Cosmic Web Reconstruction through Density Ridges

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Outline

- Introduction to Cosmic Web
- Model and Algorithm
- Analysis
- Summary

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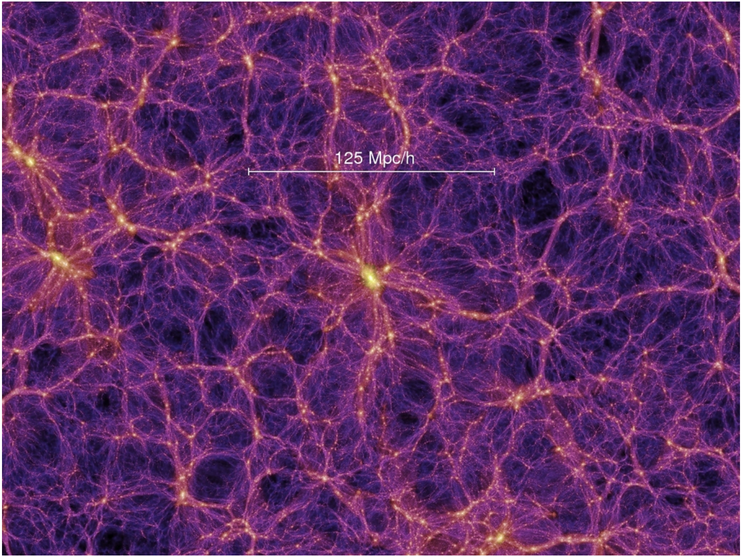
Cosmic Web: What Does Our Universe Look Like



Credit: Millennium Simulation

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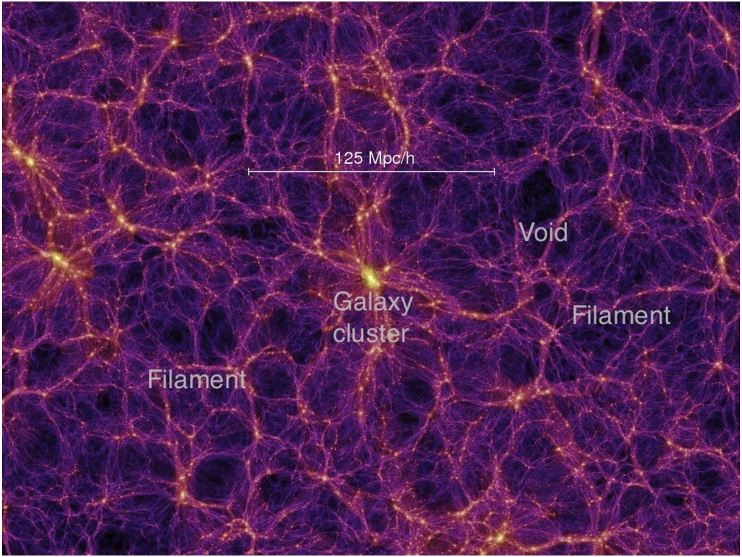

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Focus of the Research: Filaments

Why filament?

- Galaxies tend to concentrate around filaments.

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- Several properties of a galaxy are influenced by filaments.

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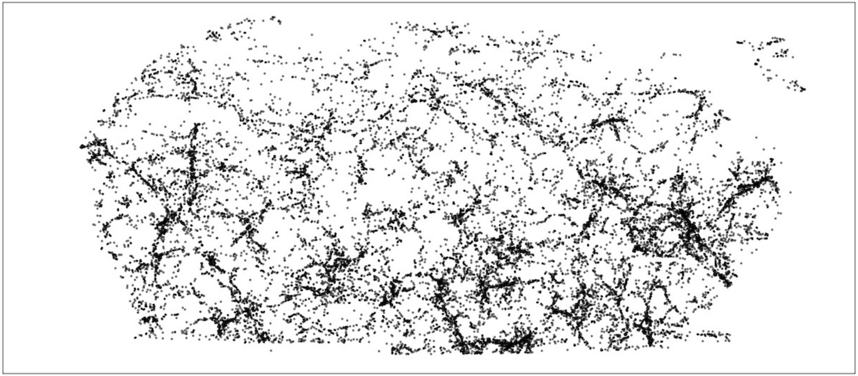
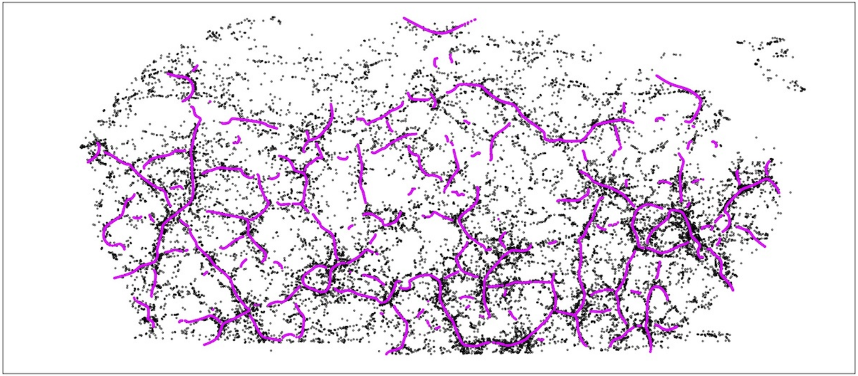
An Example



An Example

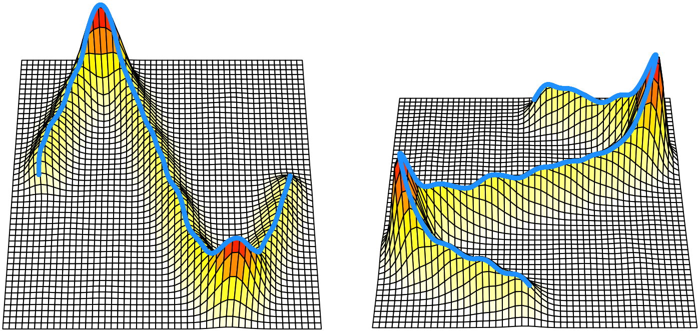


Formally, we define a filament to be a **ridge** of the density.

Example: Ridges in Mountains

Credit: Google

Example: Ridges in Smooth Functions



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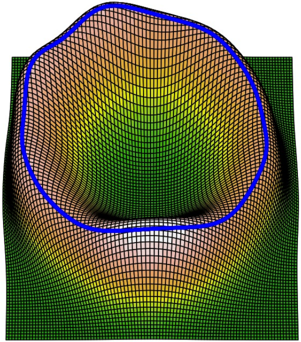
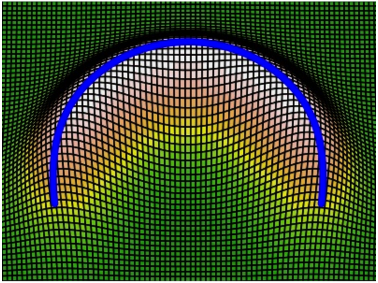
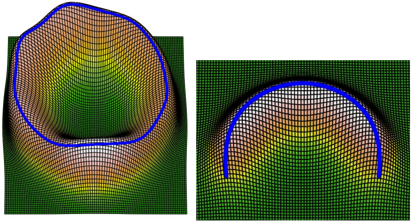
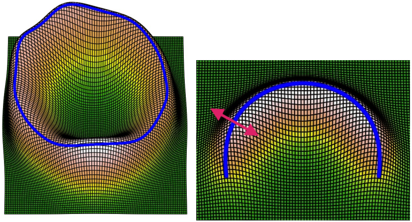






- A generalized local mode in a specific 'subspace'.

Ridges: Local Modes in Subspace



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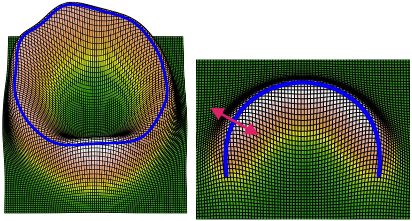

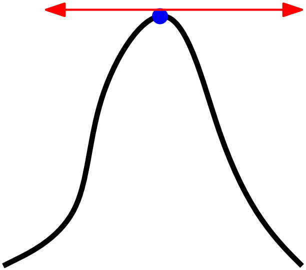

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Formal Definition of Density Ridges

- $p(x)$: a density function, $x \in \mathbb{R}^d$.

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- Local modes:

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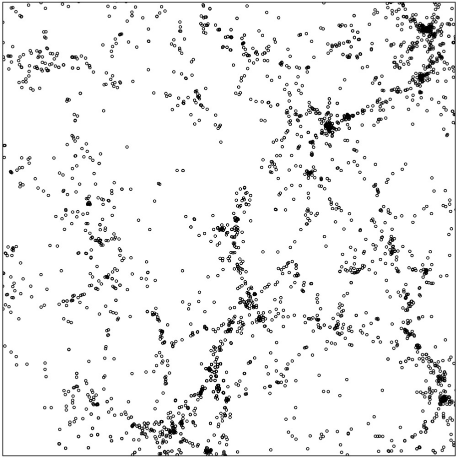
$$\widehat{p}_n(x) = \frac{1}{nh^d}\sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right).$$
- $\rightarrow$Subspace Constrained Mean Shift Algorithm [Ozertem and Erdogmus 2011].

Algorithm

1. Rawdata



Algorithm

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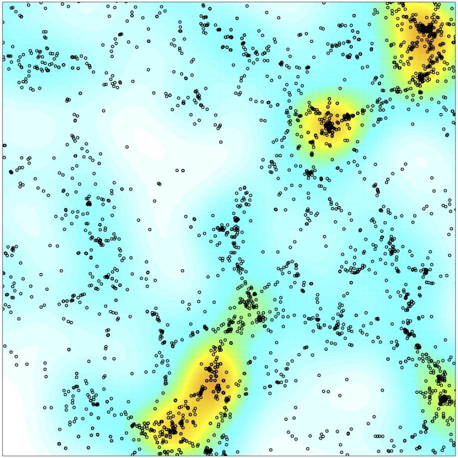
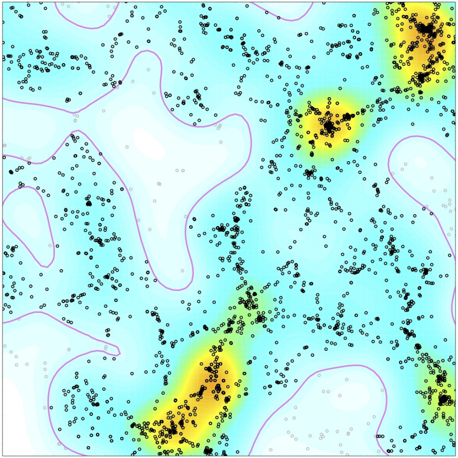
2. Density Reconstruction



Algorithm

1. Rawdata
2. Density Reconstruction
3. Thresholding



Algorithm

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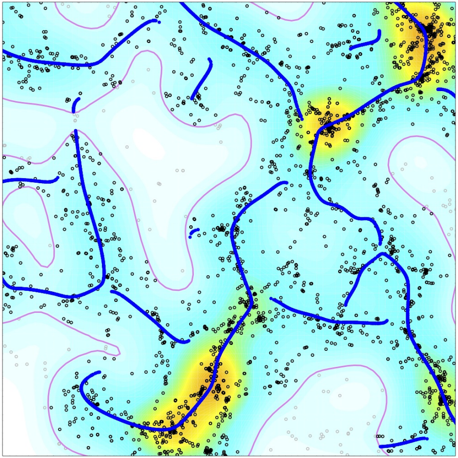
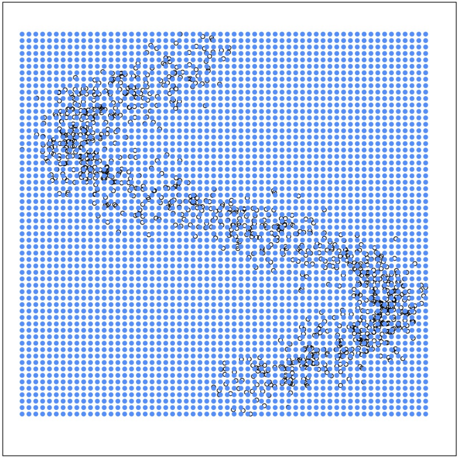
2. Density Reconstruction
3. Thresholding
4. Ridge Recovery



SCMS: Ridge Recovery Algorithm



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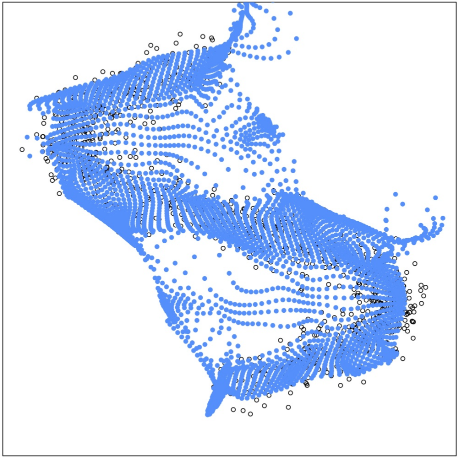

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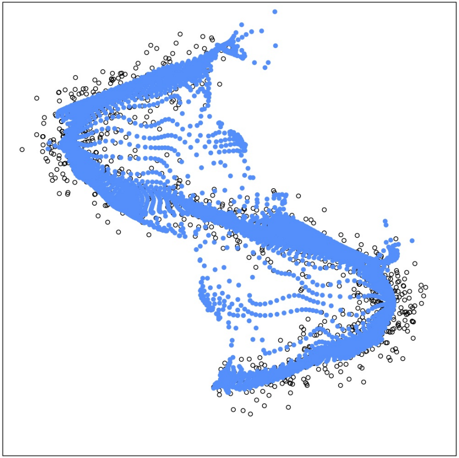

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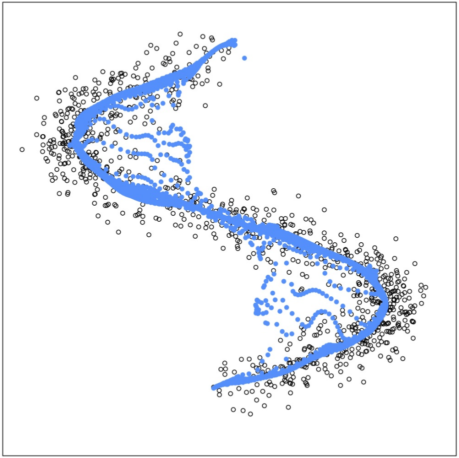

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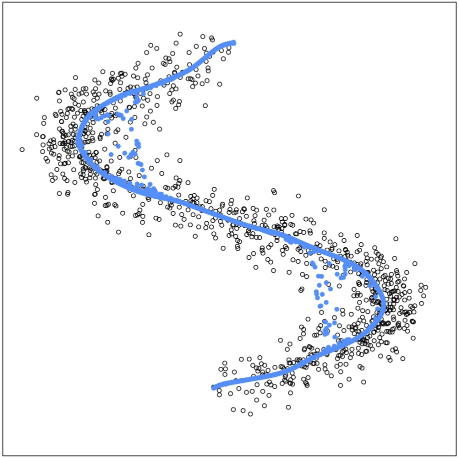

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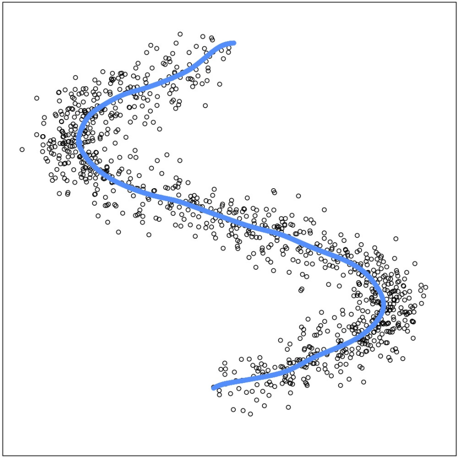
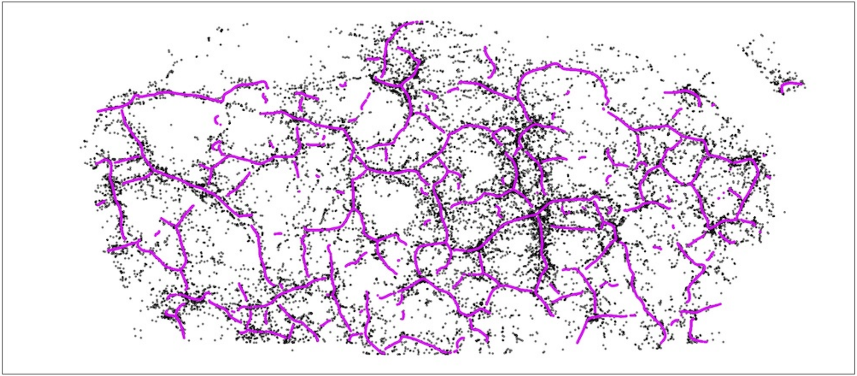


Density Ridges on an Example



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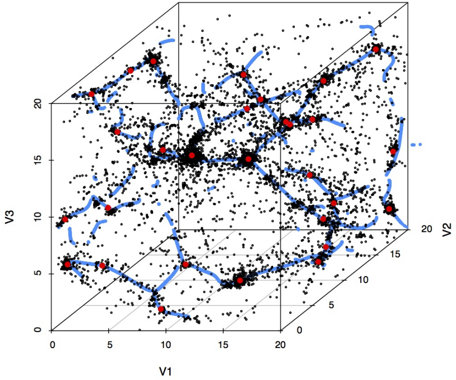
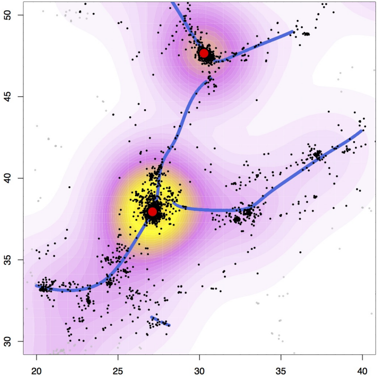
Massive Blackhole Simulation

- Method: smoothed particle hydrodynamics.



Galaxy Alignment to Filaments

- Key variable 1: Principal axes for a galaxy (μ_1, μ_2, μ_3).
- Key variable 2: Orientation of the nearest filament (μ_F).
- Key variable 3: Distance to the nearest filament (d_F).



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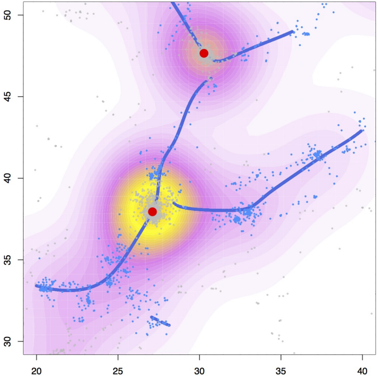

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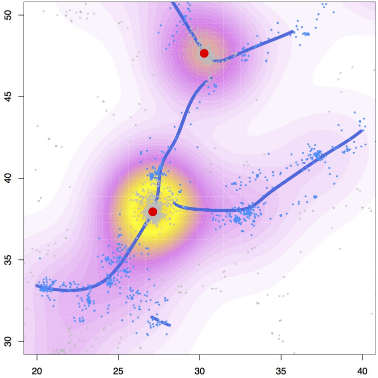

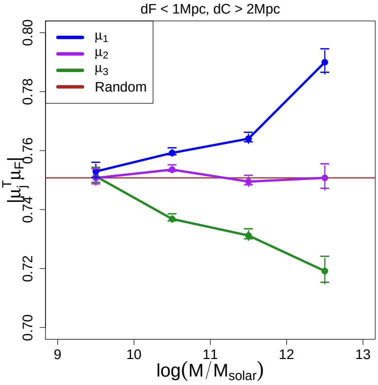

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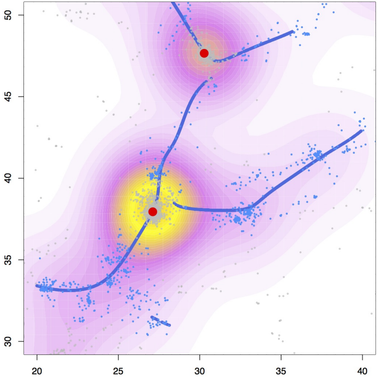
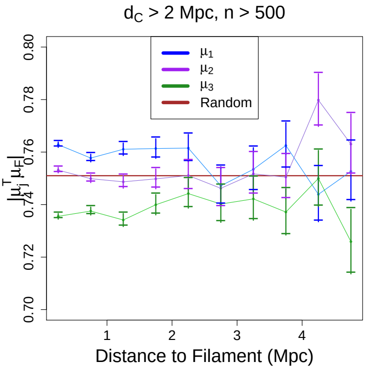
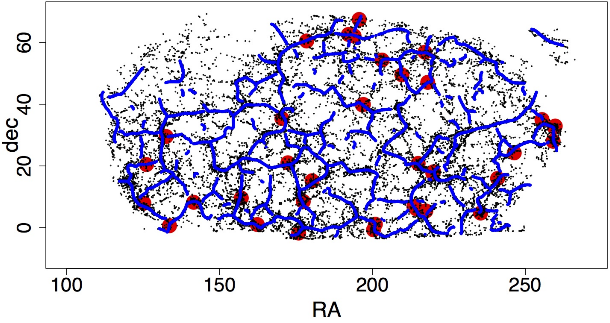




Sloan Digital Sky Survey

- Data: the Sloan Digital Sky Survey, data release 12.
- We take 2-D slices of the Universe to detect filaments ($\Delta z = 0.005$).
- Blue: filaments. Red: galaxy clusters (redMaPPer).



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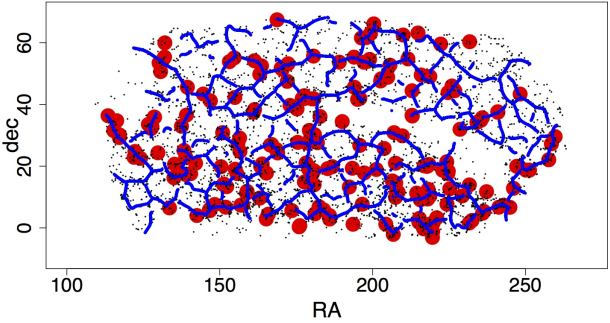

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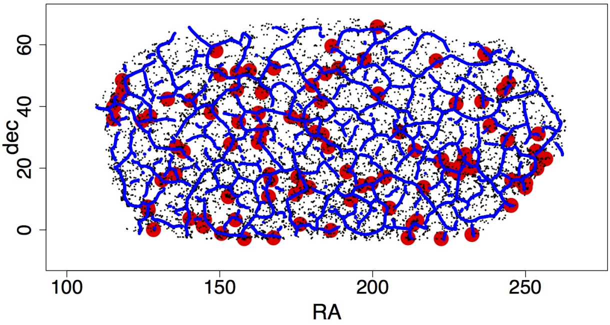
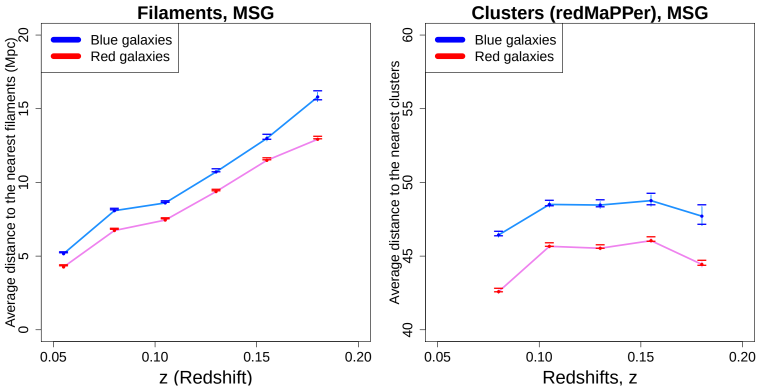


SDSS: Red and Blue Galaxies

- Redshift range: $0.05 < z < 0.20$ (main sample galaxy).
- Color cut: $(g - r) = 0.73 - 0.02(M_r + 20)$ [Masters et. al. 2010].

SDSS: Red and Blue Galaxies



SDSS: Stellar Mass of Galaxies

- Mass from Flexible Stellar Population Synthesis method [Conroy, Gunn, and White 2009].
- We partition galaxies into three groups according to their mass.
- We compare the average distance to filaments for each group.

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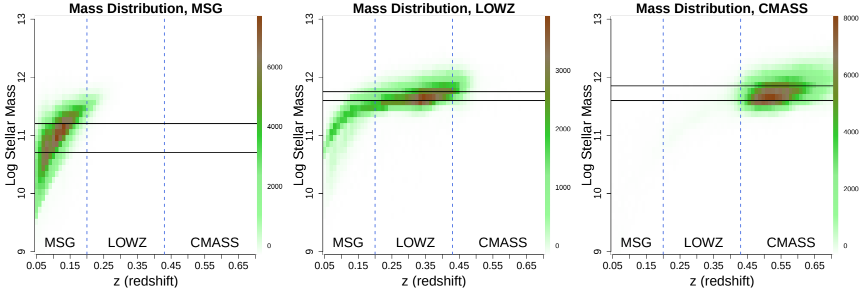

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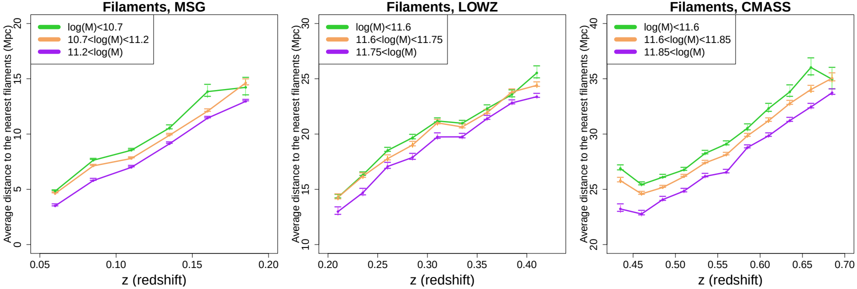

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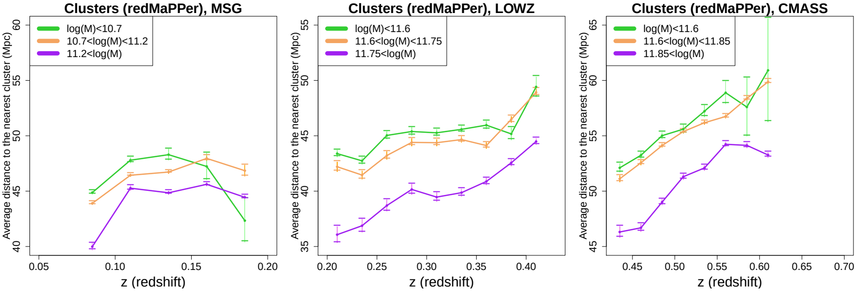

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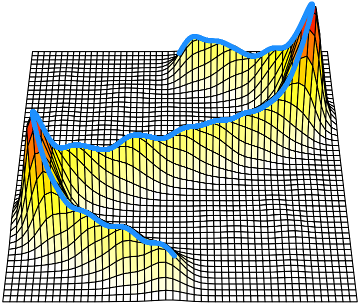

Summary

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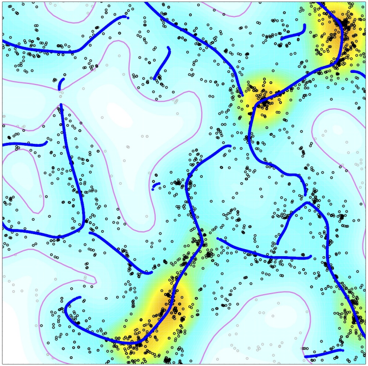
2. Algorithm: SCMS.



Summary

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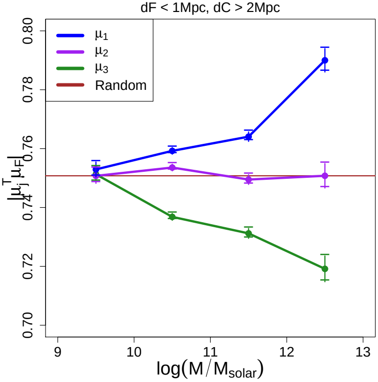
3. Works in simulation and real dataset.



Summary

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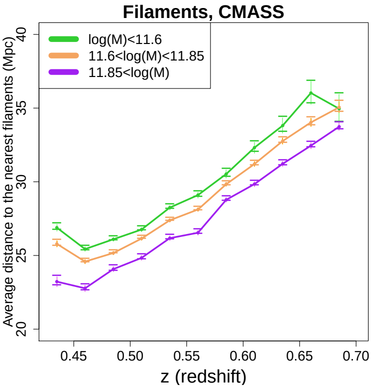
2. Algorithm: SCMS.
3. Works in simulation and real dataset.
4. Consistent with galaxy clusters.



Thank you!

reference

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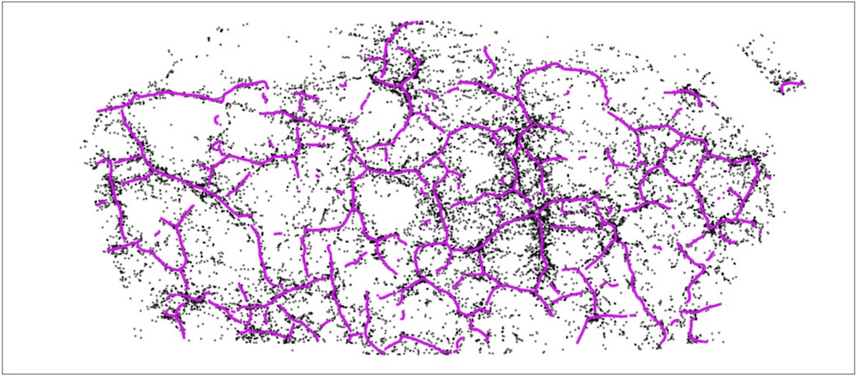
Density Ridges on the SDSS data



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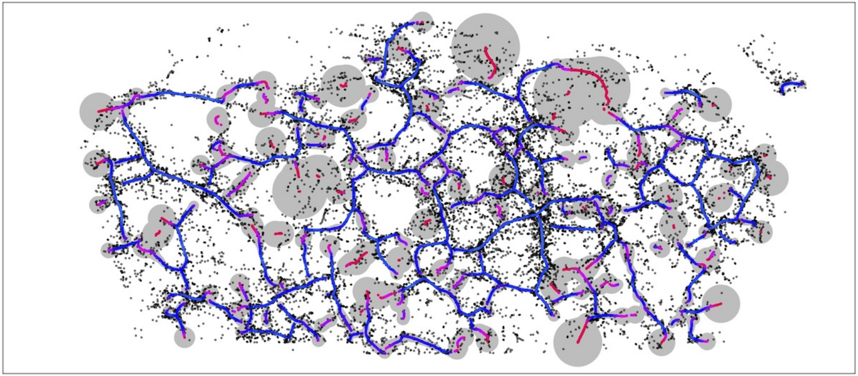
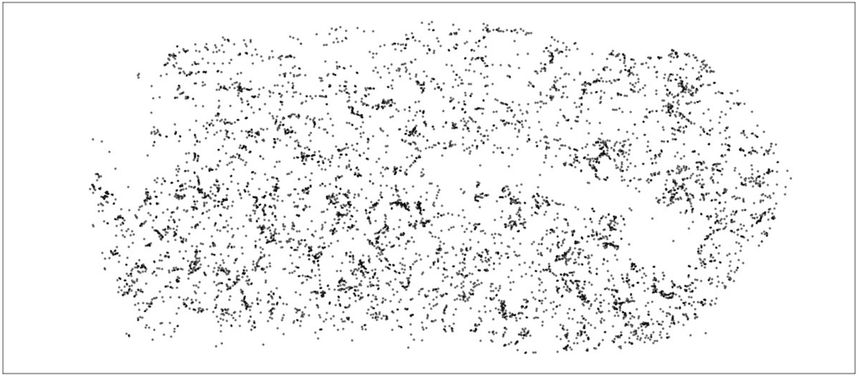


Curse of Number Density



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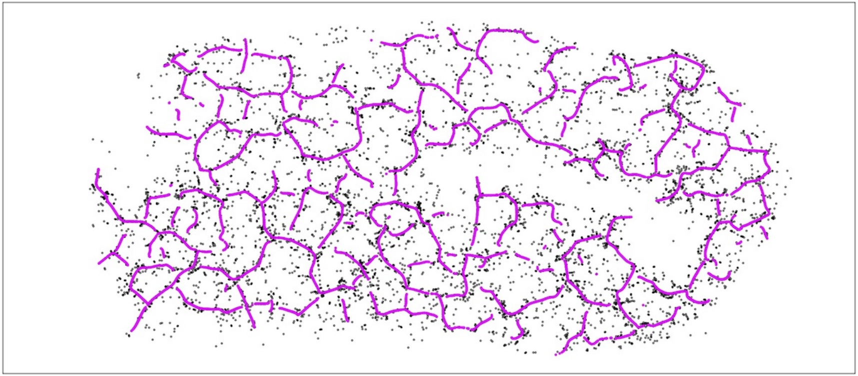

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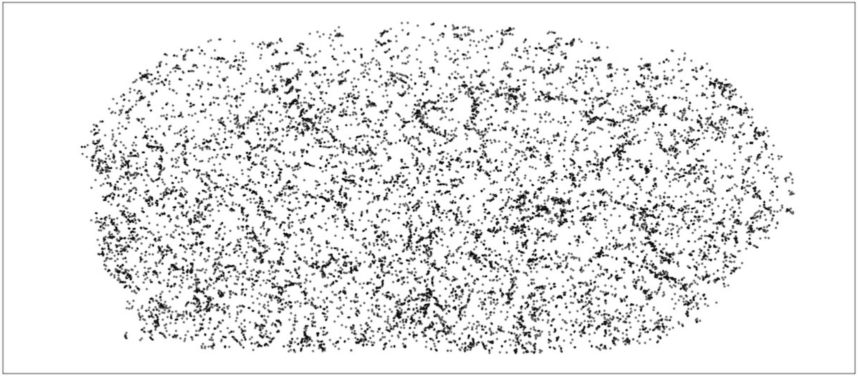

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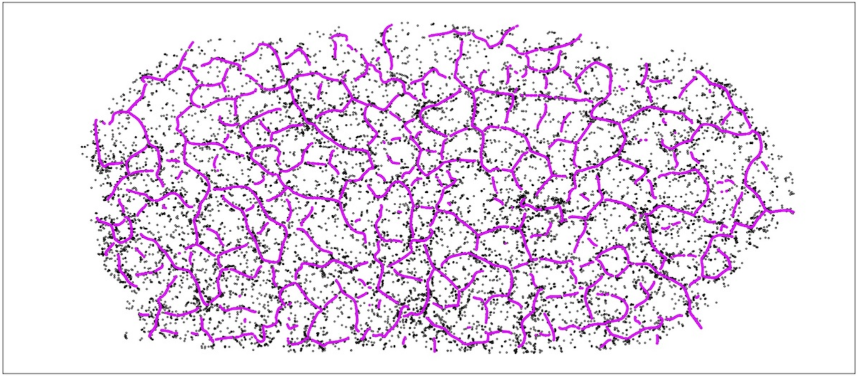
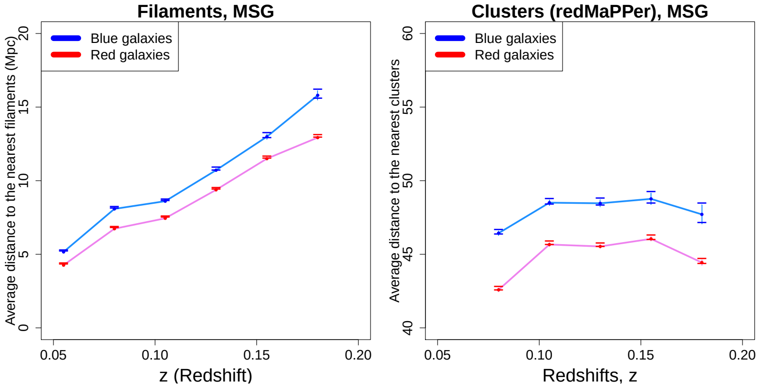


SDSS: Red and Blue Galaxies



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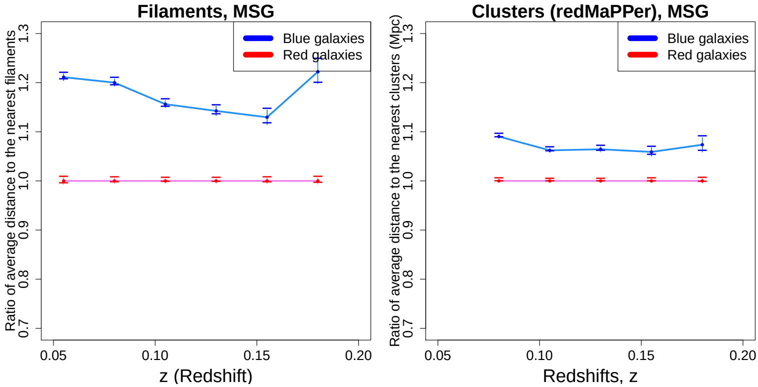
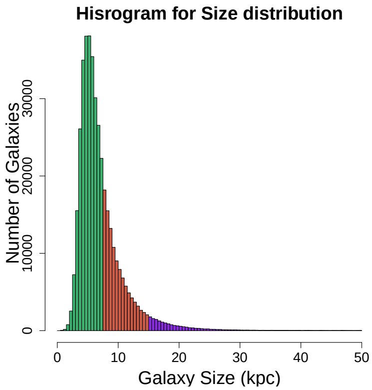


SDSS: Size for Galaxies

1. Size: 50% luminosity radii.
2. Data: LOWZ ($0.20 < z < 0.43$)
3. Partitioning galaxies into three groups according to their size.



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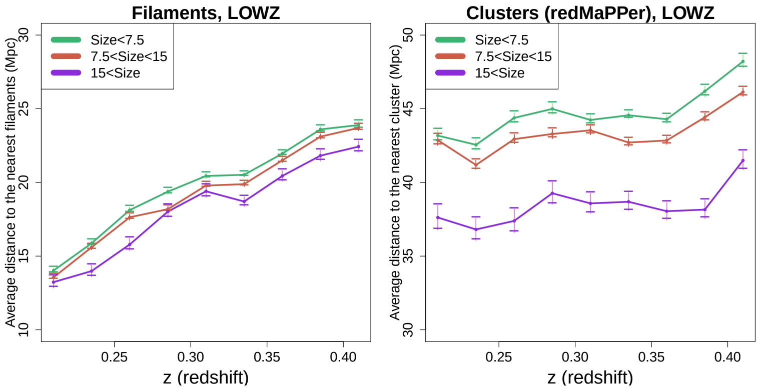

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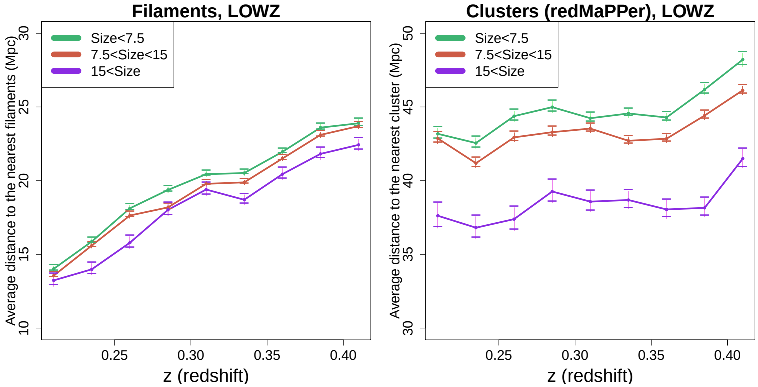
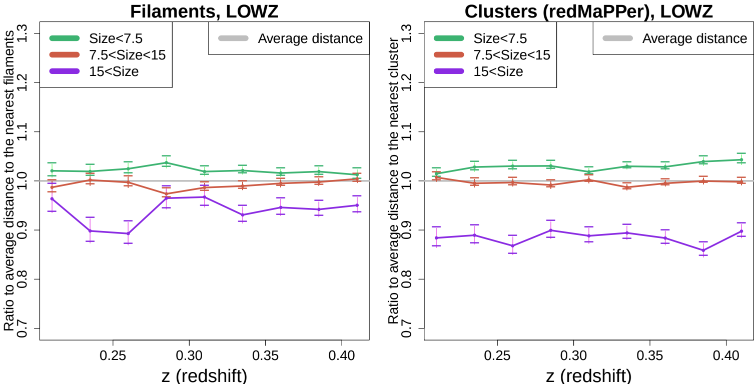
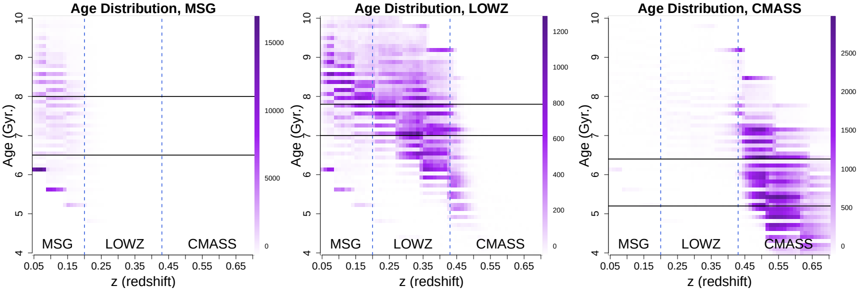


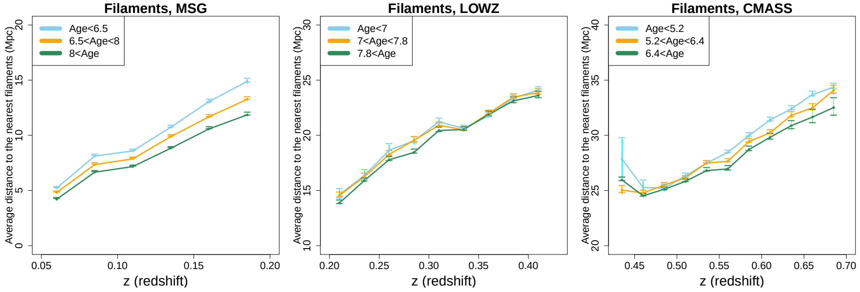
SDSS: Size for Galaxies



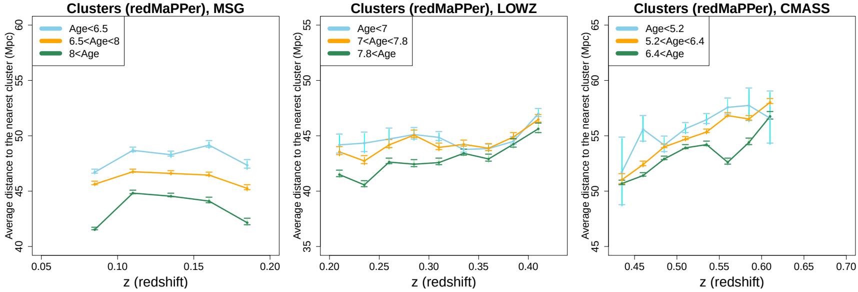
Age for Galaxies



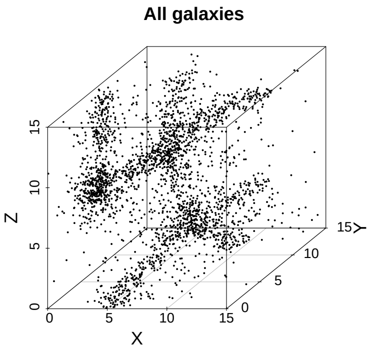
Age for Galaxies



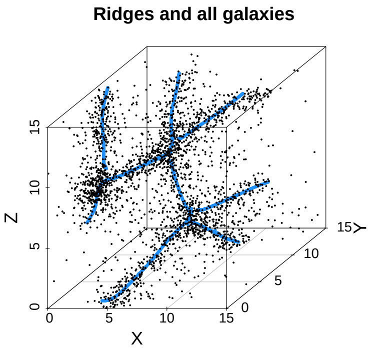
Age for Galaxies



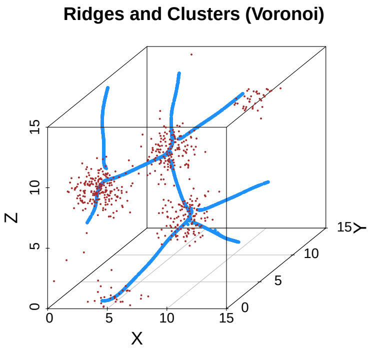
Comparison: Voronoi Model



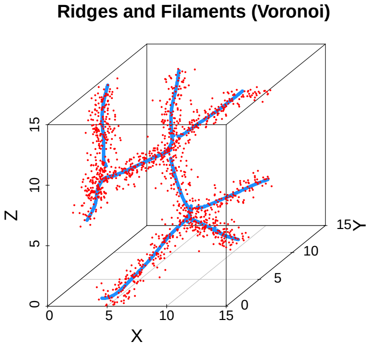
Comparison: Voronoi Model



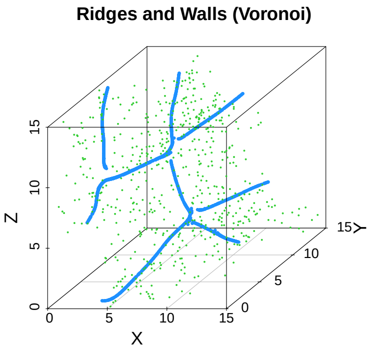
Comparison: Voronoi Model



Comparison: Voronoi Model



Comparison: Voronoi Model



Comparison: Voronoi Model