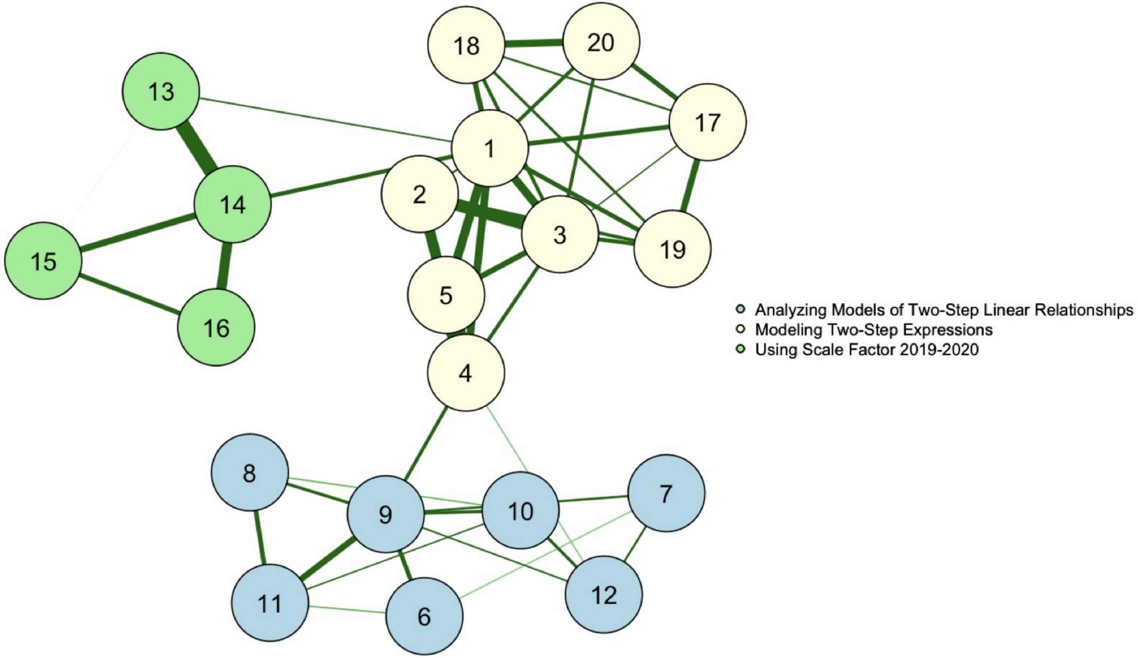
- Conduct randomized experiments to specifically determine prerequisite relationships:
  - Take pairs of workspaces for which we’ve shown a possible prerequisite relation or pairs in GGM that are strongly related.
  - Students should be randomly assigned to treatment and control groups.
  - The independent sample t-test should now give a more reliable idea about the difference in student performance between two groups.

- Educational psychologists and specialists who create content for MATHia should be made aware of pairs of KCs/workspaces for which we found prerequisite relations.
  - Using techniques in their discipline, it would be worth investigating the results of prerequisite relations that they have.

Thank you for your time!

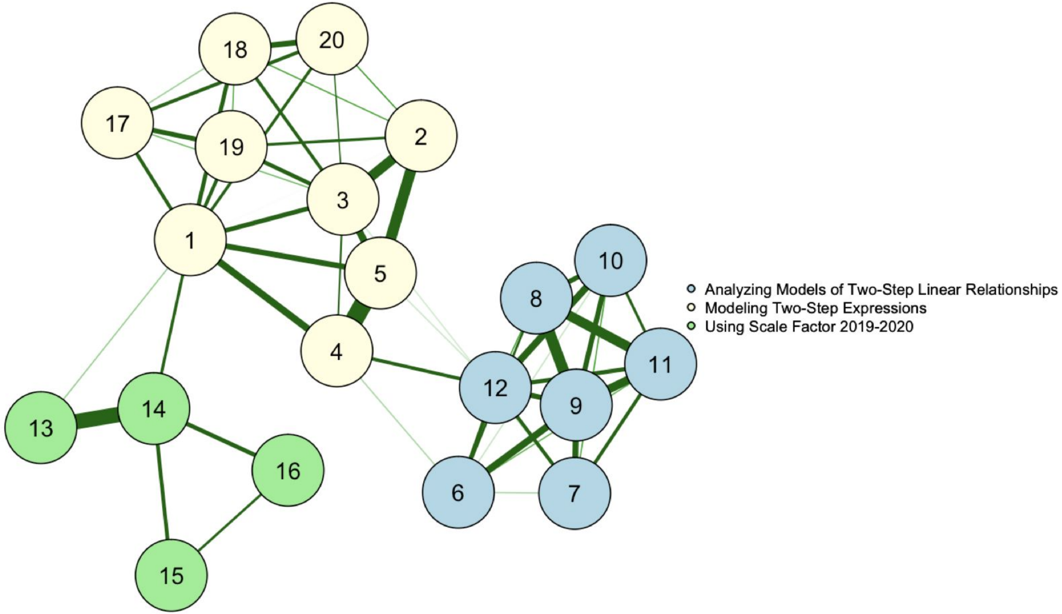
Extra Slides

When K = 1,



Extra Slides

When K = 2,



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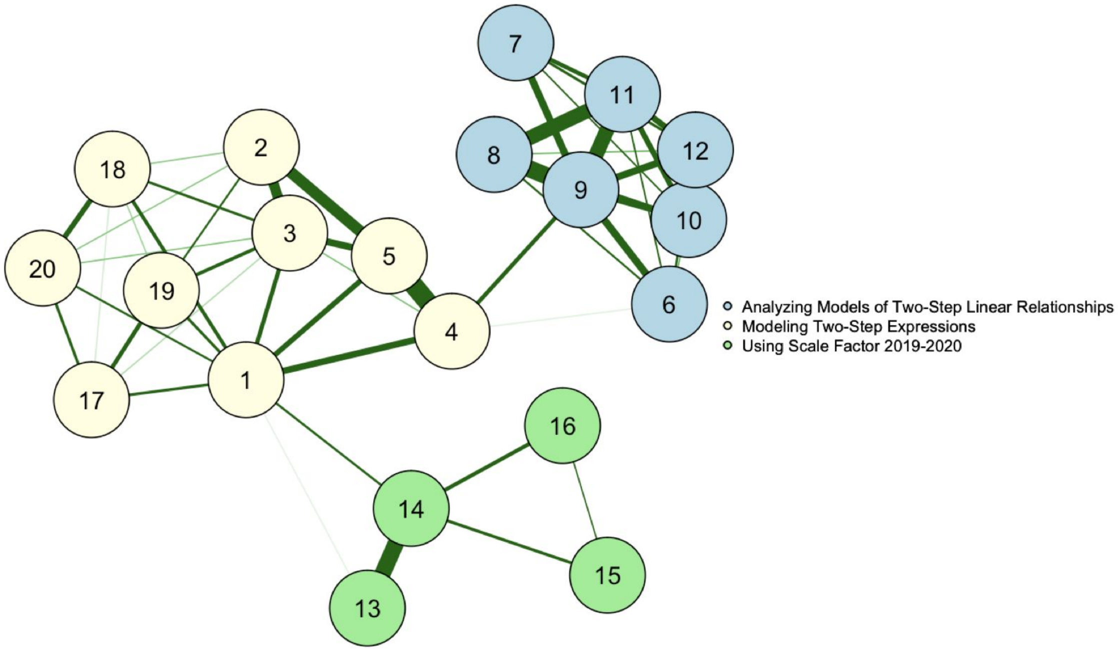
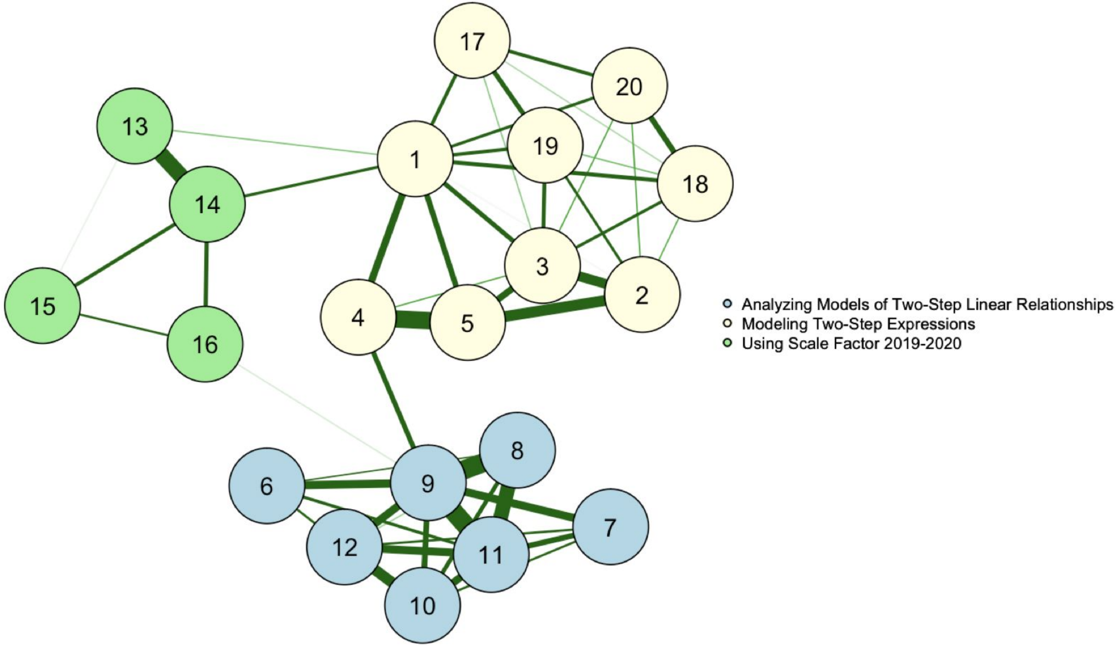
Extra Slides

When K = 3,



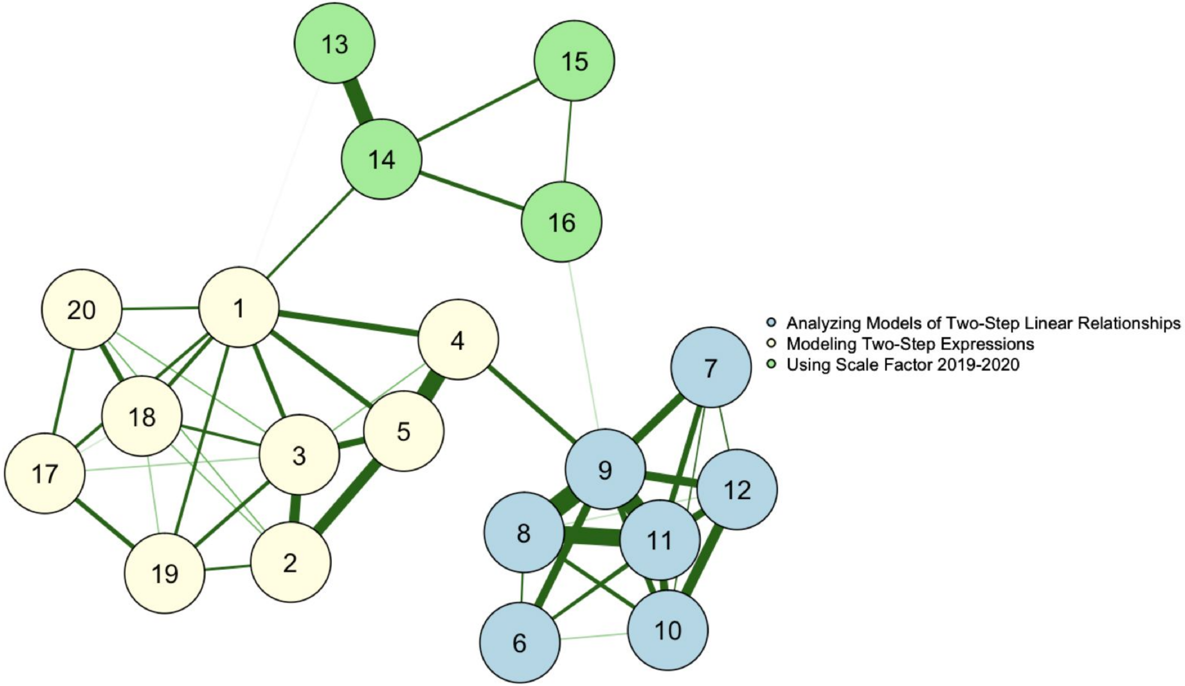
Extra Slides

When K = 4,



Extra Slides

When K = 5,



Extra Slides

When K = 6,