36-309/749
Experimental Design for Behavioral and Social Sciences

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Lecture 8: Contrasts and Multiple Comparisons

Contrasts (in general)

- Context: An ANOVA rejects the "overall" null hypothesis that all *k* means of some factor are not equal, i.e., H_0: $\mu_1 = \cdots = \mu_k$. When k>2, this is not satisfying (scientifically).
- Contrasts let us ask "which are significantly different?"
- Terminology: Define a "**contrast**" or "analytic comparison" that is of scientific interest, e.g. compare μ_1 to μ_5 or compare μ_2 to the average of μ_1, μ_3 and μ_5, i.e., $\left(\frac{\mu_1+\mu_3+\mu_5}{3}\right)$
- **Contrast null hypothesis**: Express as something equal to zero. For the above examples, $\mu_1 = \mu_5 \Rightarrow \mu_1 - \mu_5 = 0$ and $\mu_2 = \left(\frac{\mu_1+\mu_3+\mu_5}{3}\right) \Rightarrow \mu_2 - \left(\frac{\mu_1+\mu_3+\mu_5}{3}\right) = 0$.

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Contrasts, cont.

- Computer package form: Re-express in ***linear combination*** form: $C_1\mu_1 + \ldots + C_k\mu_k = 0$ which contains ***all*** of the parameters in the original "overall" null hypothesis ***in order***. (The C values are called "weights" or "coefficients". Some of the C's will be negative.) Enter just the coefficients into the computer.
  - Example 1: express $\mu_1 - \mu_5 = 0$ as $(1)\mu_1+(0)\mu_2+(0)\mu_3+(0)\mu_4+(-1)\mu_5=0$. Enter 1 0 0 0 –1 into SPSS or some other computer package.
  - Example 2: express $\mu_2 - \left(\frac{\mu_1+\mu_3+\mu_5}{3}\right) = 0$ as
    $(\ \ )\mu_1 + (\ \ )\mu_2 + (\ \ )\mu_3 + (\ \ )\mu_4 + (\ \ )\mu_5 = 0$.
    Enter into SPSS:
  - Check your work: ***Valid coefficient sets always add to zero***.

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Planned comparisons (contrasts)

- Planned comparisons **maintain type-1 error** at α (experiment-wise) only when:
  - They are chosen in advance, i.e., truly planned.
  - They are only used if the corresponding ***overall*** p-value is $\le\alpha$.
  - They number no more than the F numerator df.
    - One-way (k level) ANOVA: k-1 planned contrasts
    - Two-way (k x m) ANOVA:
      - Interaction expected: plan (k-1)(m-1) contrasts like $\mu_{A1,B1} - \mu_{A2,B3} = 0$ or the often more informative form, such as $(\mu_{A1,B1} - \mu_{A2,B1}) - (\mu_{A1,B3} - \mu_{A2,B3}) = 0$.
      - Interaction not expected: k-1 planned contrasts for factor A; m-1 for factor B.
  - They are orthogonal (often ignored): the sum of products of corresponding coefficients equal zero, i.e., they ask ***independent*** questions.

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Planned comparisons, cont.

- ***Optional*** technical details: See gray boxes in textbook and/or ask Howard.
- In SPSS, comparisons made using the "Contrasts" button or the LMATRIX subcommand under GeneralLinearModel/Univariate are assumed to be planned, and the p-values are wrong otherwise.

Multiple (post hoc, unplanned) comparisons

- Example 0: Darts game
- Example 1: In a study of twenty chocolate lovers vs. non-chocolate eaters (freaks of nature), researchers claimed that "higher levels of phenylacetylglutamine and citrate in the chocolate-desiring group suggest that these individuals may regulate the citric acid cycle slightly differently than those who don't fancy a daily dose of chocolate." (J Proteome Research, 6(11):4469-4477, 2007) Consider performing t-tests to see if the groups differ for each detectable compound. For 50 compounds (a low, but reasonable number), if they are all unrelated to chocolate, the chance of ***avoiding*** a false positive at a rate of 0.95 each is 0.95^{50}=0.077. [Chance of getting 1 or 2 FP is 20% and 26% respectively.] Conclusion:

Multiple Comparisons, cont.

- Example 2: In a study of the effect of magic beans on health, a carefully done, well powered, randomized clinical trial measures 12 health outcomes (BP, cholesterol, etc.).

  Assuming "magic beans" are useless, the chance of *avoiding* a false positive is:

  $0.95*0.95*\cdots*0.95 = 0.95^{12} = 0.54$

  So the chance of finding at least one (meaningless) finding is 1-0.54 = 46%.

Multiple Comparisons, cont.

- Math: The number of ways you can choose 2 items from a list of k items is called "k choose 2". We use the symbol $\binom{k}{2}$ and the answer is k(k-1)/2.
- Checking all $\binom{k}{2}$ pairs in a one-way ANOVA has exactly the same problem as for "magic beans".
- Less obviously, when we pick out the smallest and largest sample means out of k means to compare, we are implicitly performing multiple comparisons, thus increasing the chance of making a type-1 error.
- The problem is also referred to as ***post-hoc*** testing, unplanned comparisons, and data snooping.

Multiple Comparisons, cont.

- Most common goal: keep the **per-experiment type-1 error rate** at 0.05 (compare with FDR). The key to appropriate, honest post-hoc comparisons is to determine the size of the family of comparisons that you are considering, and handicap yourself (e.g., lower alpha or raise the p-value) to reduce the chances of a type 1 (FP) error, which, unfortunately, is at the expense of reduced power.
- Special example: In a two-way ANOVA with interaction, at least one of the three overall null hypotheses (two main effects plus interaction) are rejected at $p \le 0.05$ about 14% of the time for null experiments if the "corrected model" p-value is not used as a "screen".

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Multiple Comparisons, cont.

- Appropriate methods add a "penalty" for multiple comparisons
  - Bonferroni procedure: simplest and most general, but conservative (Holm's-Bonferroni is a tiny bit better). Set $\alpha' = \alpha/m$ where m is the number of possible comparisons in the "family", then compare the p-value to α' instead of α.
  - Based on the "degree of fishing", choose one of these methods that gives adjusted p-values and/or adjusted CIs for some specific situation (generally more power than Bonferroni):
    - Tukey's procedure: test all possible pairs for one factor.
    - Dunnet's procedure: compare a control to all possible active treatments
    - Scheffé's procedure: all possible simple and complex contrasts

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Contrasts in SPSS

- Contrasts" button gives p-values assuming that the comparisons are an appropriate set of ***planned*** comparisons.
- "Post-hoc" button gives p-values assuming post-hoc comparisons within the "family" associated with the specific post-hoc procedure. In multi-way ANOVA, a no-interaction model is assumed. Tukey (all paires) and Dunnett (baseline vs. all others) are the most useful.

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Contrasts in SPSS, cont.

Multiple Comparisons

	(I) Color	(J) Color	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Tukey HSD	white	red	-1.8000	.29804	.000	-2.6248	-.9752
		green	-2.4571	.29804	.000	-3.2819	-1.6324
		blue	.0952	.31021	.990	-.7632	.9537
	Red	white	1.8000	.29804	.000	.9752	2.6248
		green	-.6571	.29804	.152	-1.4819	.1676
		blue	1.8952	.31021	.000	1.0368	2.7537
	green	white	2.4571	.29804	.000	1.6324	3.2819
		red	.6571	.29804	.152	-.1676	1.4819
		blue	2.5524	.31021	.000	1.6939	3.4108
	Blue	white	-.0952	.31021	.990	-.9537	.7632
		red	-1.8952	.31021	.000	-2.7537	-1.0368
		green	-2.5524	.31021	.000	-3.4108	-1.6939

Homogeneous Subsets

	Color	N	Subset 1	Subset 2
Tukey HSD	blue	6	2.3333	
	white	7	2.4286	
	red	7		4.2286
	green	7		4.8857
	Sig.		.989	.164

Not necessarily non-overlapping!

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Contrasts in SPSS, cont.

- SPSS LMATRIX subcommand
  - Requires syntax pasting and manual entry of contrast coefficients
  - Very flexible: ***any*** valid contrast can be specified
  - p-values are based on the comparisons being ***planned***
  - For post-hoc, calculate t=contr./SE(contr.), F=t^2, and use, e.g., Scheffé procedure

"Paste Syntax" in SPSS

- "Paste" instead of the final "OK" in SPSS causes the "syntax" to be displayed in the "Syntax Editor" instead of running the analysis.
- You can edit the syntax (following strict rules) and then "run" the syntax to run the analysis.
- Expert SPSS users often work mainly with "syntax".
- For us, syntax is used to add features to an analysis for which there are no menu items.

Contrasts in SPSS, cont.

- 1-way ANOVA example: factor name is "treatment", level order is Placebo, Talk, Drug, Both. Paste:

```
UNIANOVA score BY treatment
  /METHOD=SSTYPE(3)
  /INTERCEPT=INCLUDE
  /CRITERIA=ALPHA(0.05)
  /DESIGN=treatment.
```

- Edit to:

```
UNIANOVA score BY treatment
  /METHOD=SSTYPE(3)
  /INTERCEPT=INCLUDE
  /CRITERIA=ALPHA(0.05)
  /LMATRIX "others - control" treatment -1 1/3 1/3 1/3
  /LMATRIX "combo - (drug+talk)/2" treatment 0 1/2 1/2 -1
  /LMATRIX "drug-talk" treatment 0 -1 1 0
  /DESIGN=treatment.
```


LMATRIX for 1-way ANOVA, cont.

```
/LMATRIX "others - control" treatment -1 1/3 1/3 1/3
```

Estimate of $\frac{\mu_D+\mu_T+\mu_B}{3} - \mu_P$

Custom Hypothesis Tests #1

Contrast Results (K Matrix)[a]

Contrast			Dependent Variable score
L1	Contrast Estimate		16.333
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		16.333
	Std. Error		6.192
	Sig.		.017
	95% Confidence Interval for Difference	Lower Bound	3.324
		Upper Bound	29.343

a. Based on the user-specified contrast coefficients (L') matrix: others - placebo

LMATRIX for 1-way ANOVA, cont.

```
/LMATRIX "combo - (drug+talk)/2" treatment 0 1/2 1/2 -1
```

Estimate of $\mu_B - \frac{\mu_D+\mu_T}{2}$

Custom Hypothesis Tests #2

Contrast Results (K Matrix)[a]

Contrast			Dependent Variable score
L1	Contrast Estimate		20.400
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		20.400
	Std. Error		6.979
	Sig.		.009
	95% Confidence Interval for Difference	Lower Bound	5.738
		Upper Bound	35.062

a. Based on the user-specified contrast coefficients (L') matrix: both - (drug+talk)/2

LMATRIX for 1-way ANOVA, cont.

```
/LMATRIX "drug-talk" treatment 0 -1 1 0
```

Estimate of $\mu_D - \mu_T$

Custom Hypothesis Tests #3

Contrast Results (K Matrix)[a]

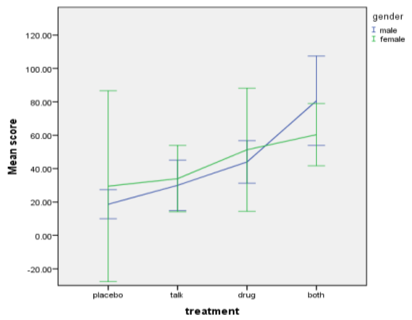
Contrast			Dependent Variable score
L1	Contrast Estimate		16.400
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		16.400
	Std. Error		7.825
	Sig.		.050
	95% Confidence Interval for Difference	Lower Bound	-.039
		Upper Bound	32.839

a. Based on the user-specified contrast coefficients (L') matrix: drug-talk

Contrasts for 1-way ANOVA without interaction

- Additive model = parallel pattern in a graph pf population means
- Valid questions: What are the effects of a specific change in level of one factor ignoring, fixing or averaging over the other factor?
- Conclusion: Analyze each factor separately as for 1-way ANOVA.

Contrasts for 2-way with Interaction



Error Bars: 95% CI

Interaction Contrasts, cont.

```
UNIANOVA score BY treatment gender
  /METHOD=SSTYPE(3)
  /INTERCEPT=INCLUDE
  /EMMEANS=TABLES(treatment*gender)
  /CRITERIA=ALPHA(.05)
  /LMATRIX "M-F for placebo" gender 1 -1
         treatment*gender 1 -1   0 0   0 0   0 0
  /LMATRIX "M-F for both" gender 1 -1
         treatment*gender 0 0   0 0   0 0   1 -1
  /LMATRIX "M-F for (talk+drug)/2" gender 1 -1
         treatment*gender 0 0   1/2 -1/2   1/2 -1/2  0 0
  /DESIGN=treatment gender treatment*gender.
```

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Interaction Contrasts, cont.

```
/LMATRIX "M-F for both" gender 1 -1
 treatment*gender 0 0  0 0  0 0  1 -1
```

H_0: $\mu_{BM} = \mu_{BF}$ Estimate: $\mu_{BM} - \mu_{BF}$

Contrast Results (K Matrix)[a]

Contrast			Dependent Variable score
L1	Contrast Estimate		20.333
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		20.333
	Std. Error		7.088
	Sig.		.012
	95% Confidence Interval for Difference	Lower Bound	5.130
		Upper Bound	35.537

a. Based on the user-specified contrast coefficients (L') matrix: M-F for both

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Interaction Contrasts, cont.

- Posthoc example: all pairs for the 2x4 ANOVA
- 8 groups = $\binom{8}{2}$ = 8*7/2 = 28 pairs
- Write up to 28 LMATRIX commands
- Compute Bonferroni α' = 0.05/28 = 0.0018
- Test (2-1)*(4-1)=3 planned contrasts using α=0.05
- Reject any others if p<0.0018

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Summary

- Contrasts allow more useful scientific conclusions when a rejected H_0 is vague, e.g., H_0: $\mu_1=\ldots=\mu_k$ (with k>2) or H_0: additive model is good enough.
- (Remember: main effect H_0s in the presence of a significant interaction answer the wrong questions!)
- Running multiple tests increases the chance for false rejection. Beyond "df" tests, corrections must be used to "maintain the type-1 error rate at α".
- Multiple comparisons corrections reduce power. Pre-planned contrasts should be selected before running the experiment to maximize power to where it is most needed.

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