Chaos, Complexity, and Inference (36-462)

Lecture 24: Contagion on Networks

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Contagion on Networks

Contagion

Recognition that some diseases spread by contact is very old
Epidemic disease largely (but not exclusively) associated with livestock herds
Massive role in human history (McNeill, 1976; Diamond, 1997)
New epidemic diseases: exposure to new animal or human host populations
Massive *continuing* role in human society: malaria, cholera, hepatitis, AIDS, tuberculosis. . .

Contagion on Networks

Transmission by contact or proximity $\Rightarrow$ diseases follow social networks

Diseases especially follow networks of *trade* (rapid, high intensity motion of people to come into contact with other travelers)

Outstanding example: 14th century Black Death

could also talk about 19th century cholera or 20th century AIDS

Following Abu-Lughod (1989)
by mid 14th century, the old world had:

- integrated economy based on cities
- start (under the Song) of an industrial revolution in China (Elvin, 1973; McNeill, 1982)
- substantial growth of trade, specialization, market-oriented production
- political integration of the most advanced parts (China and Islamic world) under the Mongols

  admittedly at the cost of occasional “shock and awe” campaign, destruction of Baghdad, etc.
- beginnings of true global perspective (Hodgson, 1974, vol. II)

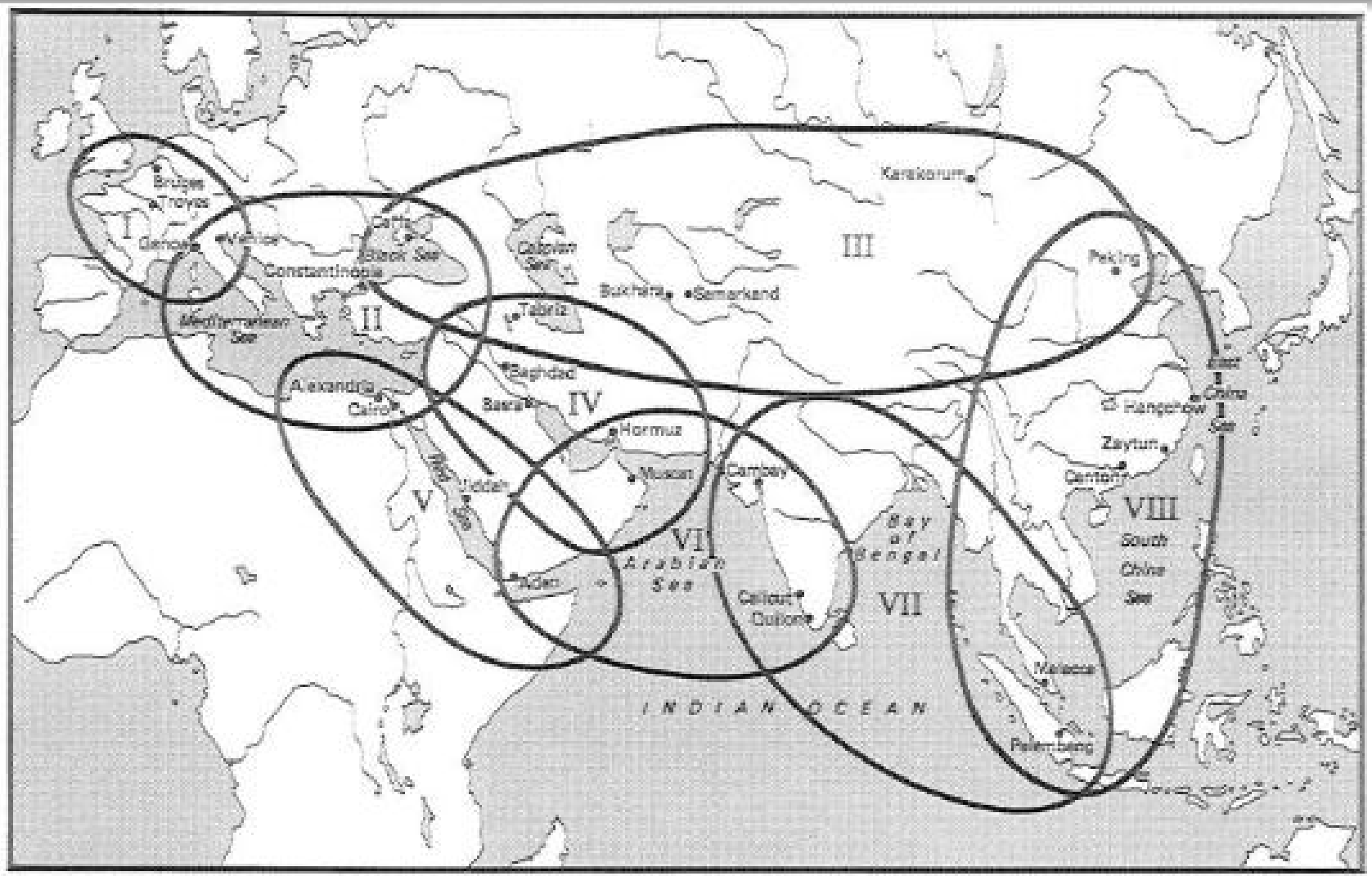


FIGURE 1. The eight circuits of the thirteenth-century world system.

Plague

Yersinia pestis, bacterium transmitted by fleas that live on rodents
apparently originating with steppe rodents in central Asia

Great Gerbil *Rhombomys opimus*

(`http://www.liv.ac.uk/science_eng_images/biology/gerbil.jpg`)

first pandemic wave may have killed as much as 25% of total human population

up to 90% in some areas

followed trade routes and general economic development

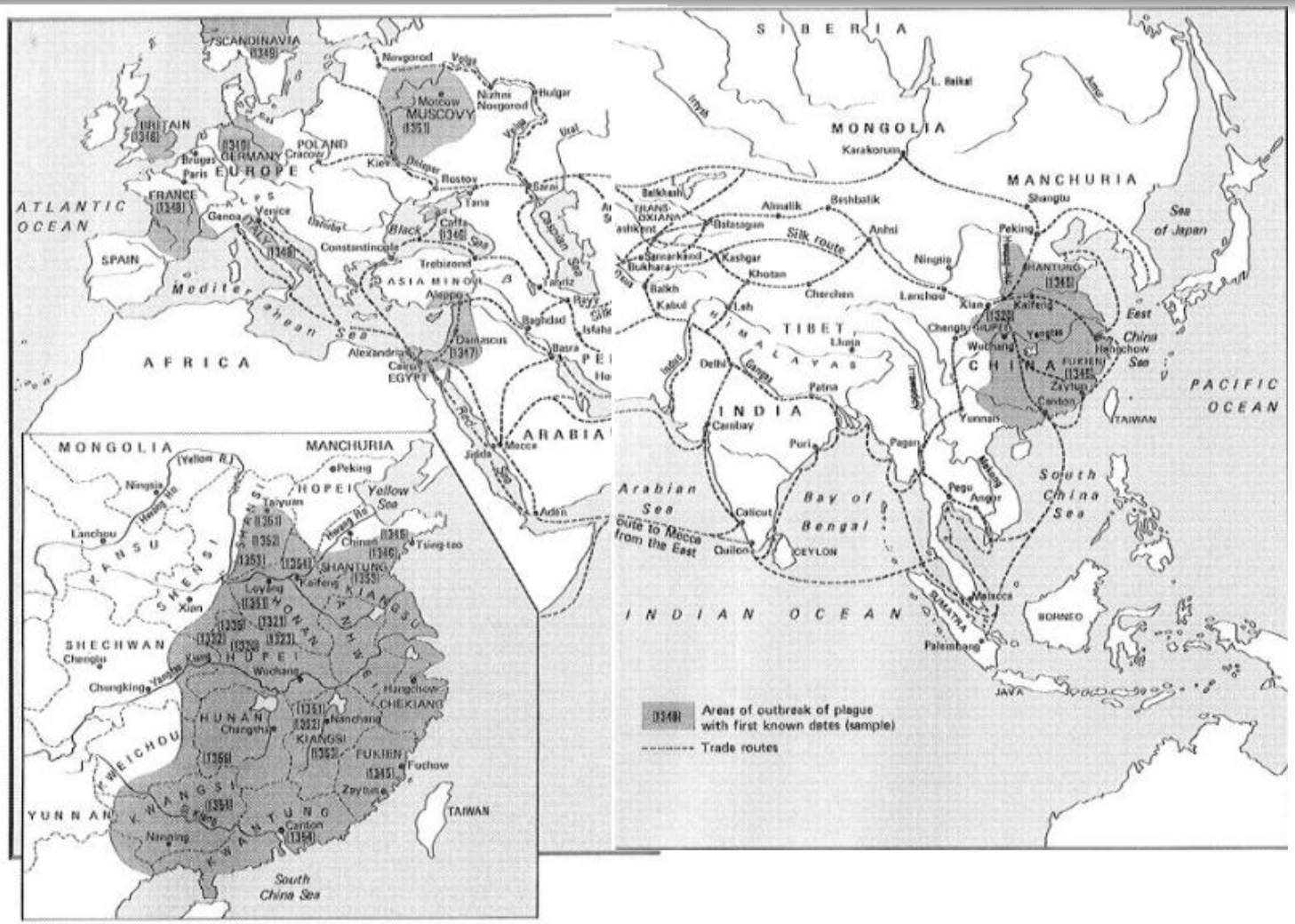
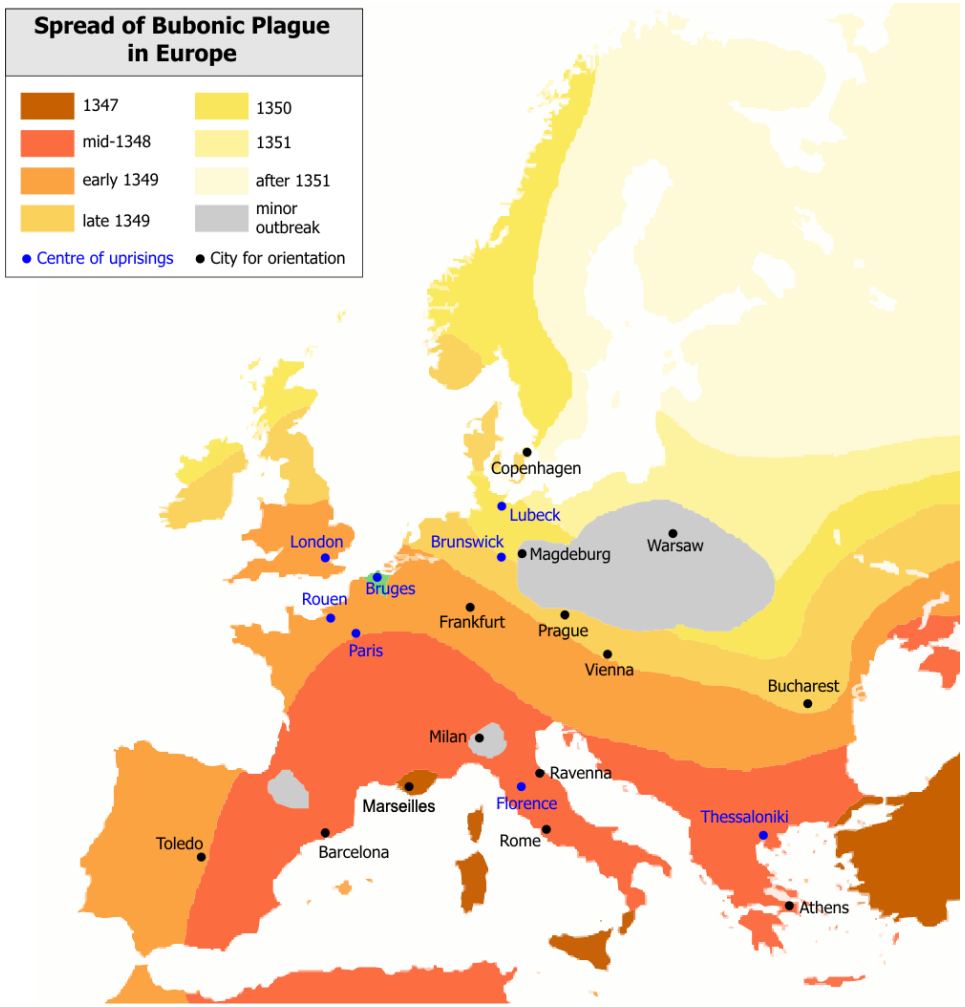


FIGURE 7. The congruence between trade routes and the spread of the Black Death circa 1350.

Europe: a peripheral part of the world economy then, like SE Asia — but part of it

FIGURE 2. The European subsystem: locations of the four Champagne fair towns, the Flemish cities of Bruges and Ghent, and the Italian ports of Genoa and Venice.

. . . and so hit by the plague (Wikipedia s.v. “Black Death”)



Contagion of Ideas

Ideas, beliefs, habits, practices, religions, etc., also spread from person to person

Analogy to spread of disease goes back to at least Roman times

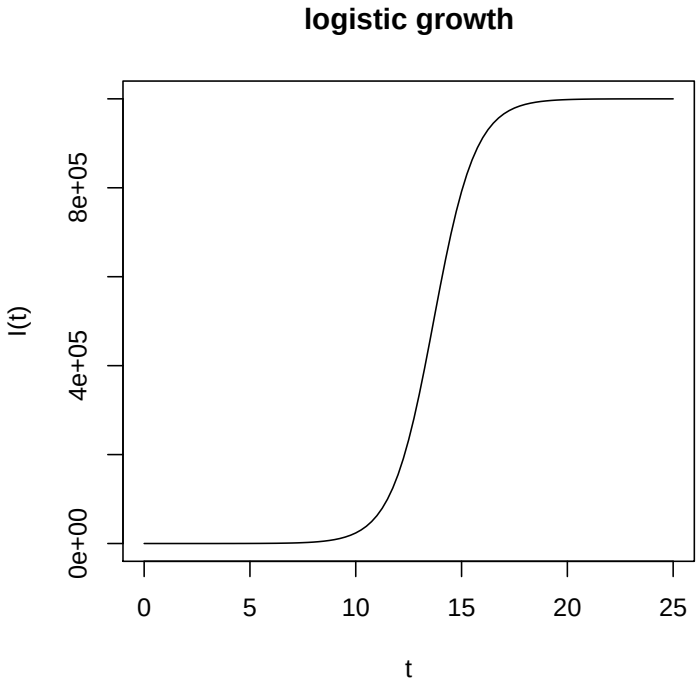
The routes of transmission are often strikingly parallel to the routes of disease transmission (Siegfried, 1960/1965)

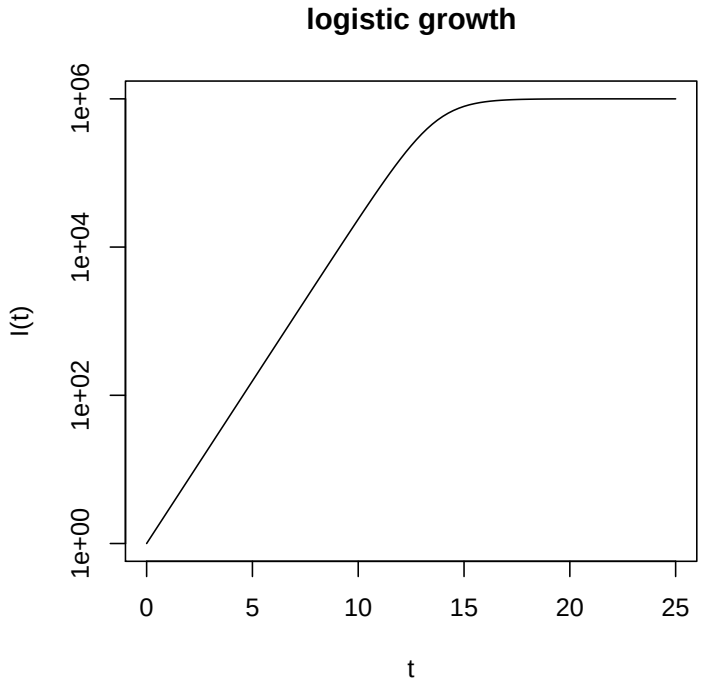
Lots of social-scientific work on this, especially on influence of network structure

- "social influence", "personal influence" (Katz and Lazarsfeld, 1955)
- "diffusion of innovations" (Rogers, 2003)

One characteristic finding: **logistic curve** for diffusion

$$\begin{aligned} \frac{dI}{dt} &= rI(N - I) \\ I(t) &= \frac{NI_0 e^{rt}}{N + I_0(e^{rt} - 1)} \end{aligned}$$





$N = 10^6$, $I_0 = 1$, $r = 1.01$

Simple Epidemic Models

Two states: S, susceptible; I, infectious
Possible third: R, recovered or removed
$\mathrm{S} \to \mathrm{I} \to \mathrm{R}$: SIR model
$\mathrm{S} \to \mathrm{I} \to \mathrm{S}$: SIS model
times taken and probabilities adjustable
Also: $\mathrm{S} + \mathrm{I} \to 2\mathrm{I}$ (infection!)

fully-mixed models: every S could be infected by any I
Deterministic limit (SIR):

$$\begin{aligned} \frac{dS}{dt} &= -\beta IS \\ \frac{dI}{dt} &= \beta IS - \gamma I \\ \frac{dR}{dt} &= \gamma I \end{aligned}$$

(for SIS, remove last equation, add γI to dS/dt)
stochasticity and discreteness: individuals, probabilities per time step (Poisson noise)

Over-All Dynamics

Basic reproductive number R_0: average number of infections caused by adding I to an all-S population

Epidemic transition: does contagion die out at vanishing fraction of population or spread to positive fraction of total size?

$R_0 > 1$ is *invasion* criterion for epidemic spreading

invasion criteria very important in biology, e.g., evolutionary game theory

Epidemic threshold: critical rate of infection for epidemic transition

Basic Branching Process Model

Z_t particles at time t

$$Z_{t+1} = \sum_{i=1}^{Z_t} \eta_{i,t+1}$$

$\eta_{i,t}$ all ≥ 0 and IID, $\mathbf{E}[\eta] = \mu$

Models asexual reproduction

bacteria, neutrons in U or Pu, interstellar civilizations (Kinouchi, 2001)

or when one sex doesn't count

mitochondria, aristocratic titles

Subcritical $\mu < 1$, dies out

Super-critical $\mu > 1$, $\mathbf{E}[Z_t] \propto \mu^t$

Critical $\mu = 1$ (dies out but long-lived fluctuations)

Learn about branching processes!

Epidemics and Branching Processes

Z_t = number infected in stage t
$\eta_{i,t+1}$ = number of new infections caused by individual i
= some but not all neighbors
Initially epidemic well-described by branching process
differences grow due to:
finite-sized populations
network structure ($\eta_{i,t}$ not IID)
Mapping epidemic on to **percolation process** does not have this issue

Percolation

(Stauffer and Aharony, 1994; Grimmett, 1999)
Given: network with nodes and edges a.k.a. bonds
Edges are either open or blocked at random (**bond percolation**)

site percolation: nodes are open or blocked

Desired: distribution of sizes of connected components; does there exist a component spanning the graph?
Percolation transition: critical fraction of open edges at which spanning happens/giant component appears
Another one of the major tools of applied probability

Gerbils make their own landscape, but not just as they please

Davis *et al.* (2008)
Back to the gerbils
Burrows are “large, complex constructions representing the efforts of many generations” of gerbils

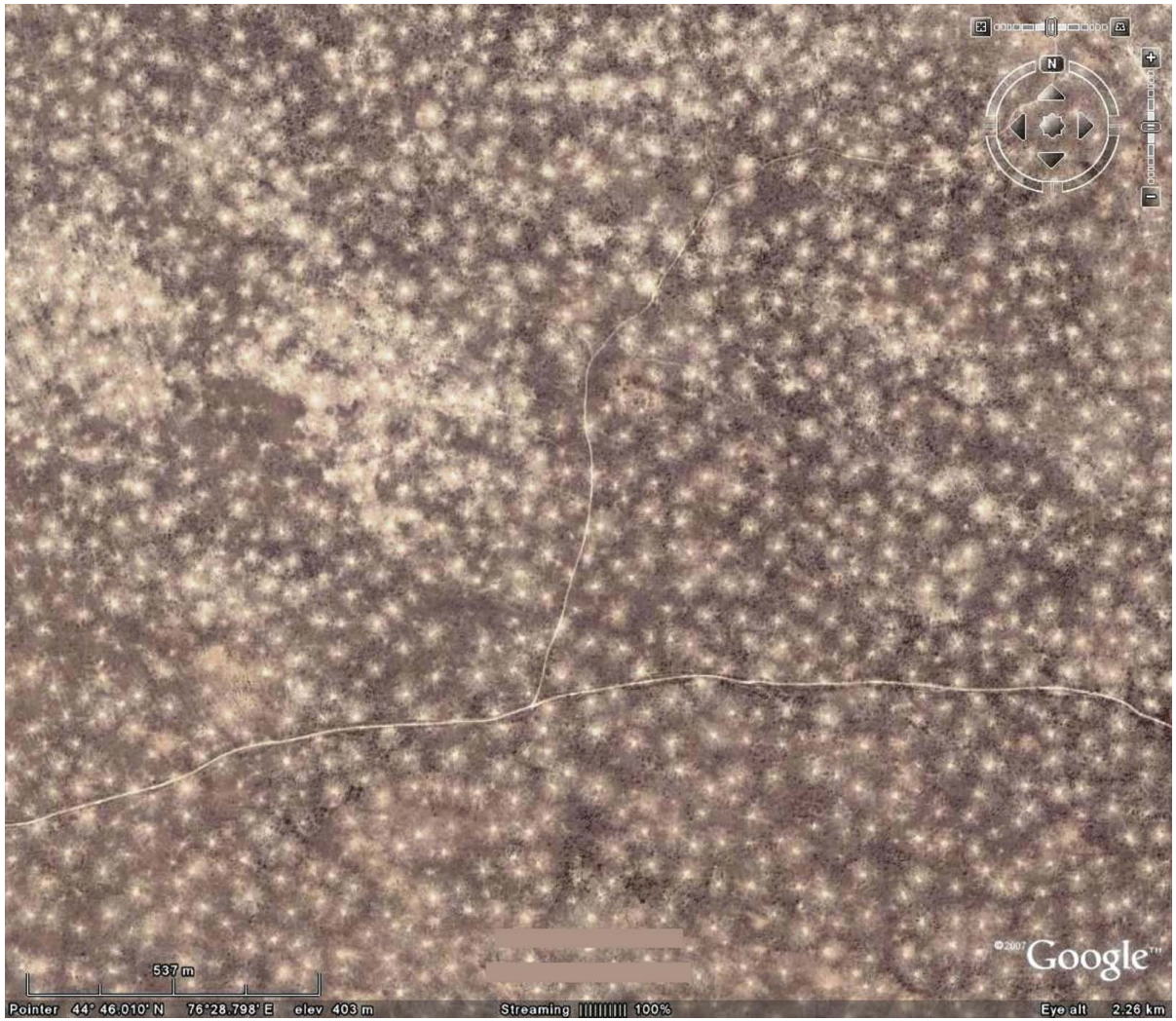


Figure 1 | The regular, star-like pattern created by burrow systems, visible on satellite images. Patches of bare earth form above and around the burrow systems dug by great gerbils and strongly reflect the sunlight. Each bright disc represents a burrow system 10–40 m in diameter. The image was captured using the publicly available software Google Earth (http://earth.google.com/). Copyright 2008 DigitalGlobe; Europa Technologies.

Burrows are either occupied or not, many are unoccupied
Abundance = fraction of occupied burrows
Percolation: adjacent occupied burrows allow for transmission of fleas
Calculated abundance threshold for eipzootic plague: 0.31
Empirical value: 0.33, 95% CI = 287–0.373

Epidemics on Structured Networks

Trick of Newman (2002) (going back to Grassberger): map epidemic into percolation

Kenah and Robins (2007) make corrections to percolation analysis

transmissibility $T \equiv$ prob. of transmitting disease from random infected individual to random susceptible

$$R_0 = T \times (\text{neighbors reached from random edge})$$

that is $\geq \mathbf{E}[K]$ because a random edge is extra likely to go to a *high* degree node

Epidemic transition: expected epidemic size diverges when $T > T_c = 1/$ expected number of neighbors of random edge
Uncorrelated transmission probabilities:

$$T_c = \frac{\sum_k k p_k}{\sum_k k(k-1)p_k} = \frac{\mathbf{E}[K]}{\mathbf{E}\left[K^2\right] - \mathbf{E}[K]} = \frac{\mathbf{E}[K]}{\mathrm{Var}[K] + \mathbf{E}[K](\mathbf{E}[K] - 1)}$$

with $p_k =$ degree distribution
Notice $T_c \to 0$ as $\mathrm{Var}[K] \to \infty$
Similar results for correlated transmission, structured populations, etc.

The Scale-Free Sexual Diseases Controversy

$$\sum_{k=1}^{\infty} k^2 C_\alpha k^{-\alpha} = \infty \text{ if } \alpha \leq 3$$

so power laws with low exponents $\leq 3 \Rightarrow$ infinite variance

Liljeros *et al.* (2001) Swedish self-report data on lifetime sexual partners $\Rightarrow$ the "web of human sexual contacts" has power-law degree distribution, $\alpha = 3.2 \pm 0.3$ $\Rightarrow$ OMG we're all gonna die!!!!!
But it'll all be OK if we just "destroy the hubs"

Handcock and Jones (2004) + related publications WTF? No way the data supports those inferences! What power law tails? Seriously, dudes, *infinite* variance? WTF?

More productively...

Add heterogeneity to the models:

Patterns of contacts

Modes of transmission

Rates of transmission

- Treatment

Disease evolution

Much more easily done with agent-based models than with heroic generating-functionology
Good example: Jacquez *et al.* (1994); Koopman *et al.* (1997)

Homophily or Contagion?

Recall **homophily**: "bird of a feather flock together"
Closeness in network $\Rightarrow$ contagion $\Rightarrow$ similarity
Similarity $\Rightarrow$ homophily $\Rightarrow$ closeness in network
These can be very hard to distinguish (especially if you don't try)

Example: "The Spread of Obesity in a Large Social Network over 32 Years"

Christakis and Fowler (2007); much reported

Results a friend becoming obese increases your risk of become obese by 57% (CI 6–123)
sibling, 40% (CI 21–60)
spouse, 37% (CI 7–73)

Confounding friends and family are similar people (homophily), likely to have similar habits of diet and exercise

Control for this confounding none

They claim to, but procedure makes no sense

Evidence for contagion of obesity weak

because we could easily get these results if obesity is not contagious, due to homophily

Conclusion Someone should study this question

Mistaking Contagion and Homophily for Causation

Example: differences in beliefs and values/ideology/culture across social groups

Traditionally attributed to social position/experience somehow shaping beliefs

(another prize for the first student to identify the authors of the next quote)

The production of ideas, of conceptions, of consciousness, is at first directly interwoven with the material activity and the material intercourse of men, the language of real life. Conceiving, thinking, the mental intercourse of men, appear at this stage as the direct efflux of their material behaviour. The same applies to mental production as expressed in the language of politics, laws, morality, religion, metaphysics, etc. . . . Men are the producers of their conceptions, ideas, etc. — real, active men, as they are conditioned by a definite development of their productive forces and of the intercourse corresponding to these . . . [O]n the basis of [men's] real life-process we demonstrate the development of the ideological reflexes and echoes of this life-process. The phantoms formed in the human brain are . . . sublimates of their material life-process, which is empirically verifiable and bound to material premises. Morality, religion, metaphysics, all the rest of ideology and their corresponding forms of consciousness, thus no longer retain the semblance of independence. They have no history, no development; but men, developing their material production and their material intercourse, alter, along with this their real existence, their thinking and the products of their thinking. Life is not determined by consciousness, but consciousness by life.

This mode of reasoning is extremely common!

> *You go into some of these small towns in Pennsylvania, and like a lot of small towns in the Midwest, the jobs have been gone now for 25 years and nothing's replaced them. And they fell through the Clinton administration, and the Bush administration, and each successive administration has said that somehow these communities are gonna regenerate and they have not. So it's not surprising then that they get bitter, they cling to guns or religion or antipathy to people who aren't like them or anti-immigrant sentiment or anti-trade sentiment as a way to explain their frustrations.*

See also: denunciations of the "media elite", "cultural elites", "the New Class", etc., any issue of *The Economist*, etc., any pollster talking about "soccer moms", "NASCAR dads", etc.

Correlations between social position and culture/politics are real. . .

but is it $\text{social} \rightarrow \text{cultural}$?

Alternative: social homophily + contagion $\rightarrow$ correlation (Shalizi, 2007)

Nobody knows (yet) how big or important these effects might be

Homophily Reinforced by Contagion

Bell *et al.* (2006) documents spread of delusions in electronic social networks of people who think their minds are being controlled

they link because they are deluded (similarity), and then further delusions spread over the network, making them more similar

good luck using ANOVA to disentangle this

Recommended Reading

On historical role of epidemic diseases: McNeill (1976); Diamond (1997)
On history of disease, the life-cycle of the rat, the louse, public health, the nature of art, etc., etc.: Zinsser (1935) **(read this!)**
Ewald (1996) is extremely accessible on the evolutionary biology of infectious disease; some of the ideas are radical and probably wrong
On contagion of ideas: Gladwell (2000) is actually pretty good once you ignore all the bits where he pretends to theorize
Sperber (1996) is unquestionably the best thing ever written about the "epidemiology of representations" **(read this too)**

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